



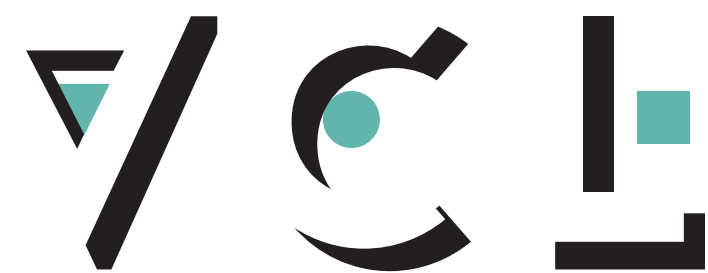
SIGGRAPH2017

AT THE  of COMPUTER | INTERACTIVE
GRAPHICS & | TECHNIQUES

Convergence Analysis for Anisotropic Monte Carlo Sampling Spectra

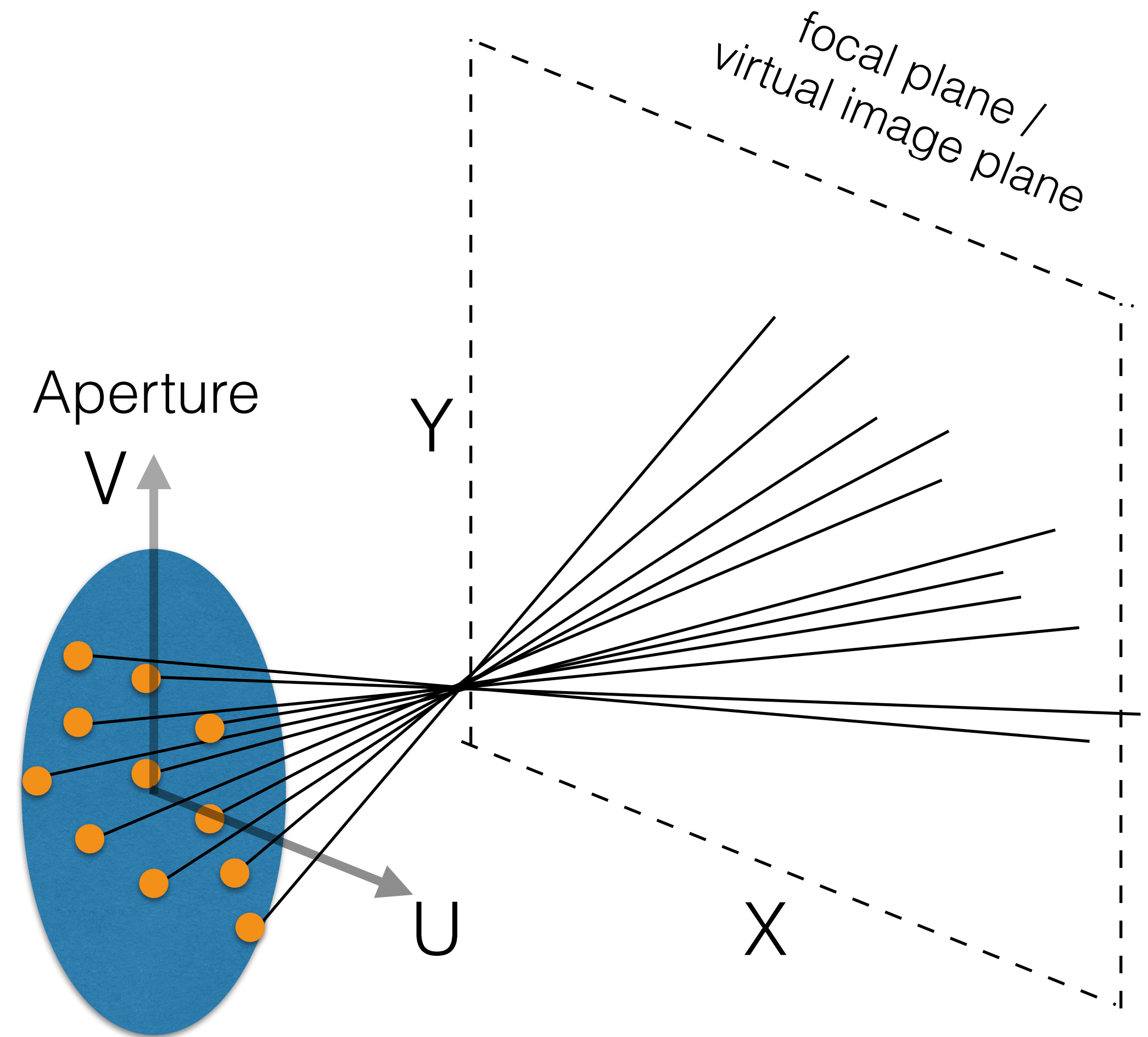
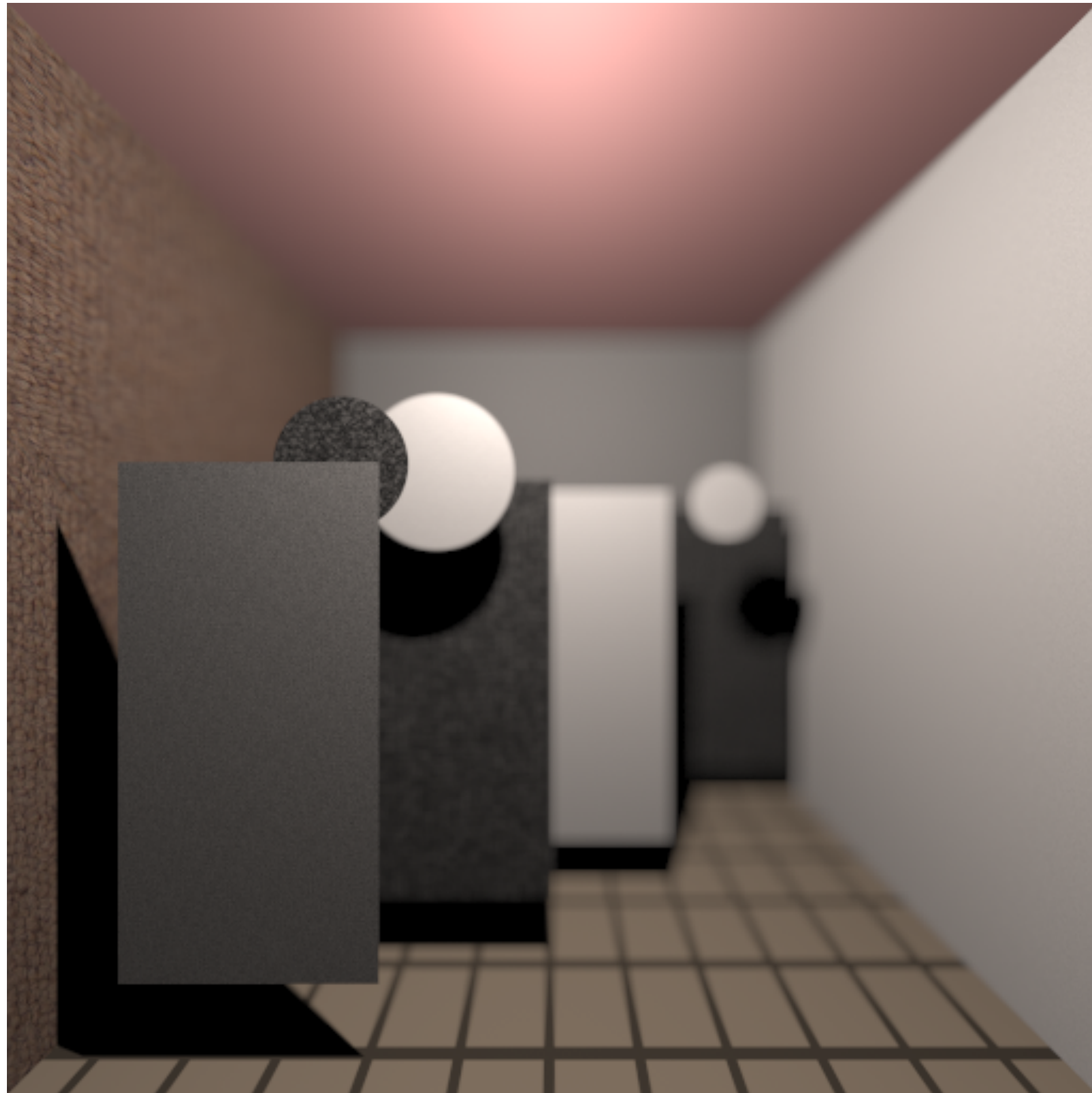
Gurprit Singh

Wojciech Jarosz



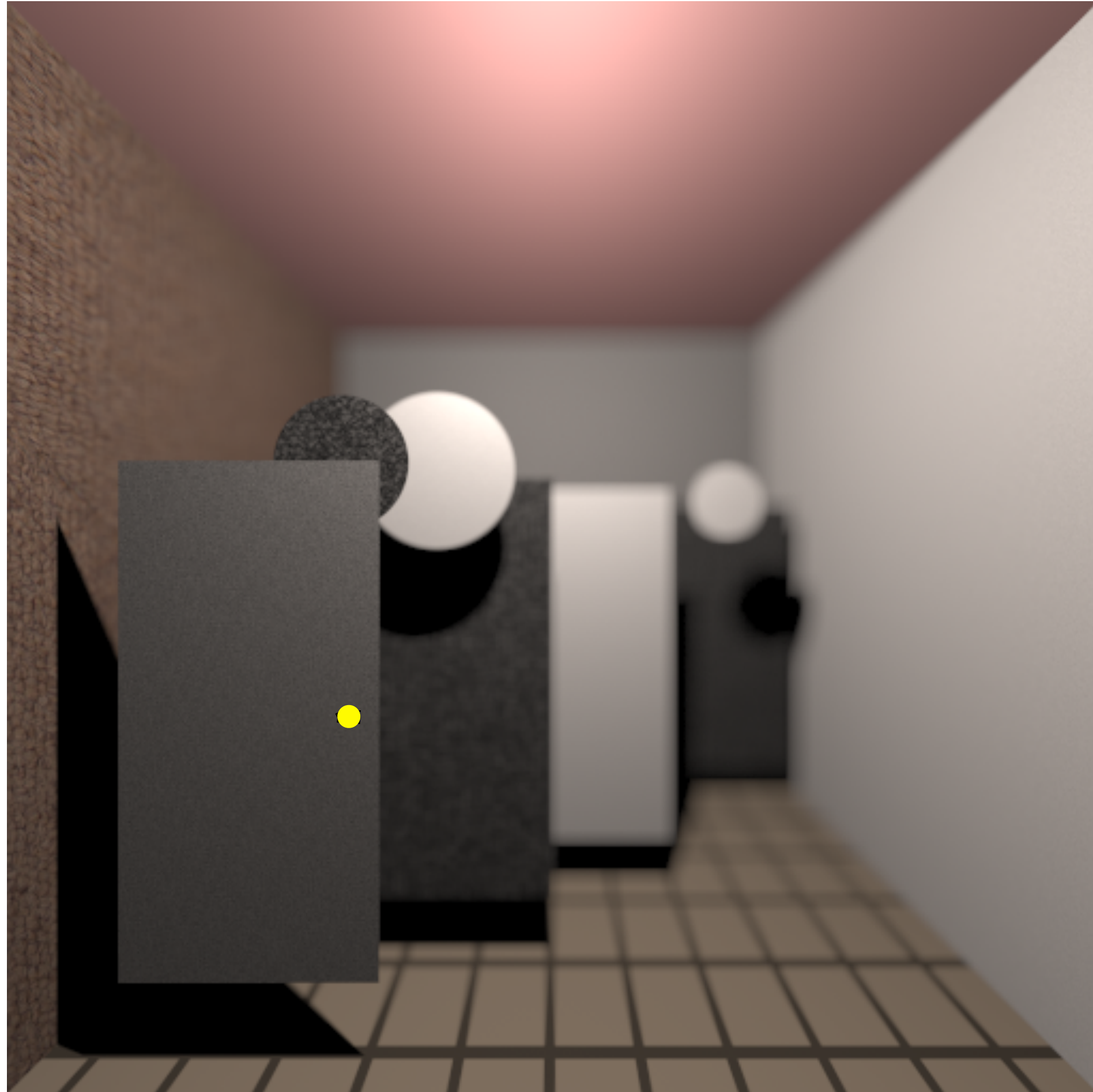
DARTMOUTH
VISUAL COMPUTING LAB

Monte Carlo Integration

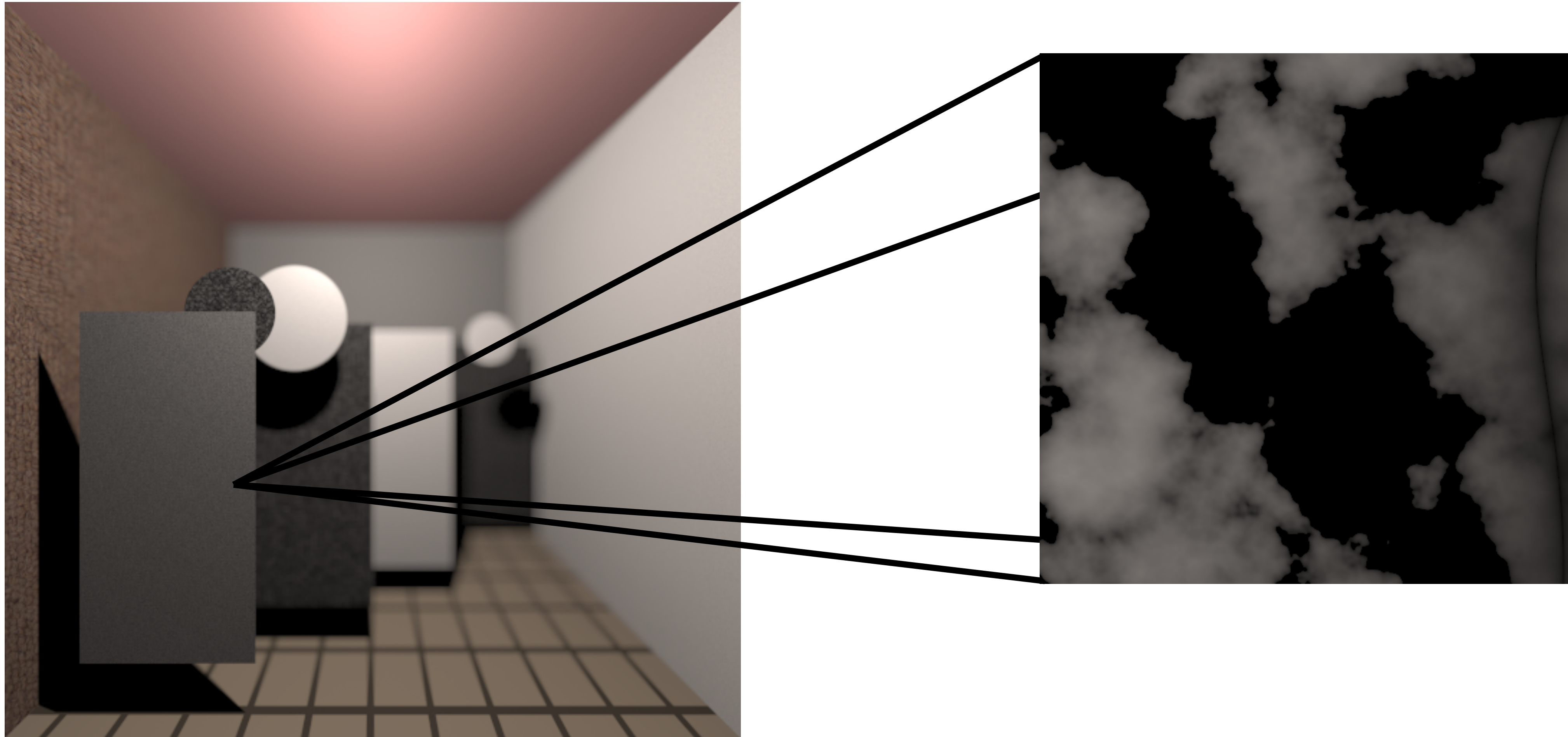


$$\int_x \int_y \int_u \int_v f(x, y, u, v) dv du dy dx$$

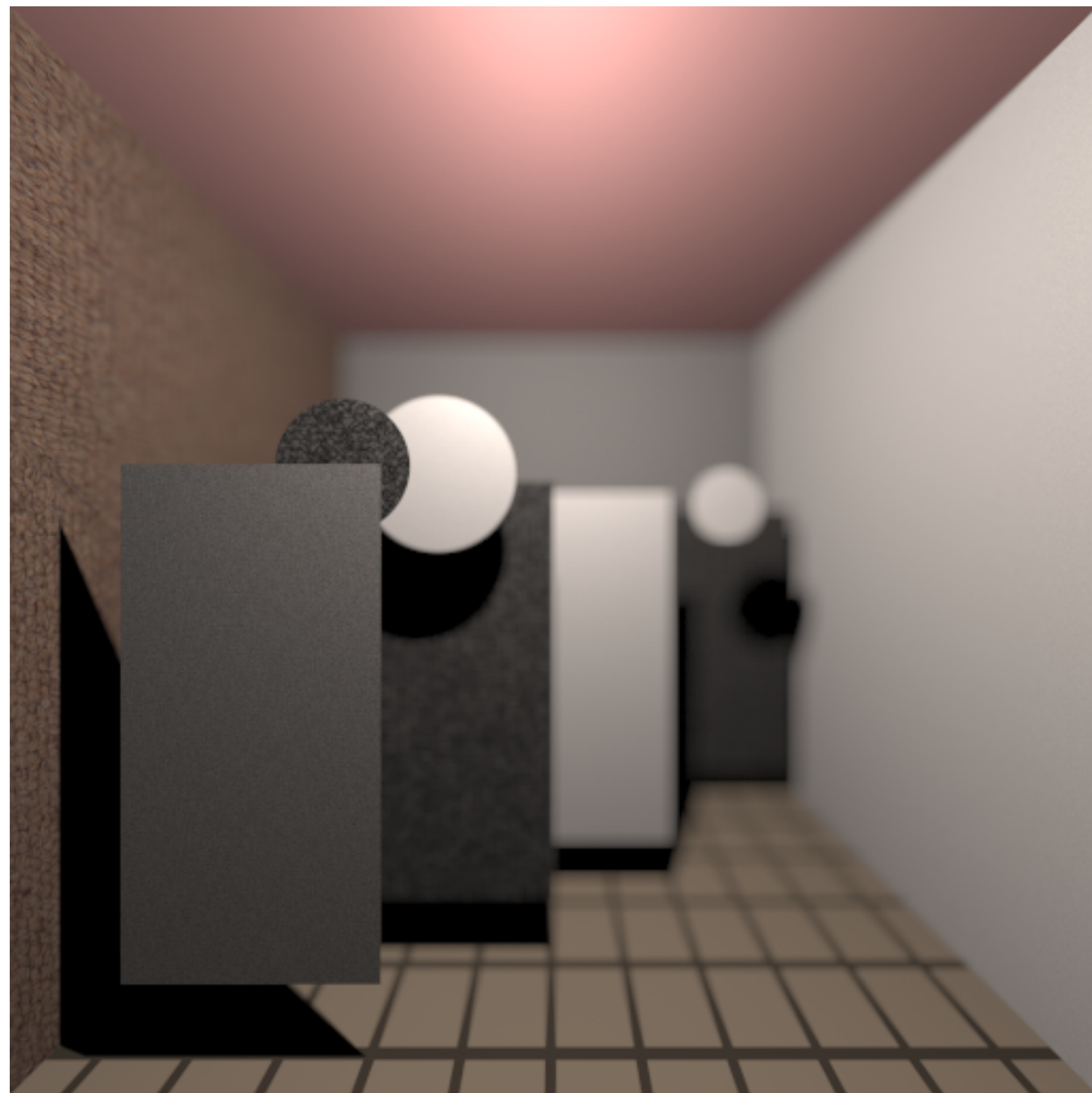
Monte Carlo Integration



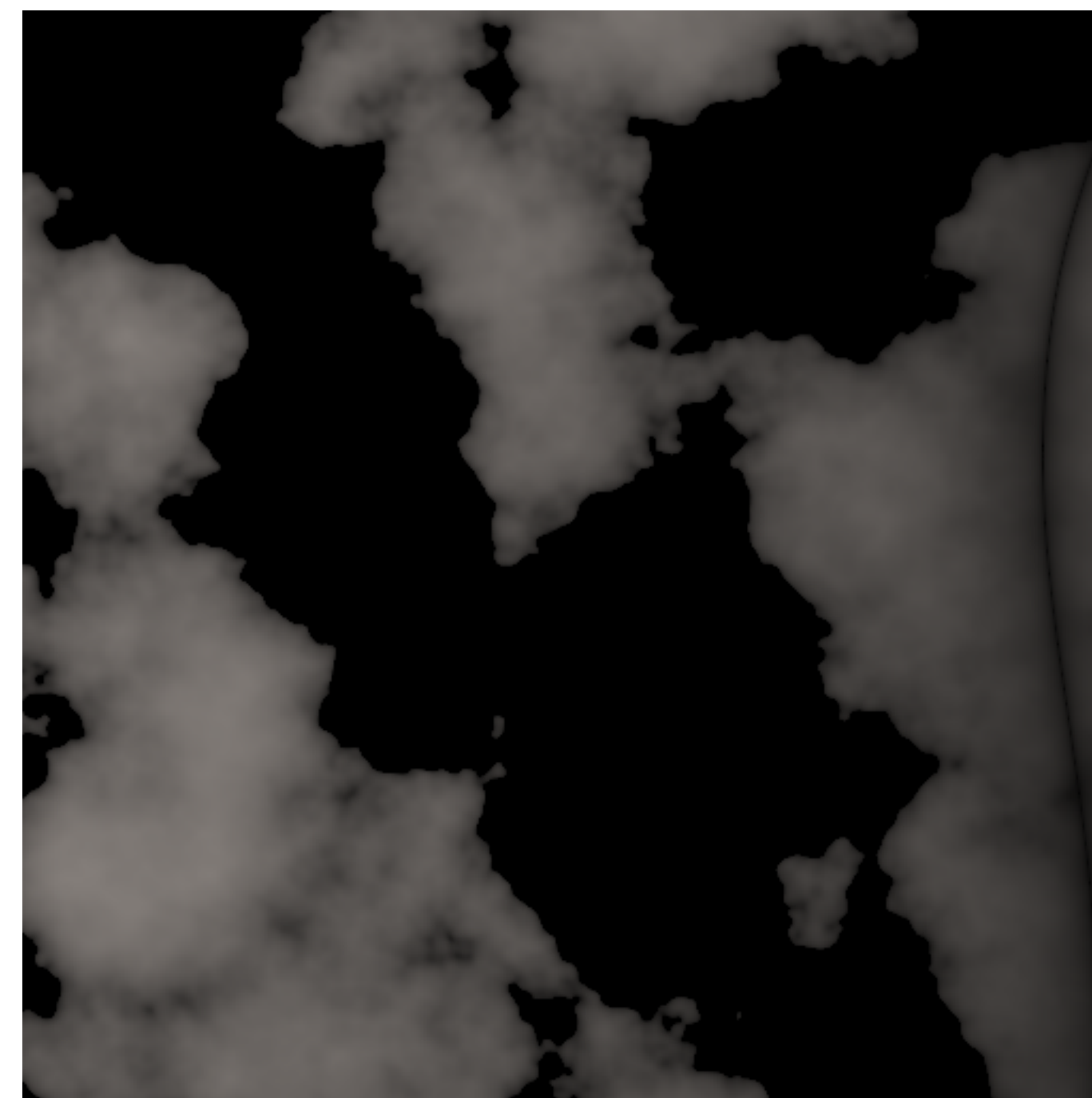
Monte Carlo Integration



Monte Carlo Integration

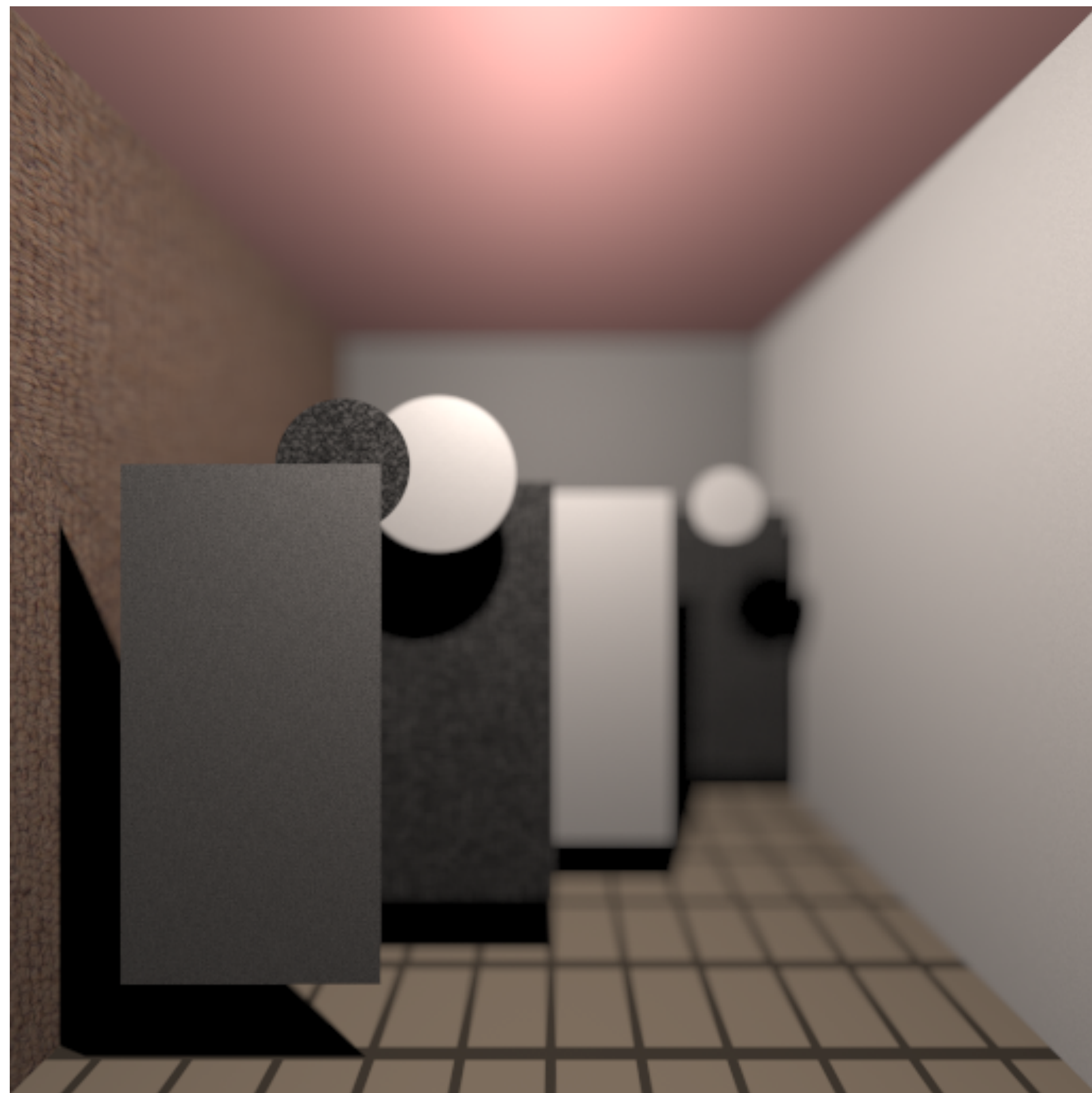


$f(\vec{x})$

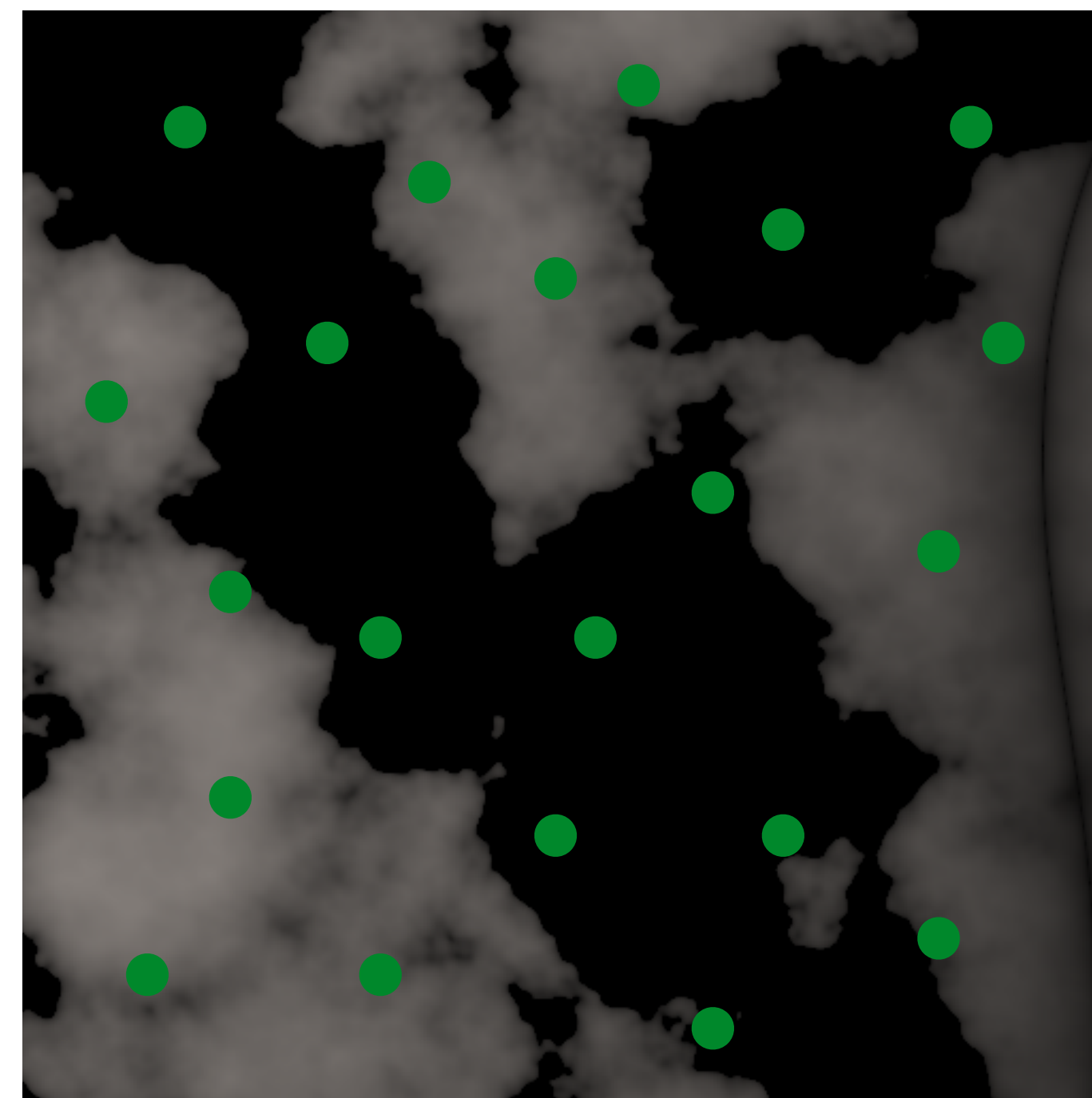


$$I = \int_0^1 f(\vec{x}) d\vec{x}$$

Monte Carlo Integration

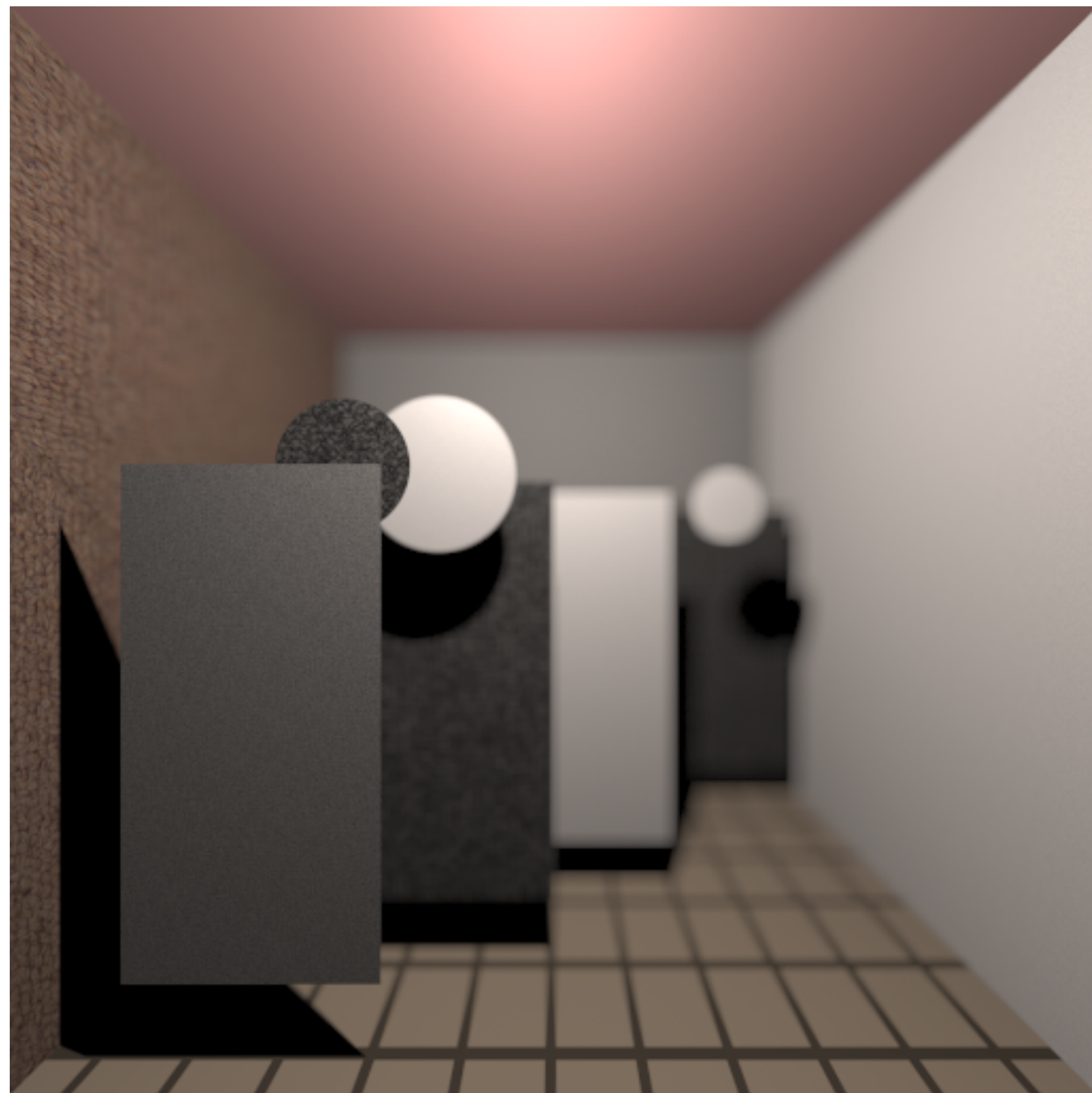


$f(\vec{x})$

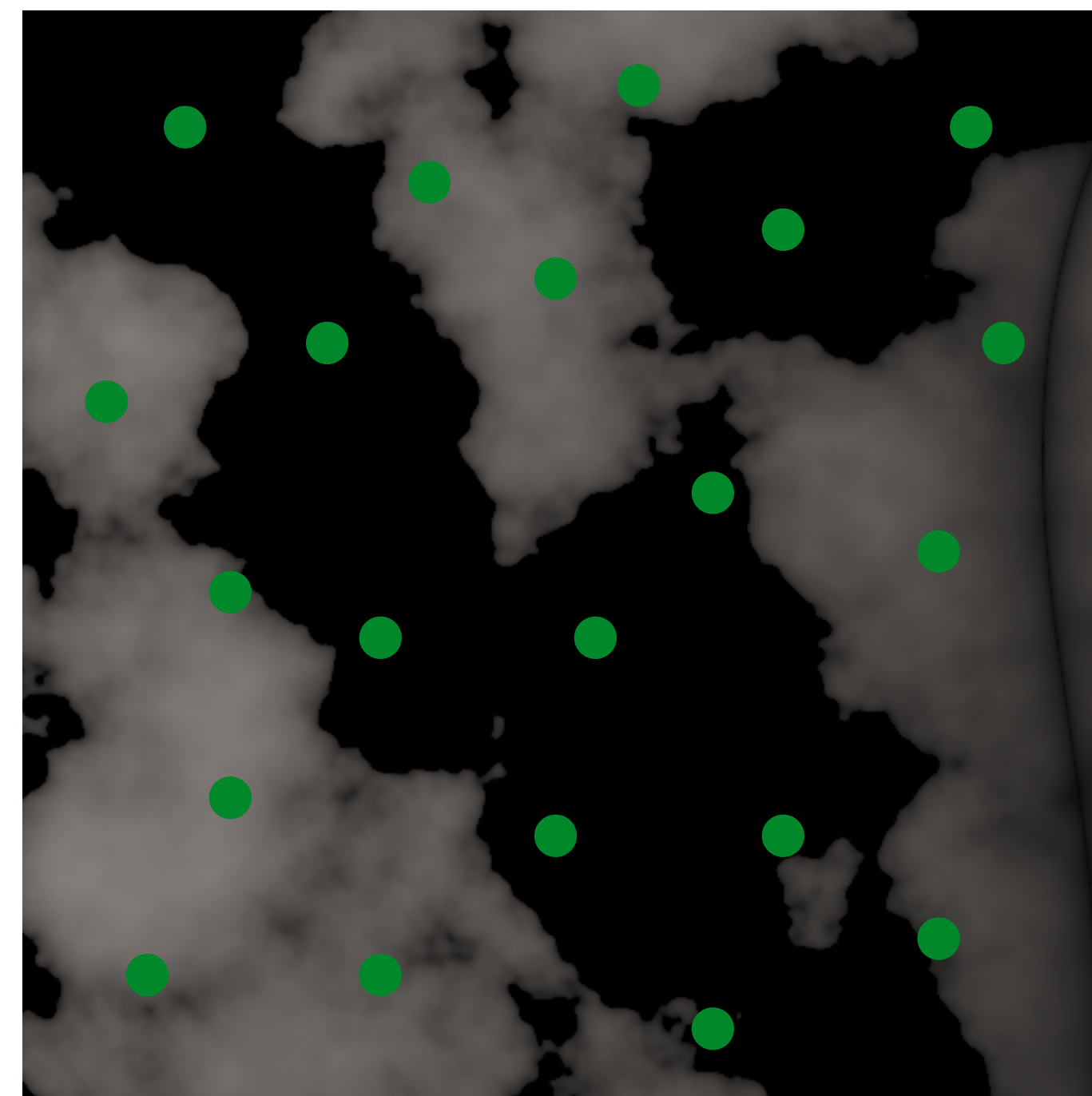


$$I = \int_0^1 f(\vec{x}) d\vec{x}$$

Monte Carlo Integration

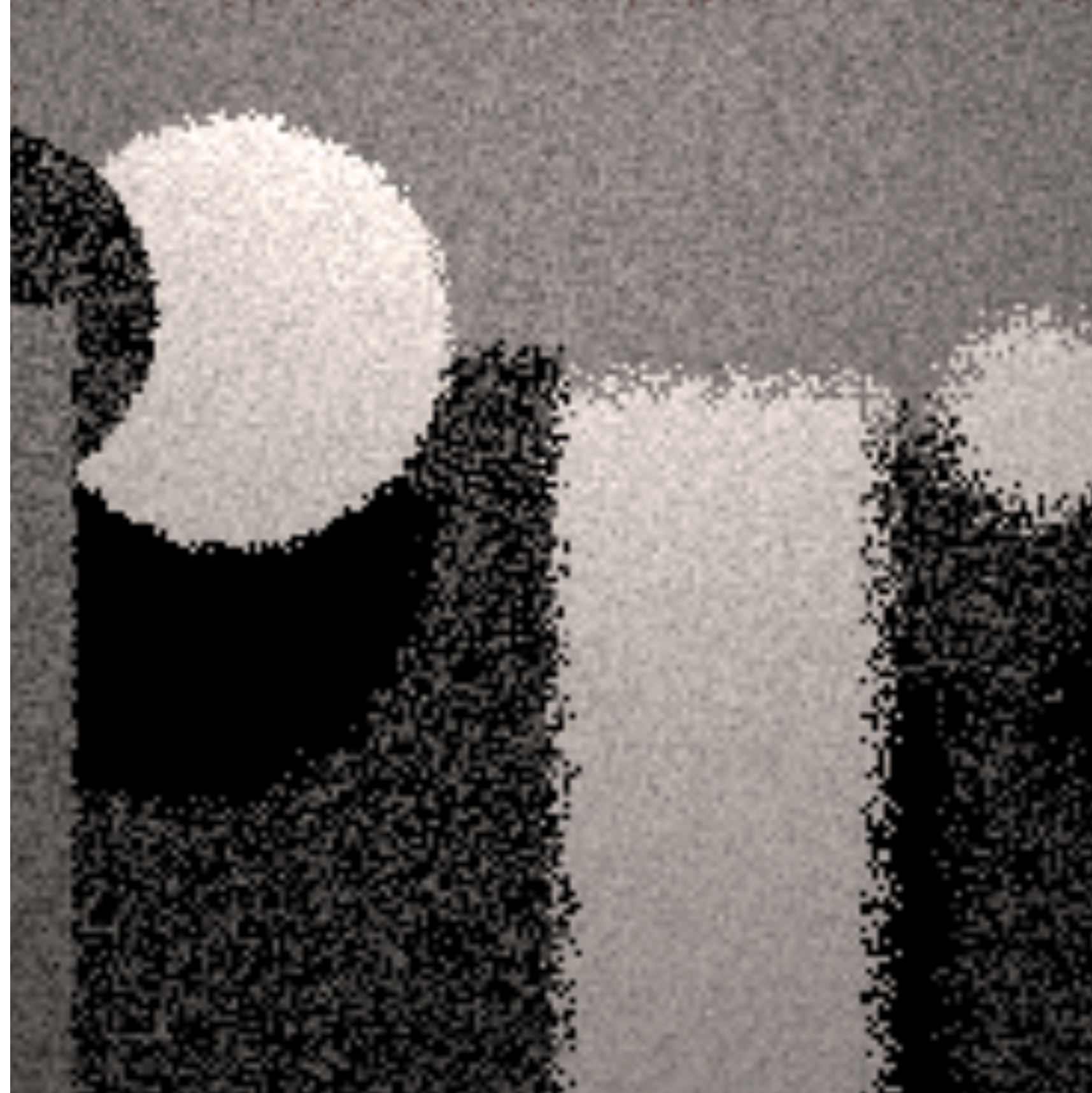


$f(\vec{x})$



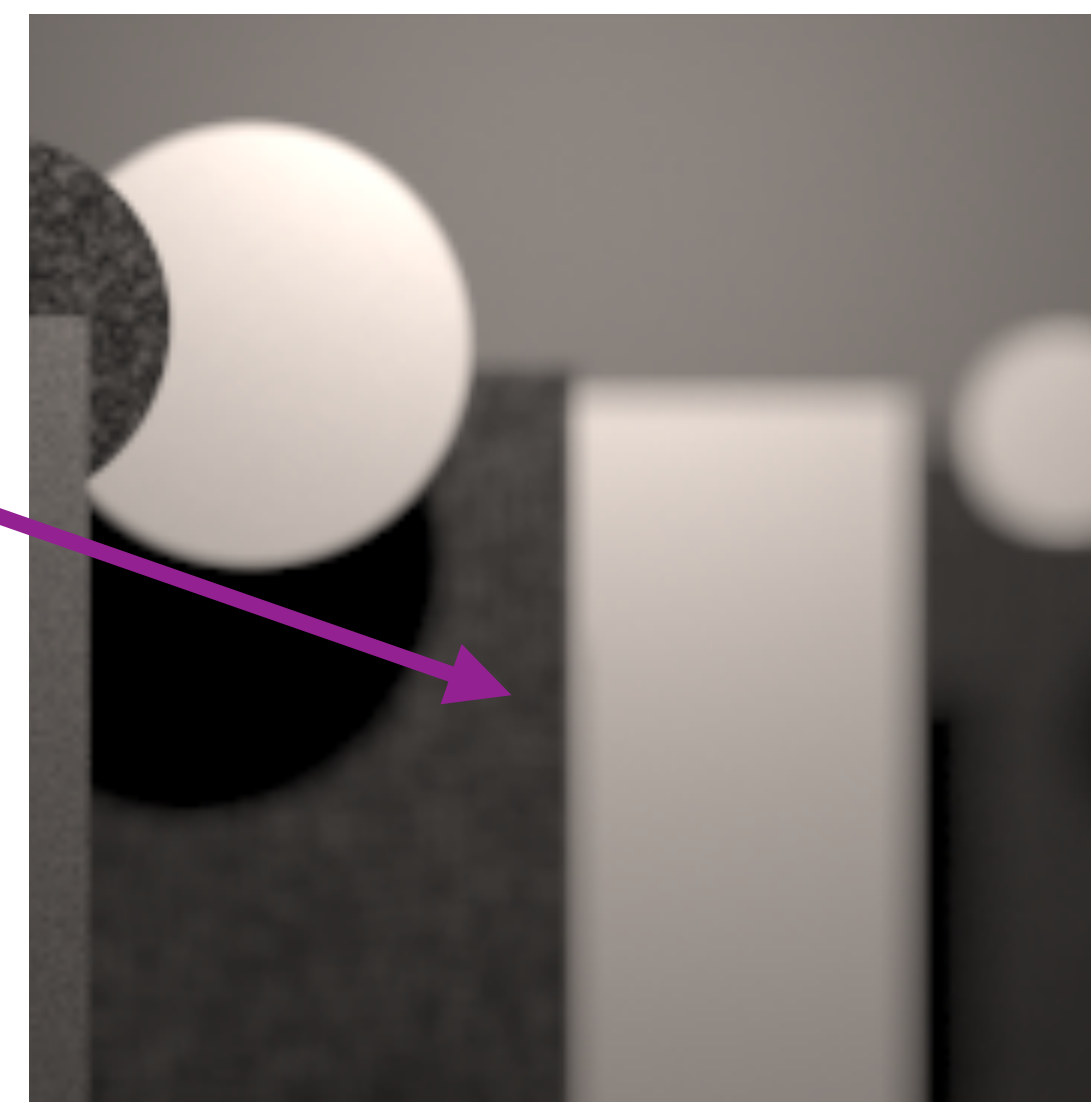
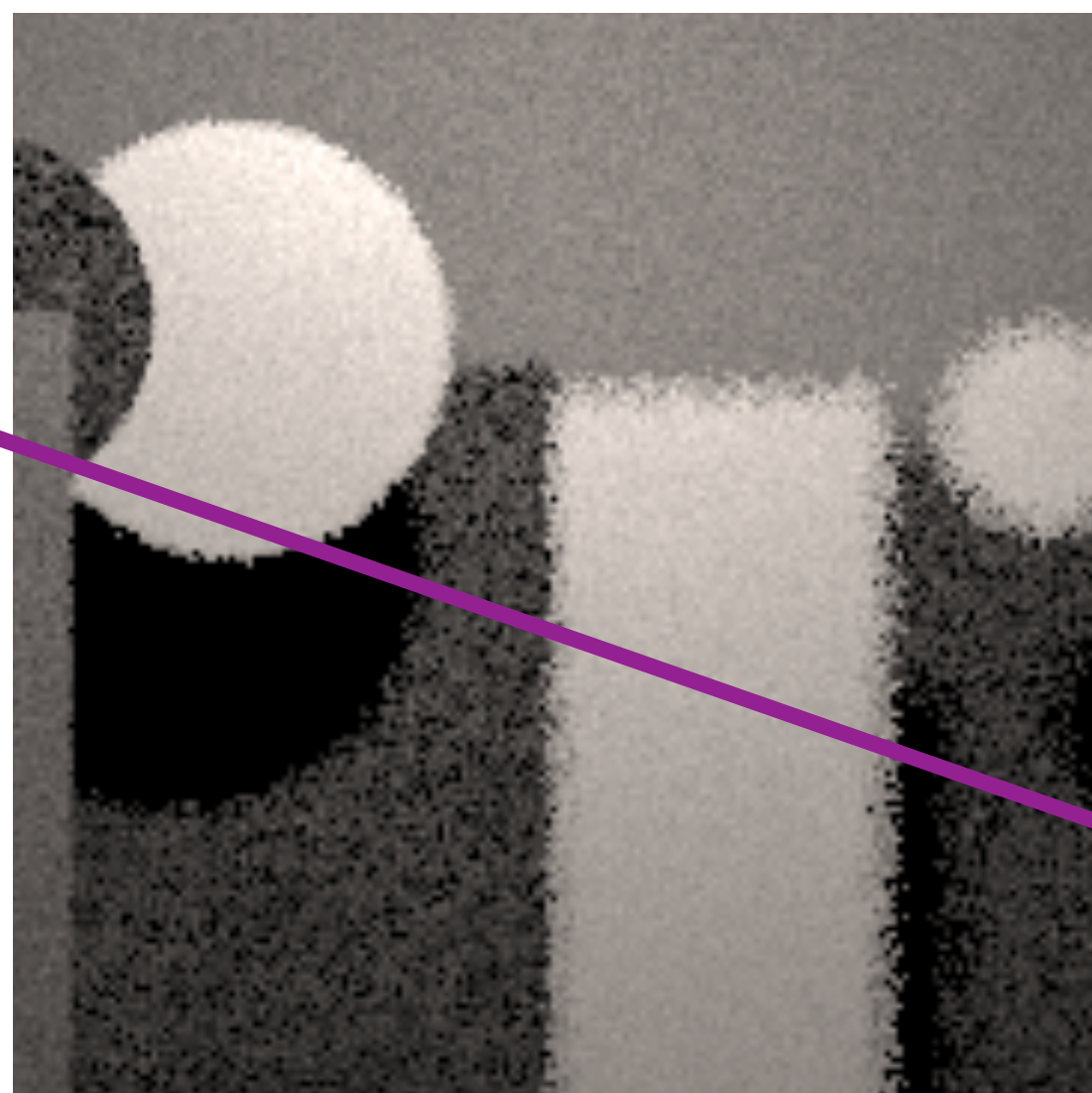
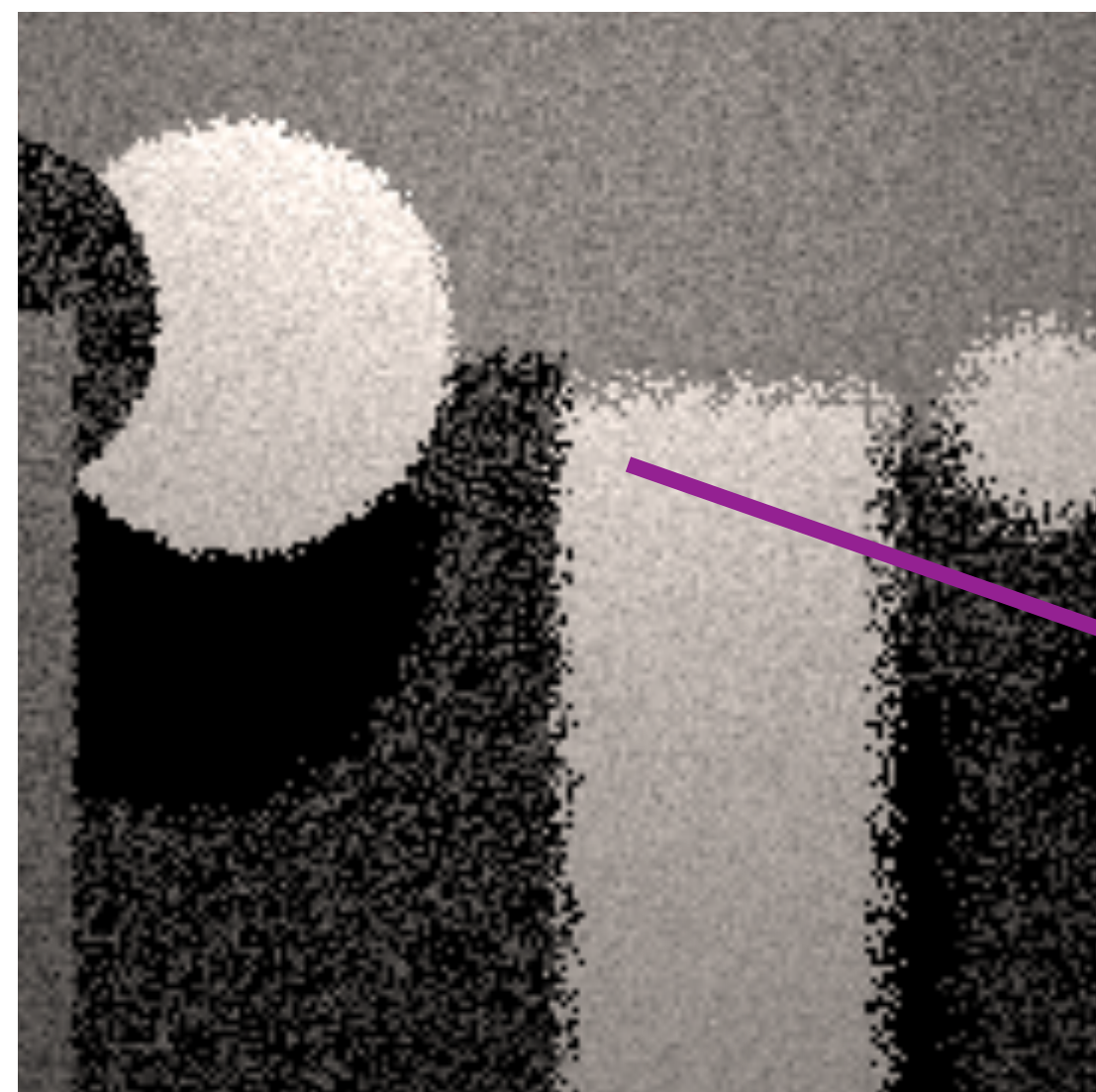
$$I_N = \frac{1}{N} \sum_{k=1}^N \frac{f(\vec{x}_k)}{p(\vec{x}_k)}$$

Variance



Variance Convergence Rate of Samplers

Variance



— 4D Jittered

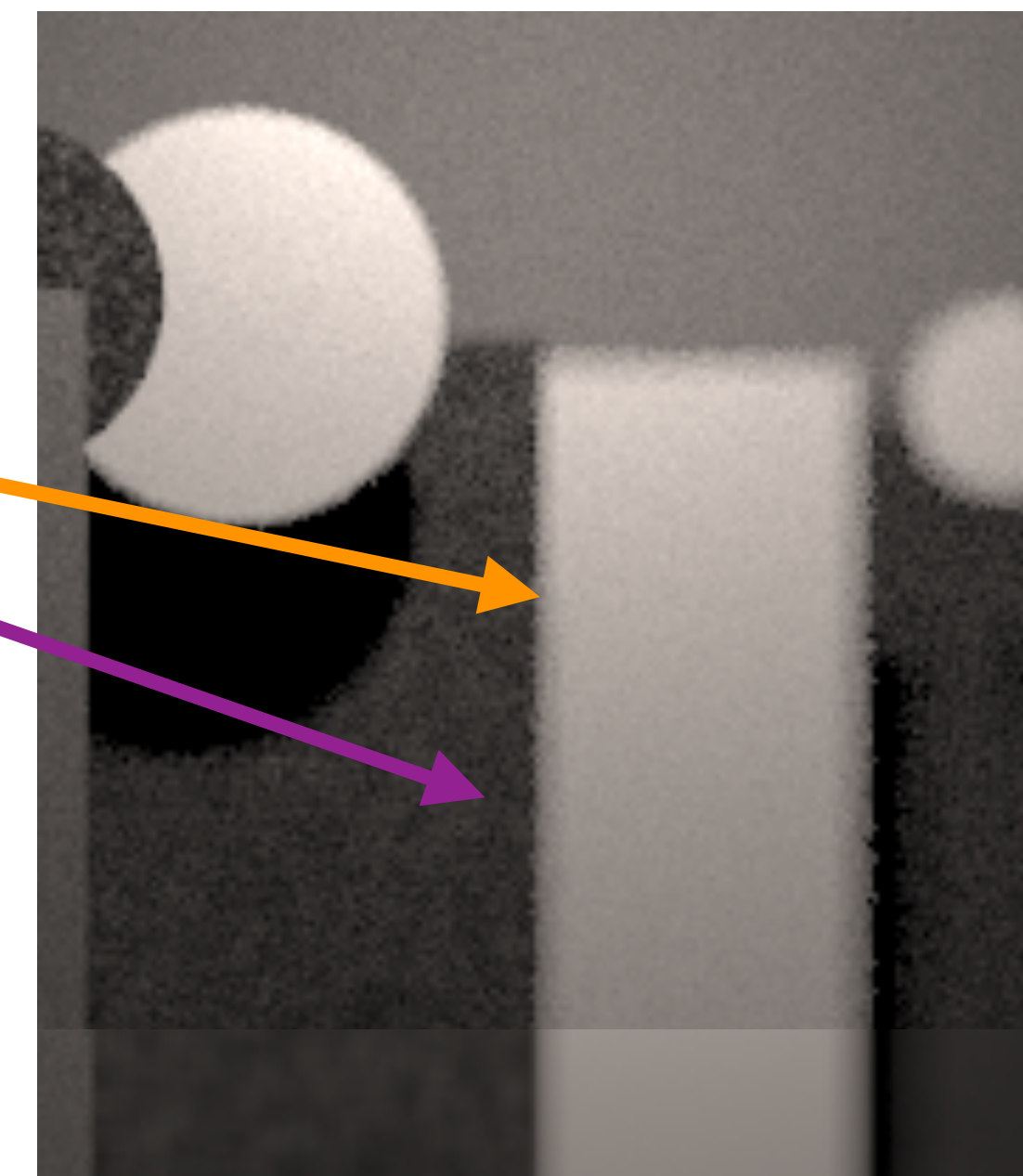
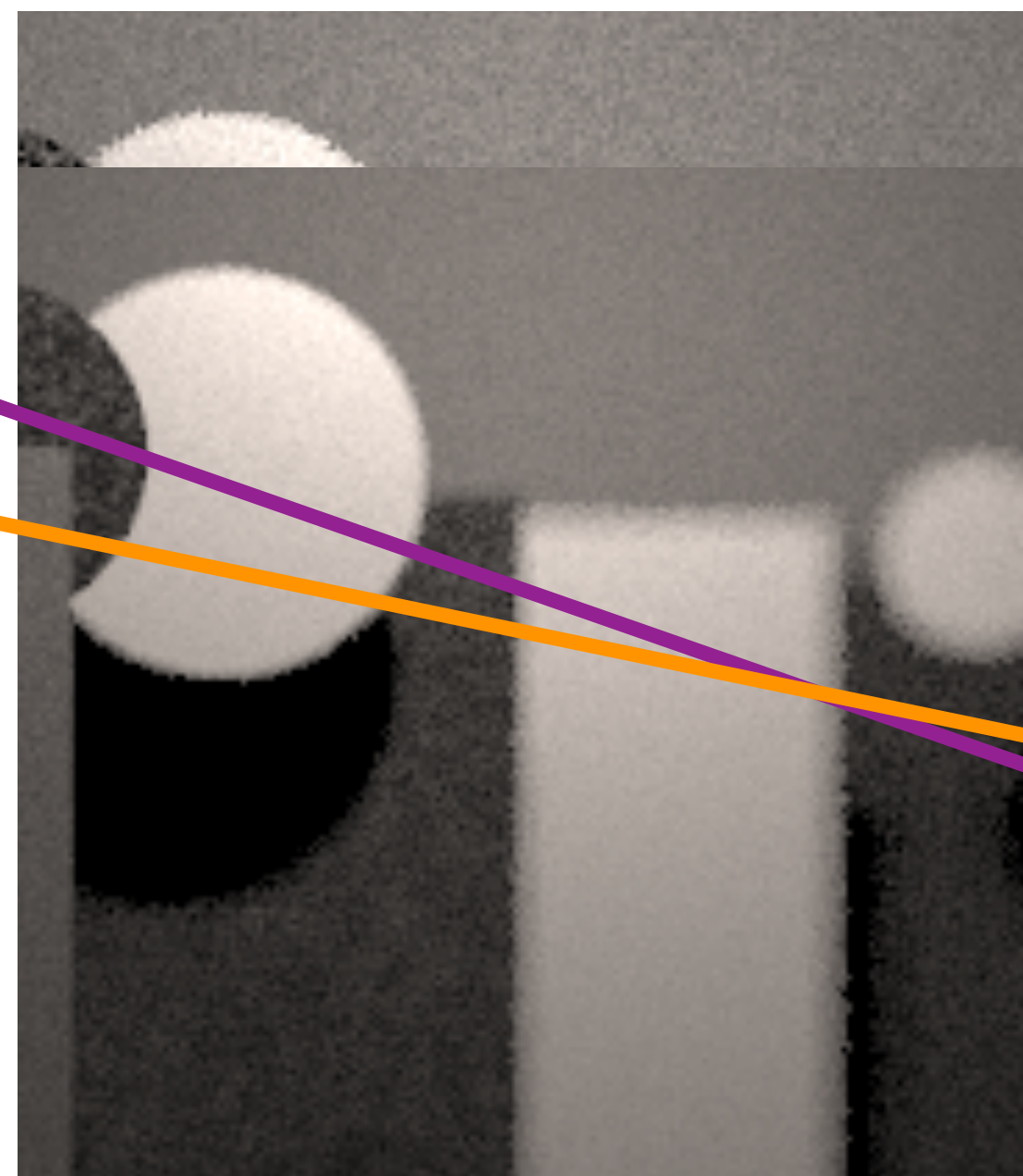
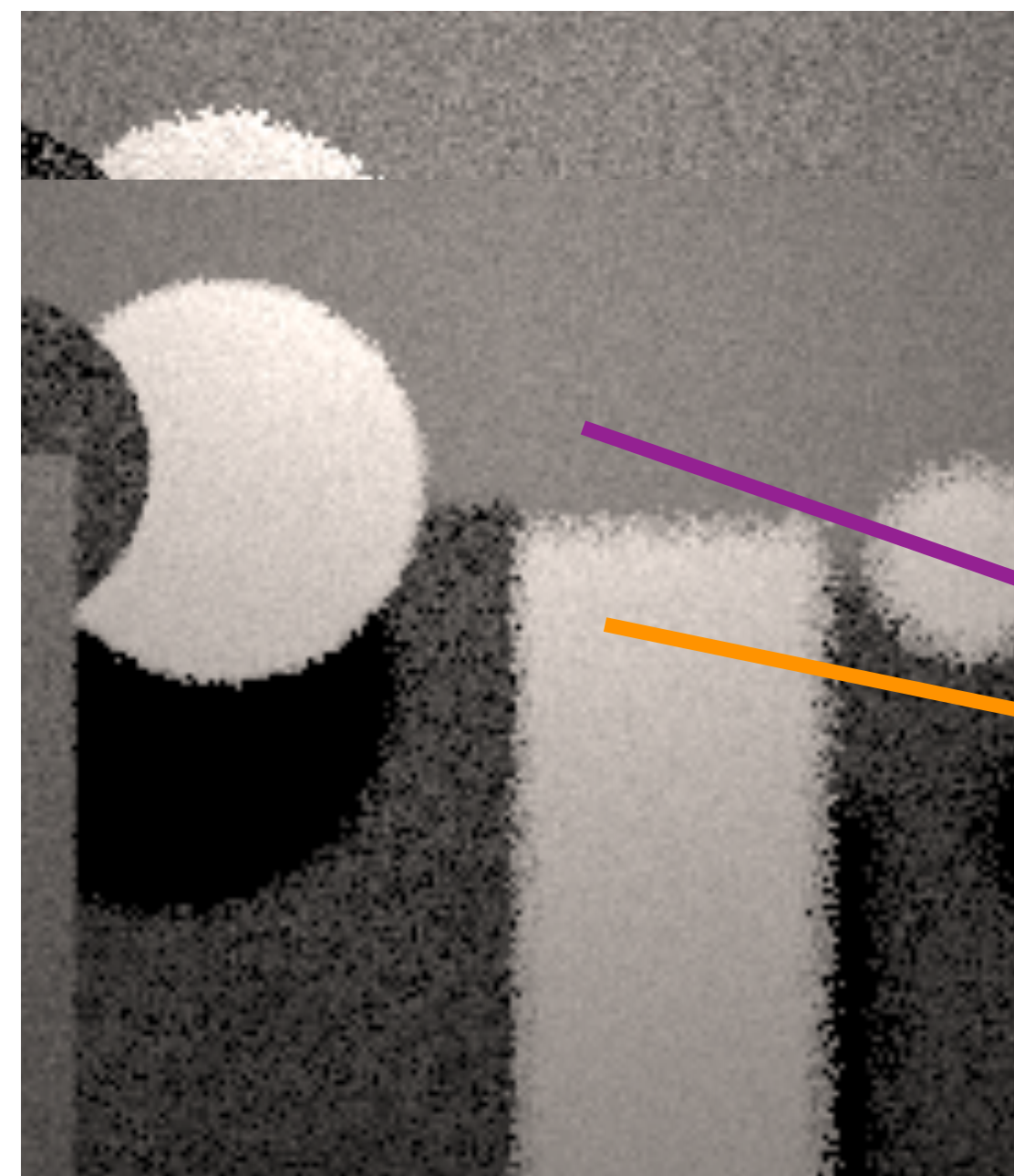
$$O(N^{-1.25})$$

■ ■ ■

Number of Samples

Variance Convergence Rate of Samplers

Variance



- 4D Jittered
- Poisson Disk

$$O(N^{-1})$$

$$O(N^{-1.25})$$

■ ■ ■

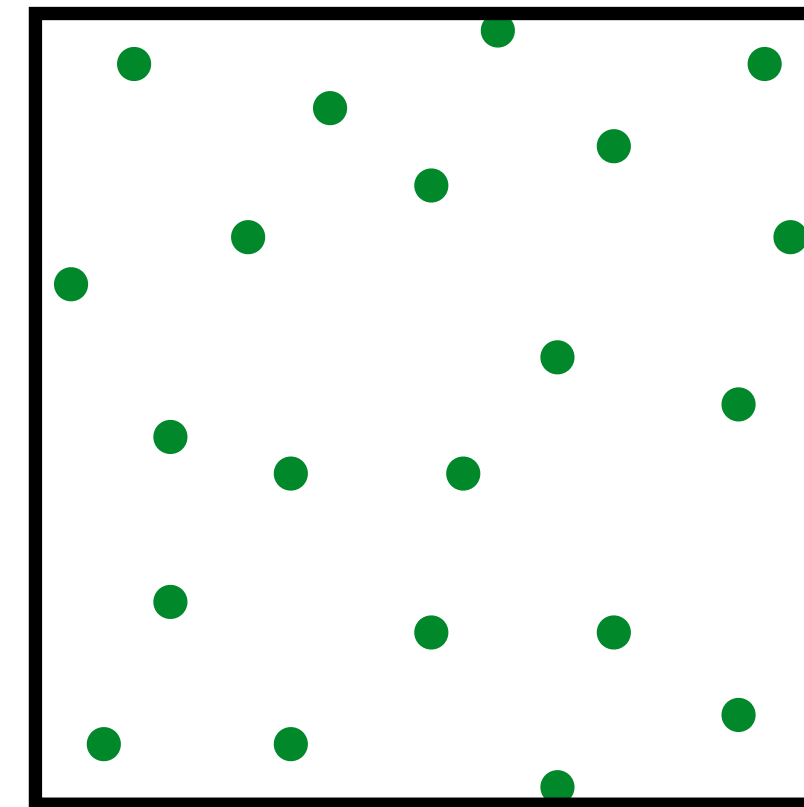
Fredo Durand [2011]
Subr & Kautz [2013]
Pilleboue et al. [2015]

Number of Samples

Monte Carlo Estimator

$$I_N = \frac{1}{N} \sum_{k=1}^N f(\vec{x}_k) = \int_0^1 \frac{1}{N} \sum_{k=1}^N \delta(\vec{x} - \vec{x}_k) f(\vec{x}) d\vec{x} = \int_0^1 S_N(\vec{x}) f(\vec{x}) d\vec{x}$$

$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^N \delta(\vec{x} - \vec{x}_k)$$

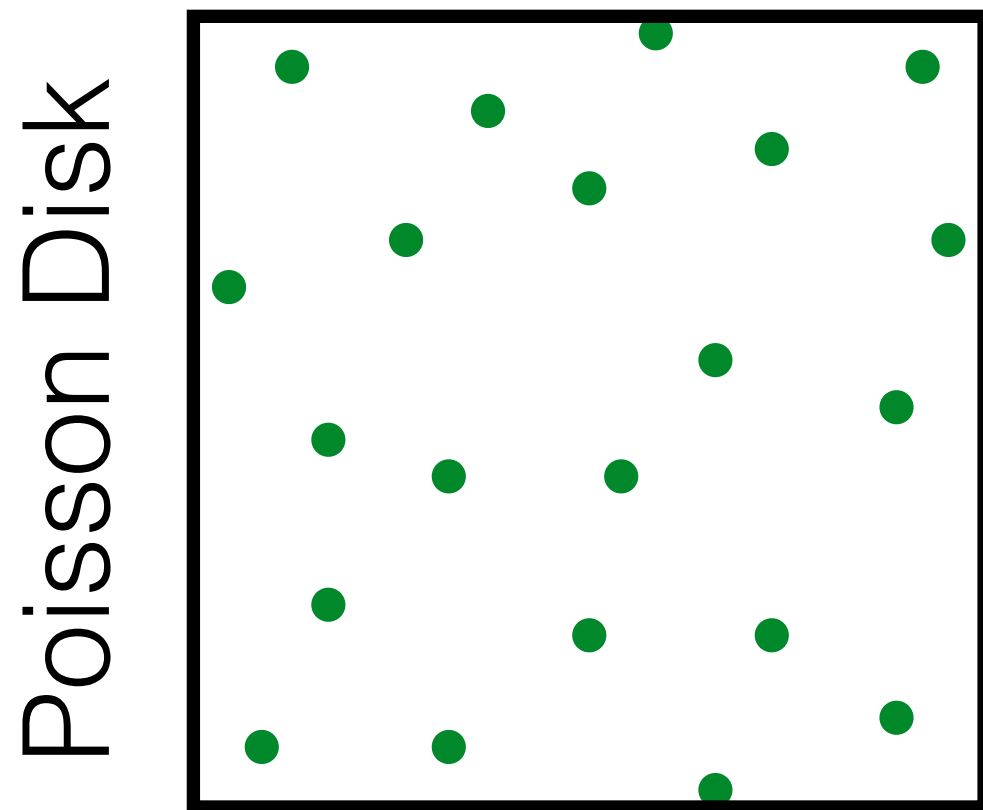


Fredo Durand [2011]

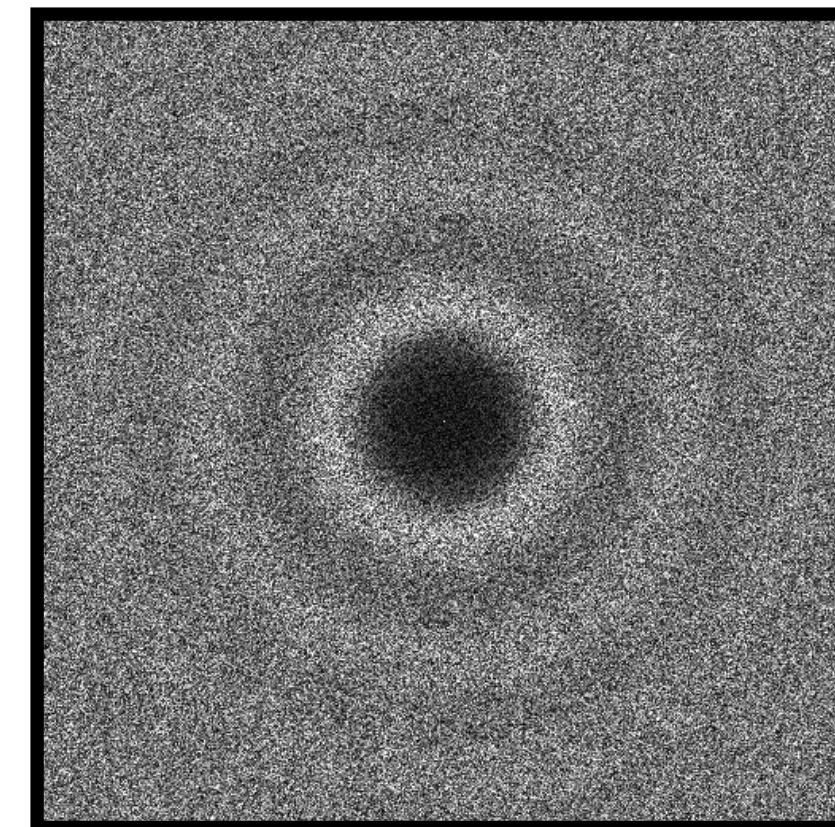
Samples Power Spectrum

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x}) d\vec{x}$$

Samples



Spectrum

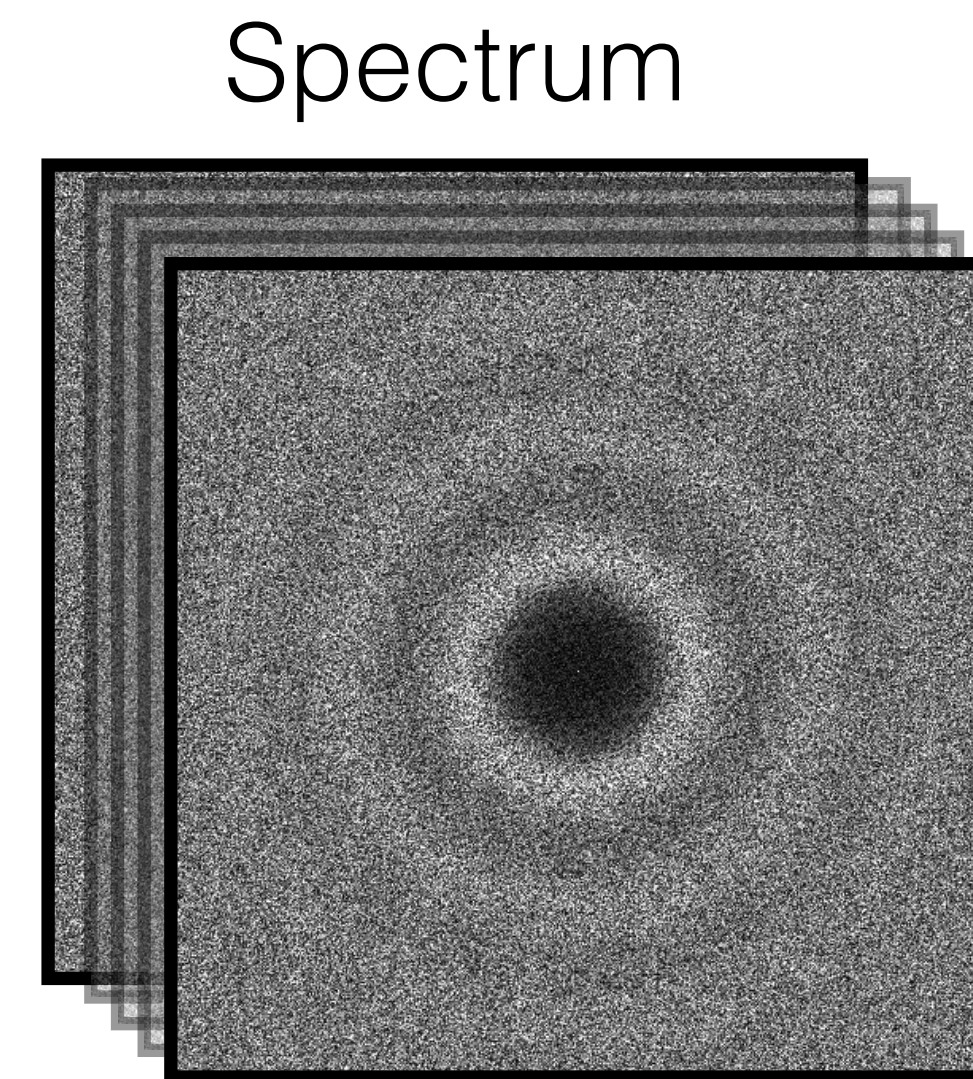
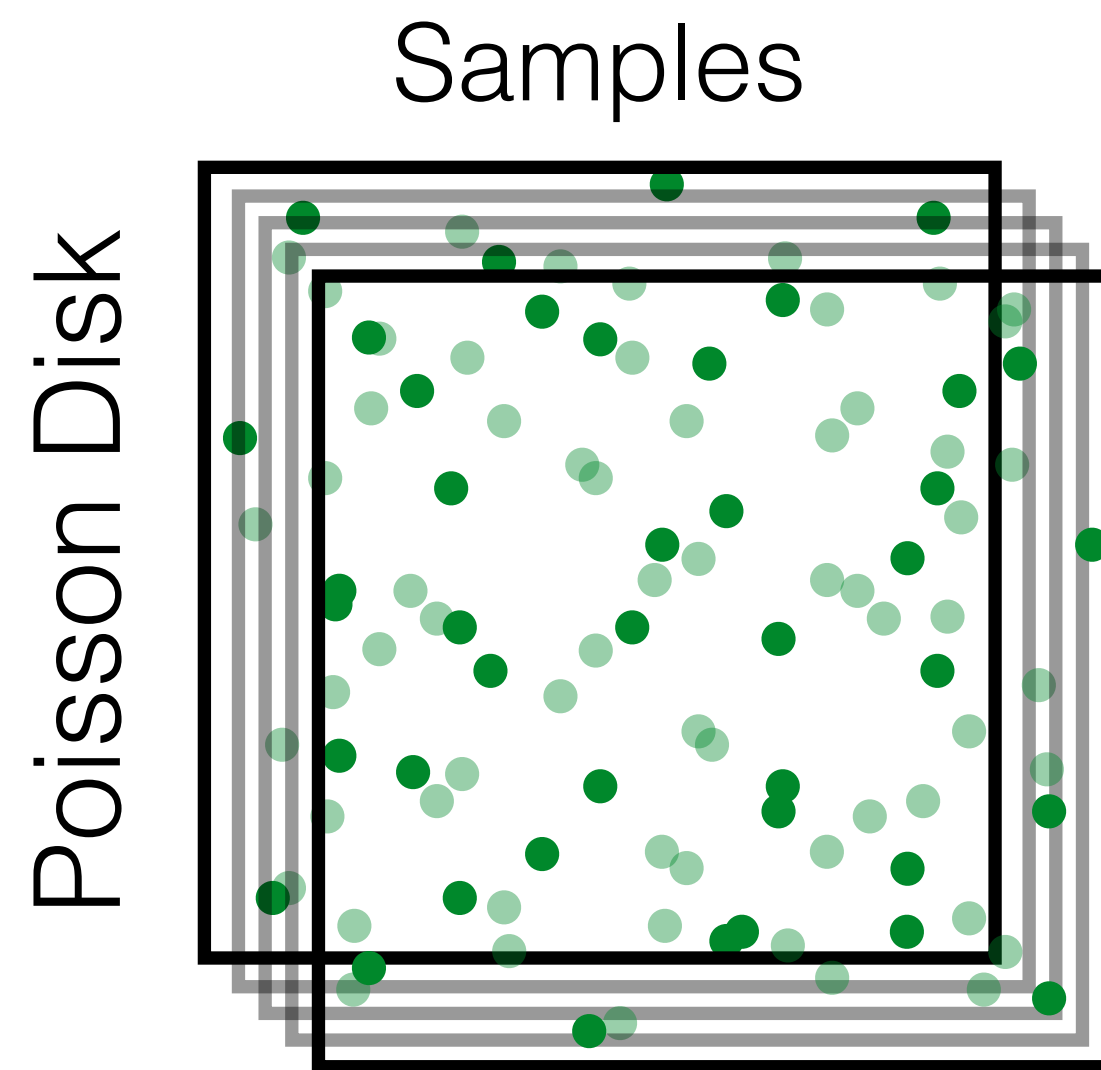


$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^N \delta(\vec{x} - \vec{x}_k)$$

$$\mathcal{P}_{S_N}(\nu) = \left| \frac{1}{N} \sum_{k=1}^N e^{-i2\pi\nu \cdot \vec{x}_k} \right|^2$$

Expected Sampling Power Spectra

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x}) d\vec{x}$$

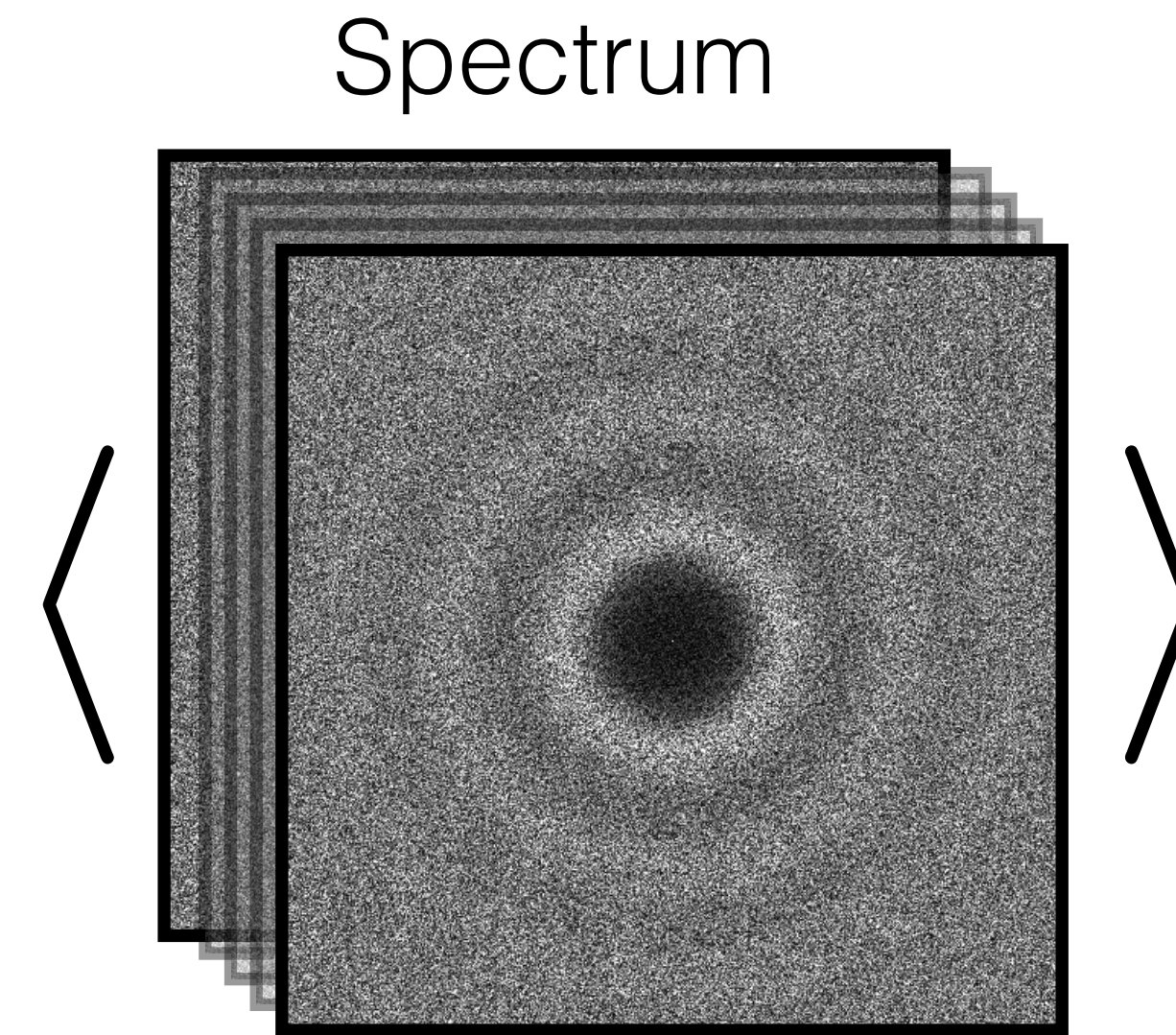
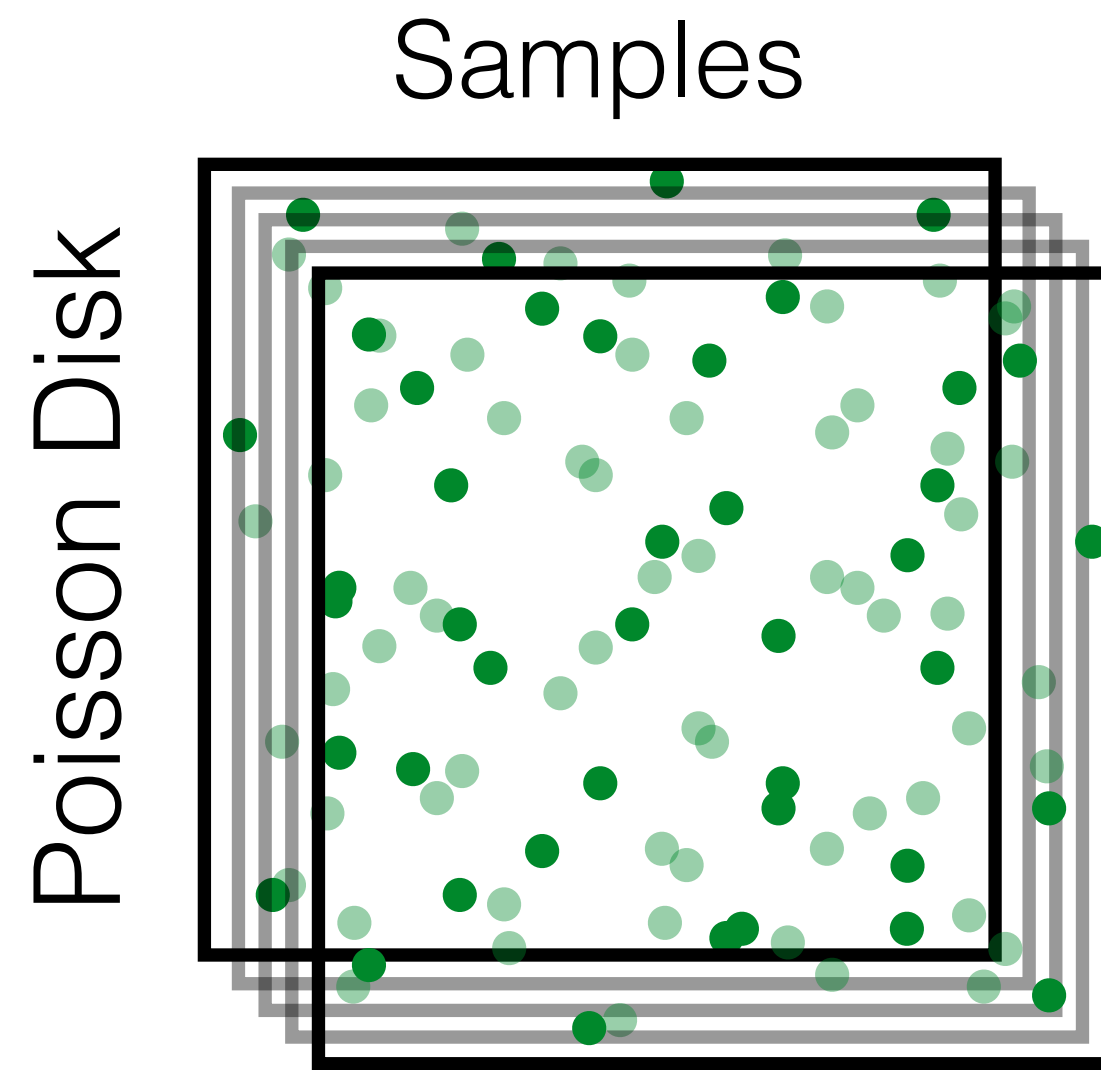


$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^N \delta(\vec{x} - \vec{x}_k)$$

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Expected Sampling Power Spectra

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x}) d\vec{x}$$

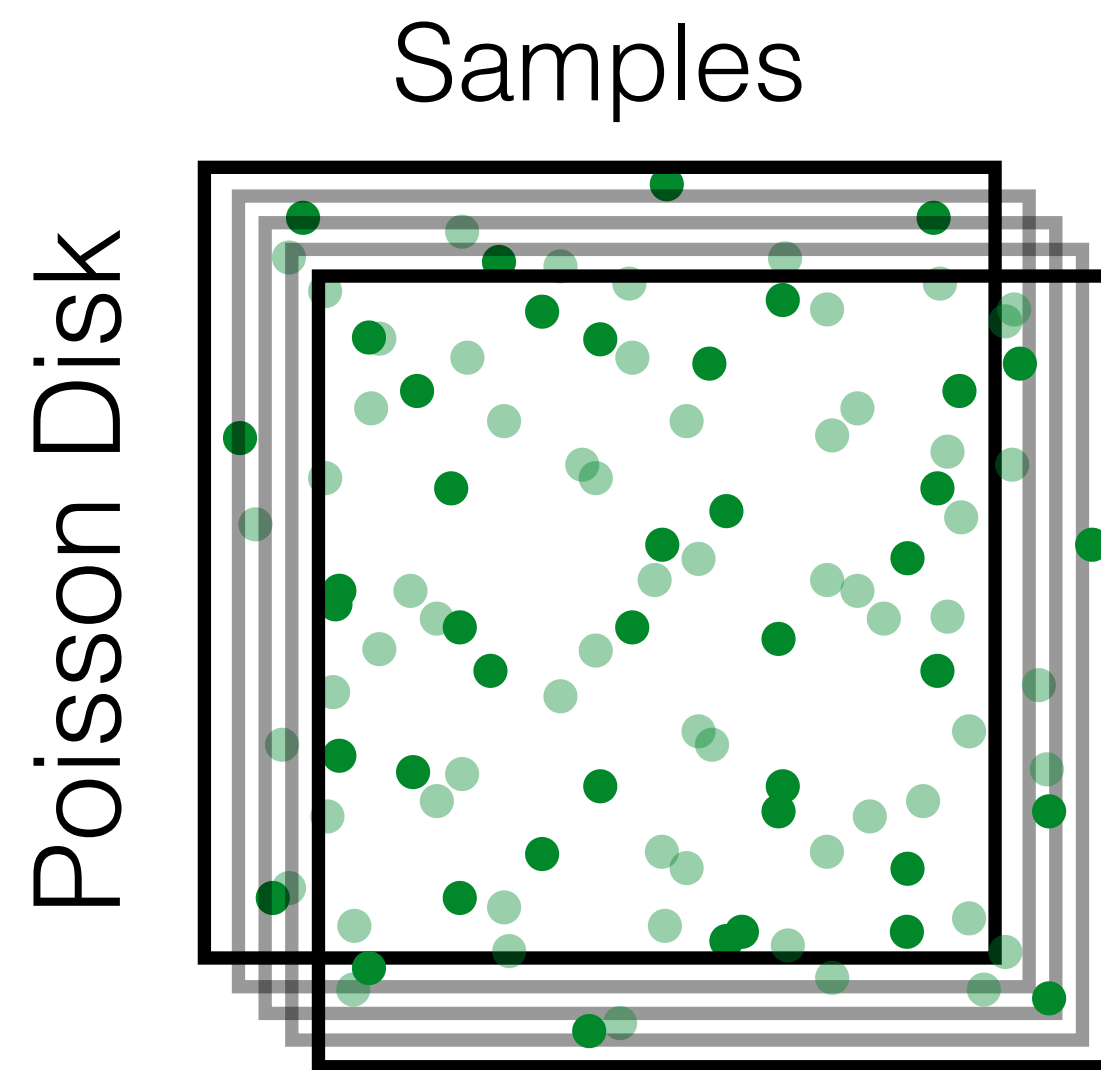


$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^N \delta(\vec{x} - \vec{x}_k)$$

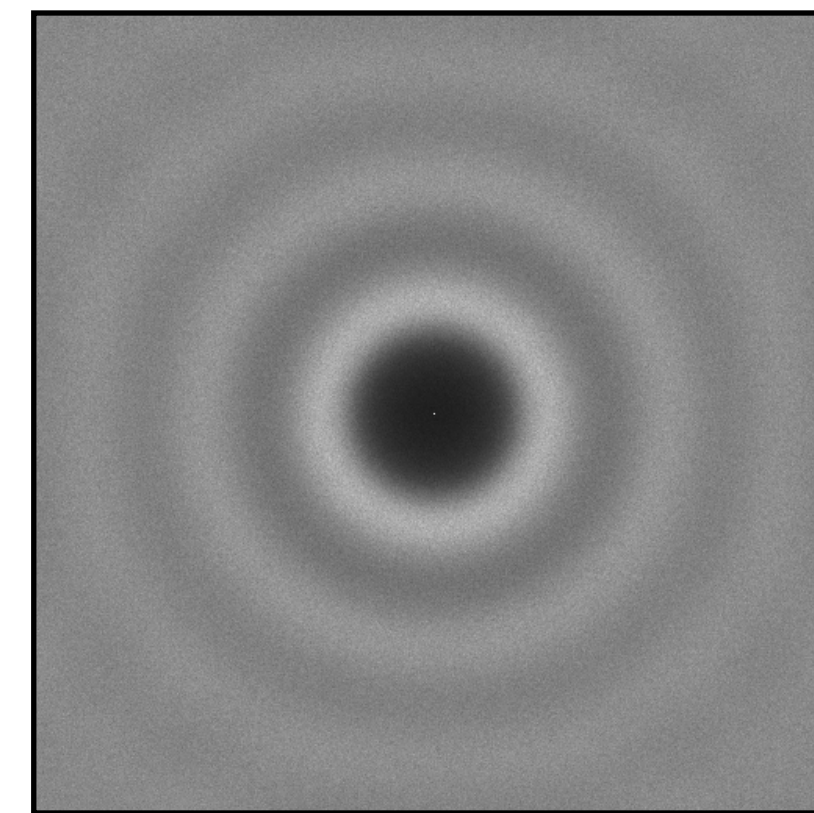
$$\langle \mathcal{P}_{S_N}(\nu) \rangle = \left\langle \left| \frac{1}{N} \sum_{k=1}^N e^{-i2\pi\nu \cdot \vec{x}_k} \right|^2 \right\rangle$$

Expected Sampling Power Spectra

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x}) d\vec{x}$$



Expected Spectrum

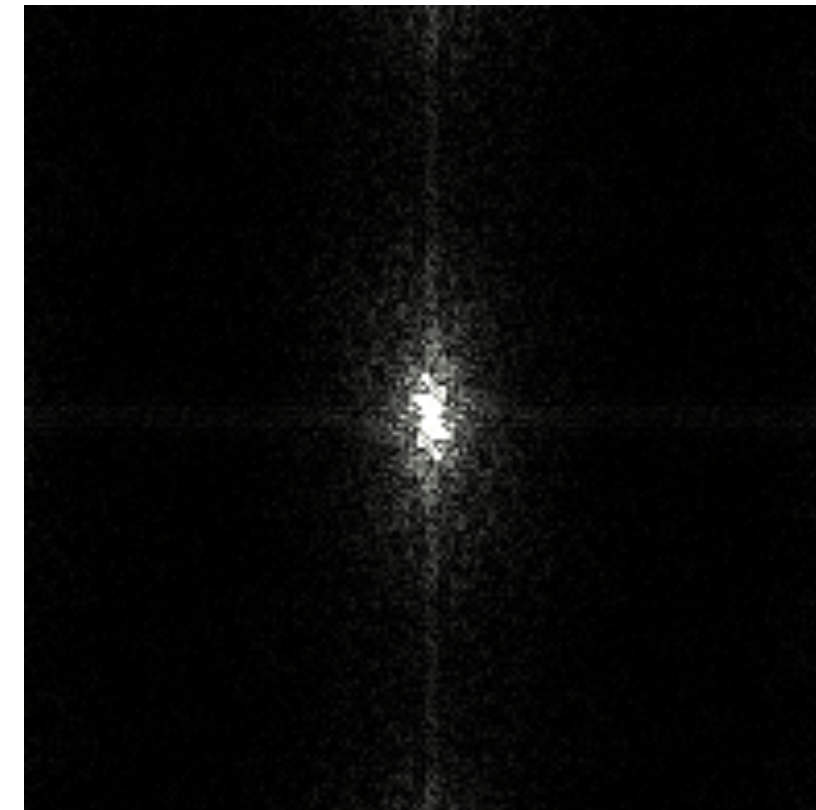
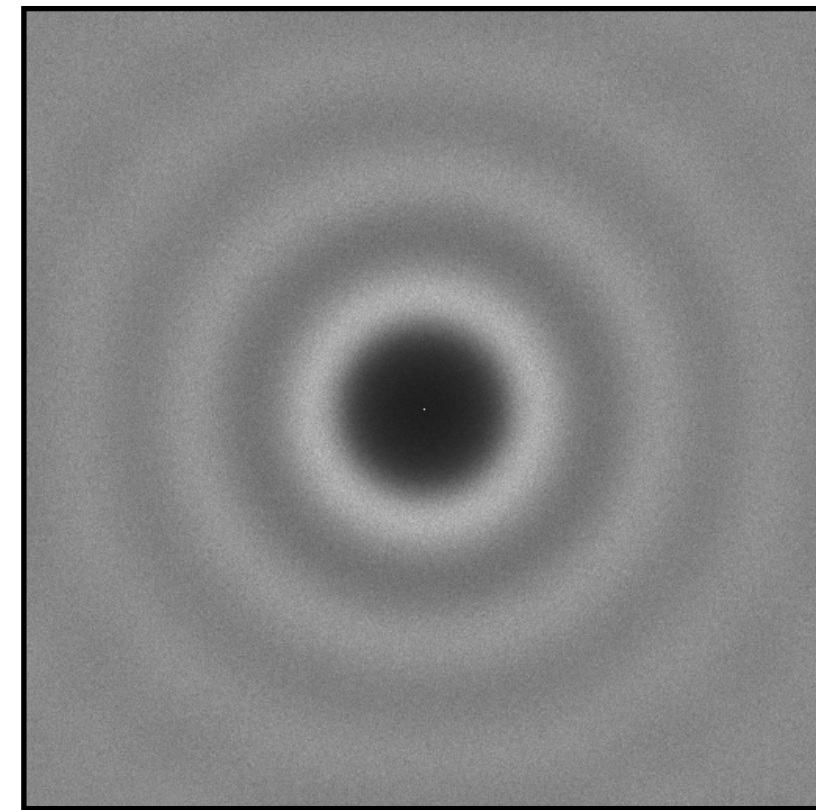


$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^N \delta(\vec{x} - \vec{x}_k)$$

$$\langle \mathcal{P}_{S_N}(\nu) \rangle = \left\langle \left| \frac{1}{N} \sum_{k=1}^N e^{-i2\pi\nu \cdot \vec{x}_k} \right|^2 \right\rangle$$

Variance of Monte Carlo Estimator

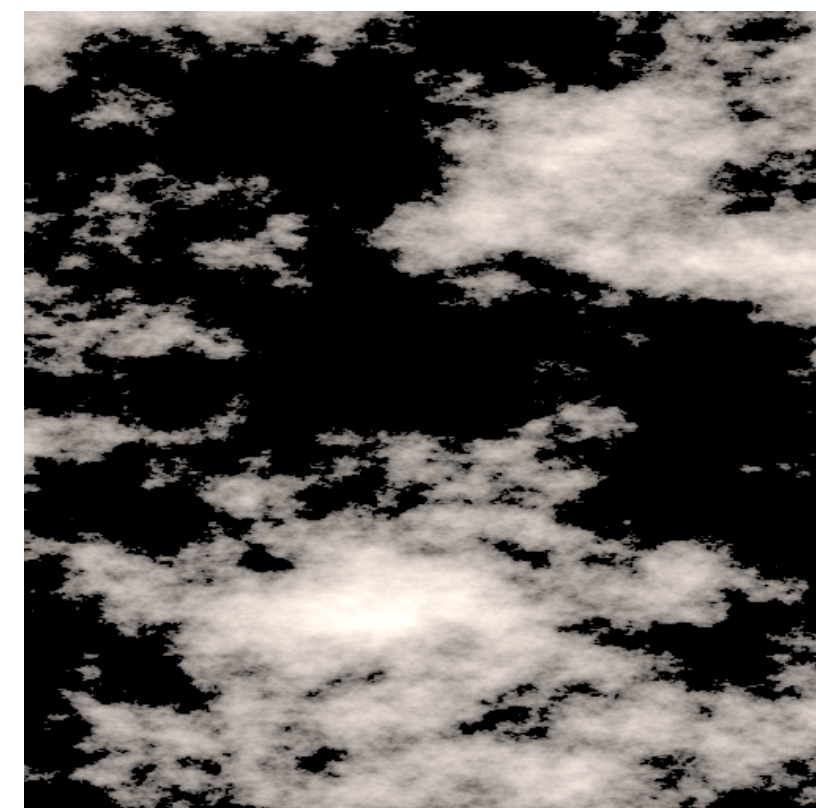
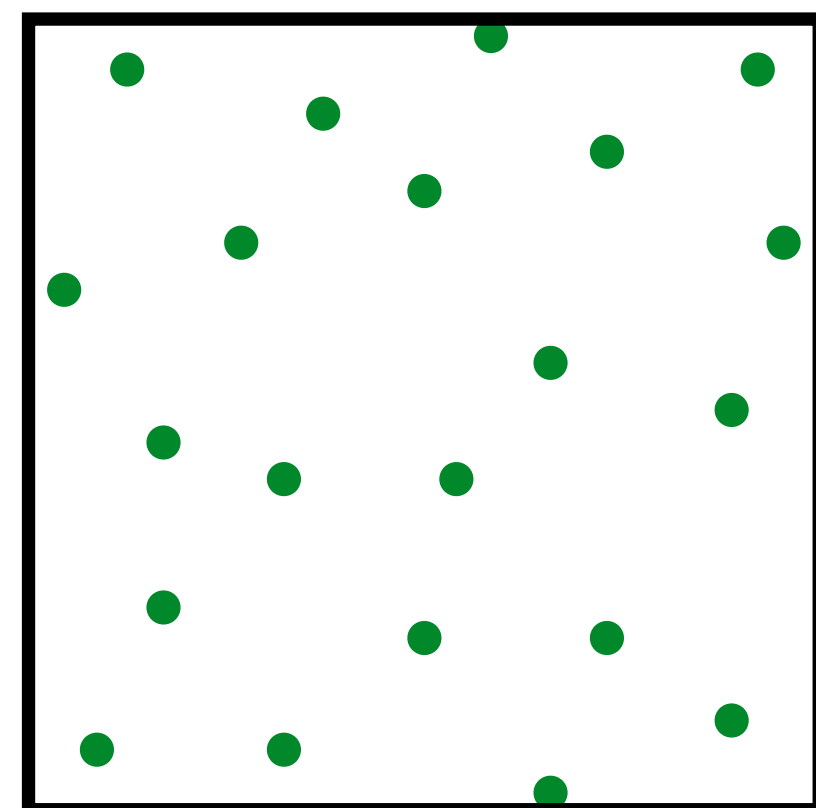
$$\text{Var}(I_N) = \int_{\Omega} \langle \mathcal{P}_{S_N}(\nu) \rangle \times \mathcal{P}_f(\nu) d\nu$$



$S_N(\vec{x})$

$f(\vec{x})$

Poisson Disk

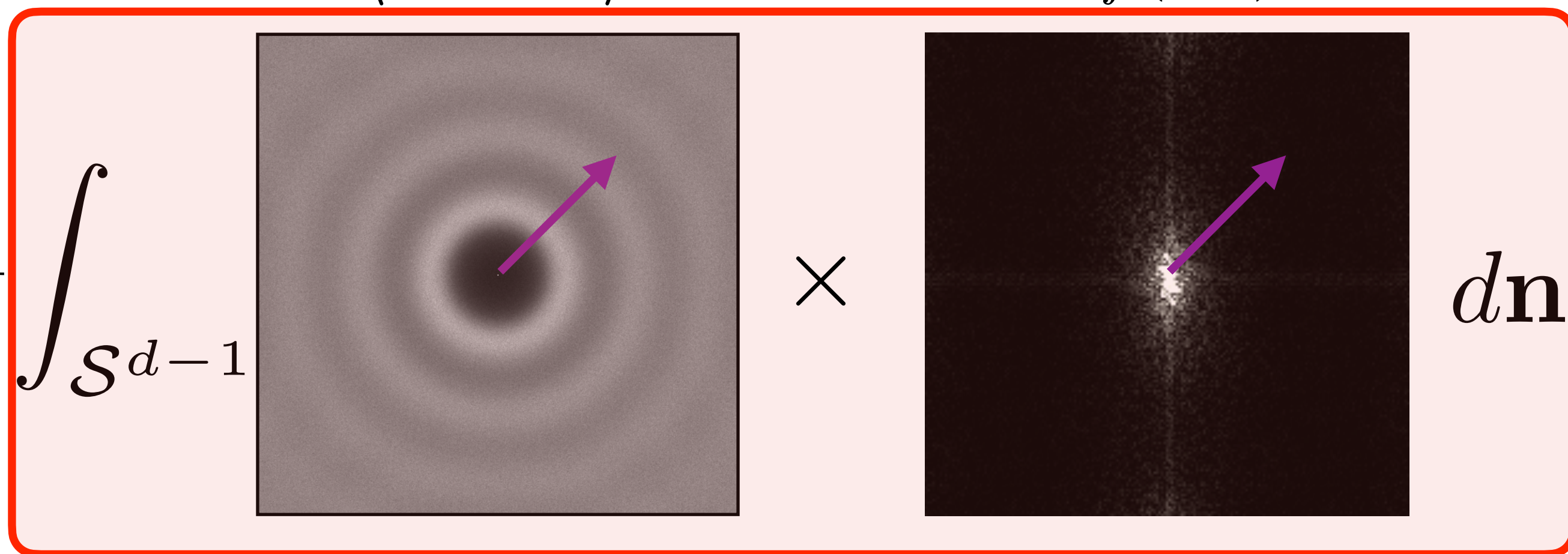


Fredo Durand [2011]

Subr & Kautz [2013]

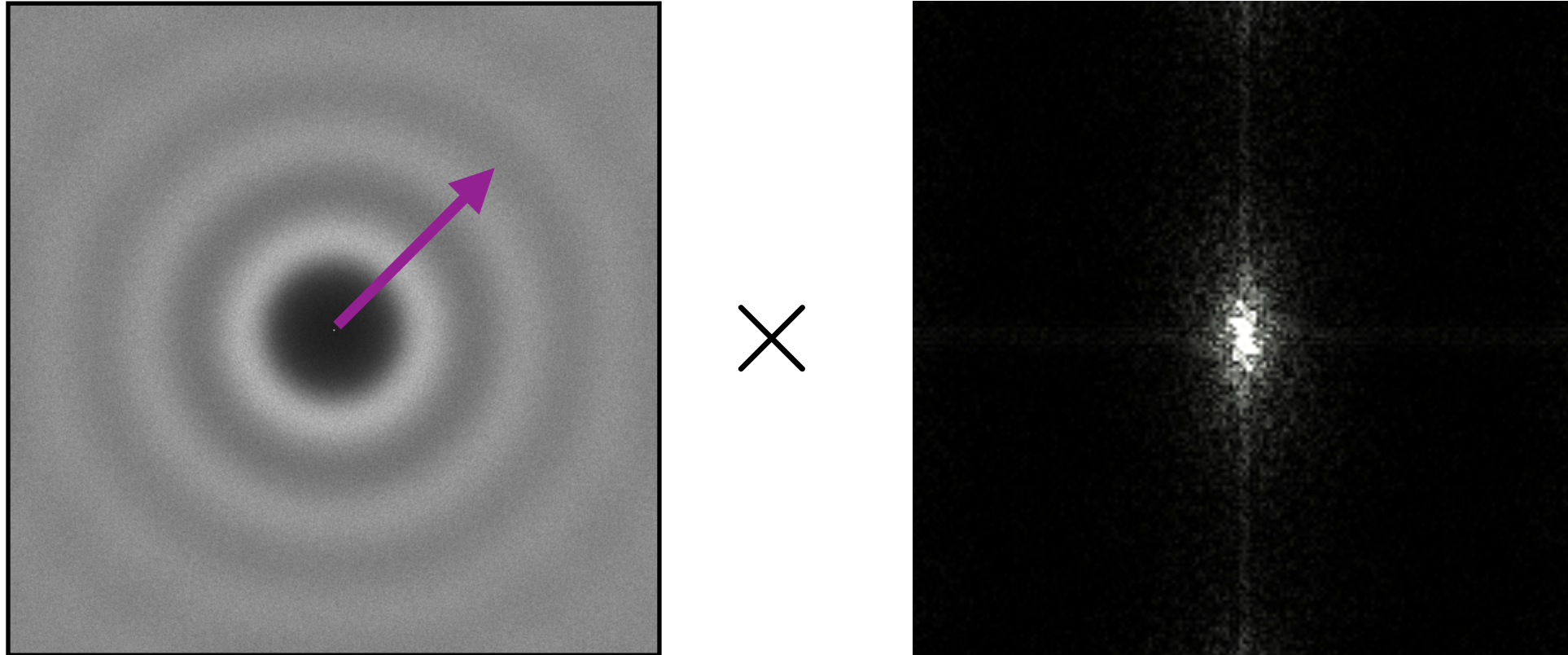
Pilleboue et al. [2015]

Variance of Monte Carlo Estimator in Polar Coordinates

$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \int_{\mathcal{S}^{d-1}} \langle \tilde{\mathcal{P}}_{S_N}(\rho \mathbf{n}) \rangle \times \mathcal{P}_f(\rho \mathbf{n}) d\mathbf{n} d\rho$$


Pilleboue et al. [2015]

Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \int_{\mathcal{S}^{d-1}} \langle \tilde{\mathcal{P}}_{S_N}(\rho \mathbf{n}) \rangle \times \mathcal{P}_f(\rho \mathbf{n}) d\mathbf{n} d\rho$$


Pilleboue et al. [2015]

Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

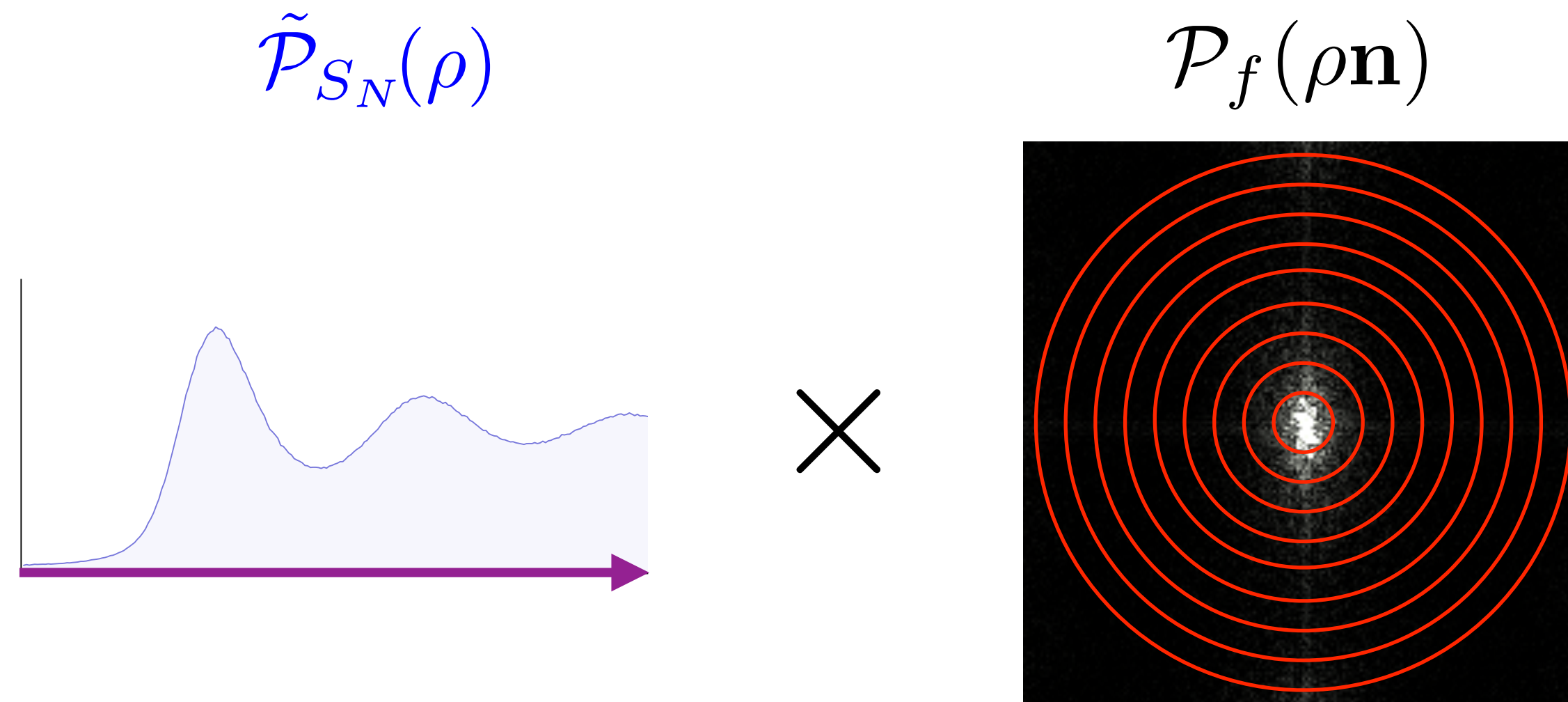
$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \tilde{\mathcal{P}}_{S_N}(\rho) \int_{\mathcal{S}^{d-1}} \mathcal{P}_f(\rho \mathbf{n}) d\mathbf{n} d\rho$$

The diagram illustrates the variance of the Monte Carlo estimator for isotropic sampling spectra. It consists of three main parts:

- Left:** The mathematical expression for the variance: $\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \tilde{\mathcal{P}}_{S_N}(\rho) \int_{\mathcal{S}^{d-1}} \mathcal{P}_f(\rho \mathbf{n}) d\mathbf{n} d\rho$.
- Middle:** A plot of the probability density function $\tilde{\mathcal{P}}_{S_N}(\rho)$ as a function of ρ . The curve is blue and has a light blue shaded area underneath it. A purple arrow points from this plot towards the right.
- Right:** A 2D plot of the power spectrum $\mathcal{P}_f(\rho \mathbf{n})$ as a function of direction \mathbf{n} and radius ρ . The plot is enclosed in a red rounded rectangle. It shows a central bright spot with a radial gradient, surrounded by a dark background. The integral $\int_{\mathcal{S}^{d-1}}$ is shown to the left of the plot, and $d\mathbf{n} d\rho$ is shown to the right.

Pilleboue et al. [2015]

Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \tilde{\mathcal{P}}_{S_N}(\rho) \times \mathcal{P}_f(\rho \mathbf{n}) d\rho$$


The diagram illustrates the variance of the Monte Carlo estimator for isotropic sampling spectra. It shows the equation $\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \tilde{\mathcal{P}}_{S_N}(\rho) \times \mathcal{P}_f(\rho \mathbf{n}) d\rho$. The first term, $\tilde{\mathcal{P}}_{S_N}(\rho)$, is represented by a graph of a blue curve with a light blue shaded area under it, indicating a probability density function. The second term, $\mathcal{P}_f(\rho \mathbf{n})$, is represented by a square image showing a central bright spot surrounded by concentric red circles on a dark background, representing a function of the sampling direction.

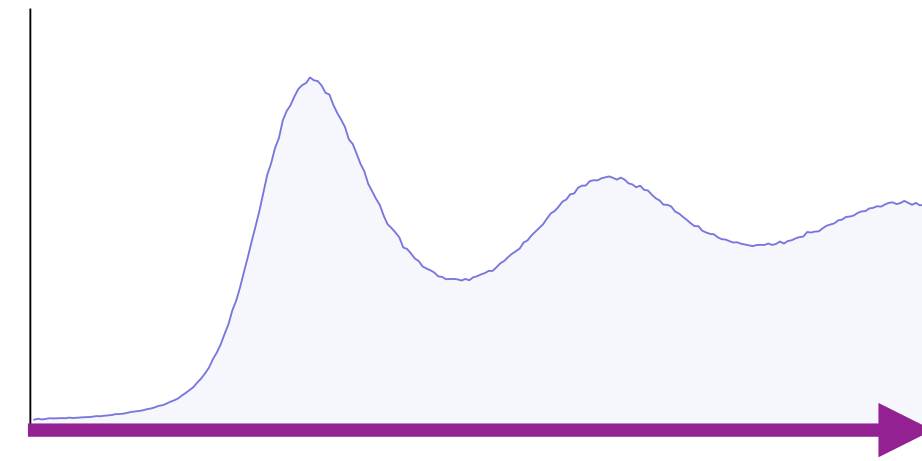
Pilleboue et al. [2015]

Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

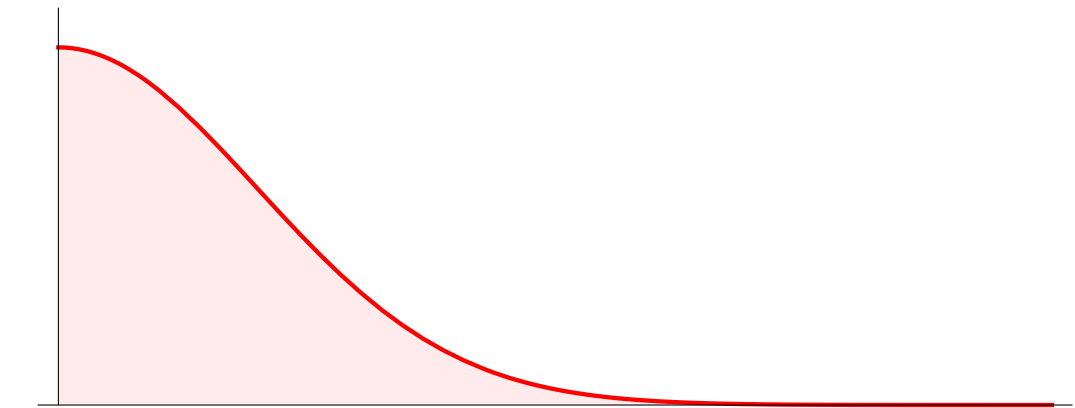
$\tilde{\mathcal{P}}_{S_N}(\rho)$

$\mathcal{P}_f(\rho)$

$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1}$$



\times



$d\rho$

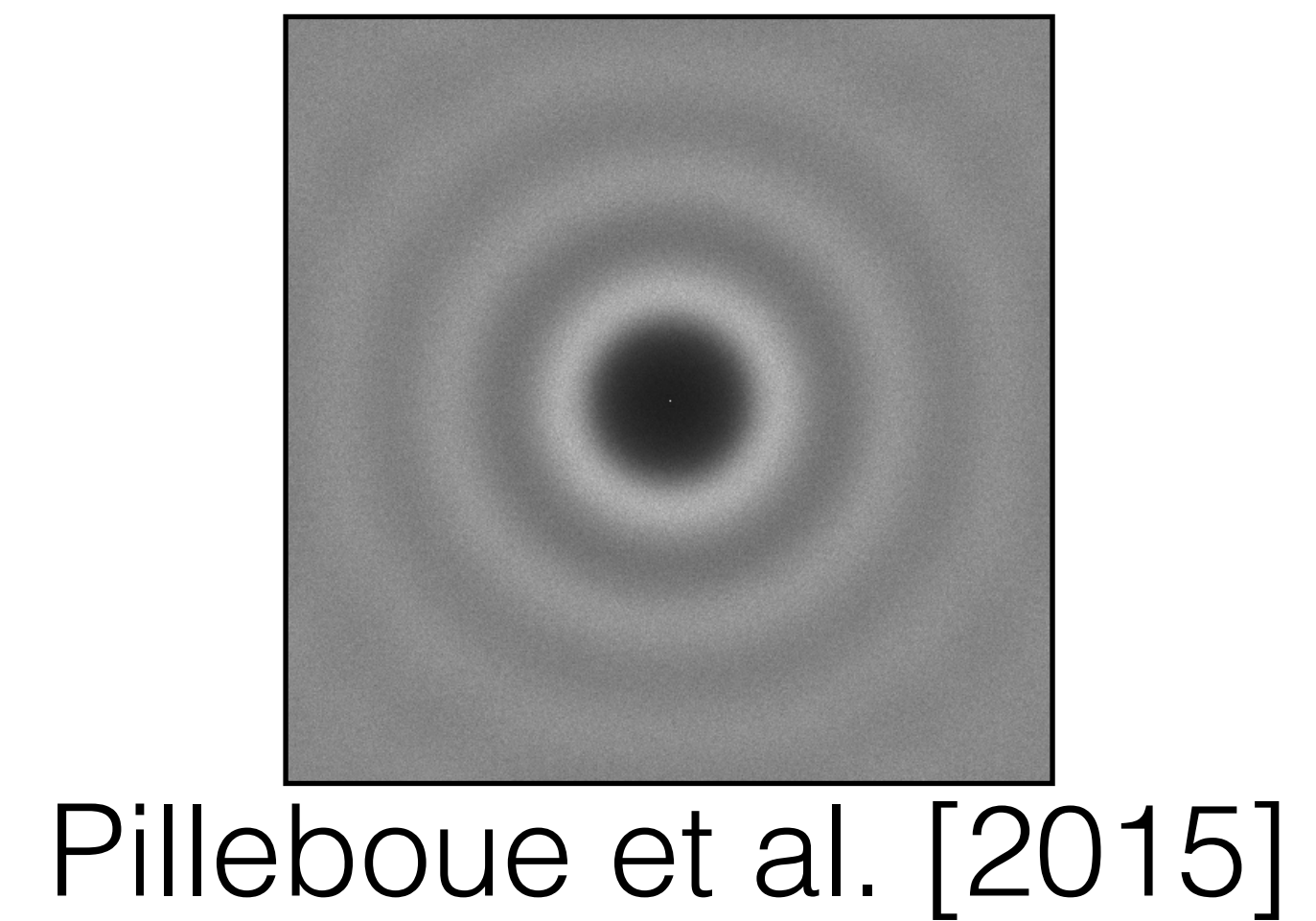
Pilleboue et al. [2015]

Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

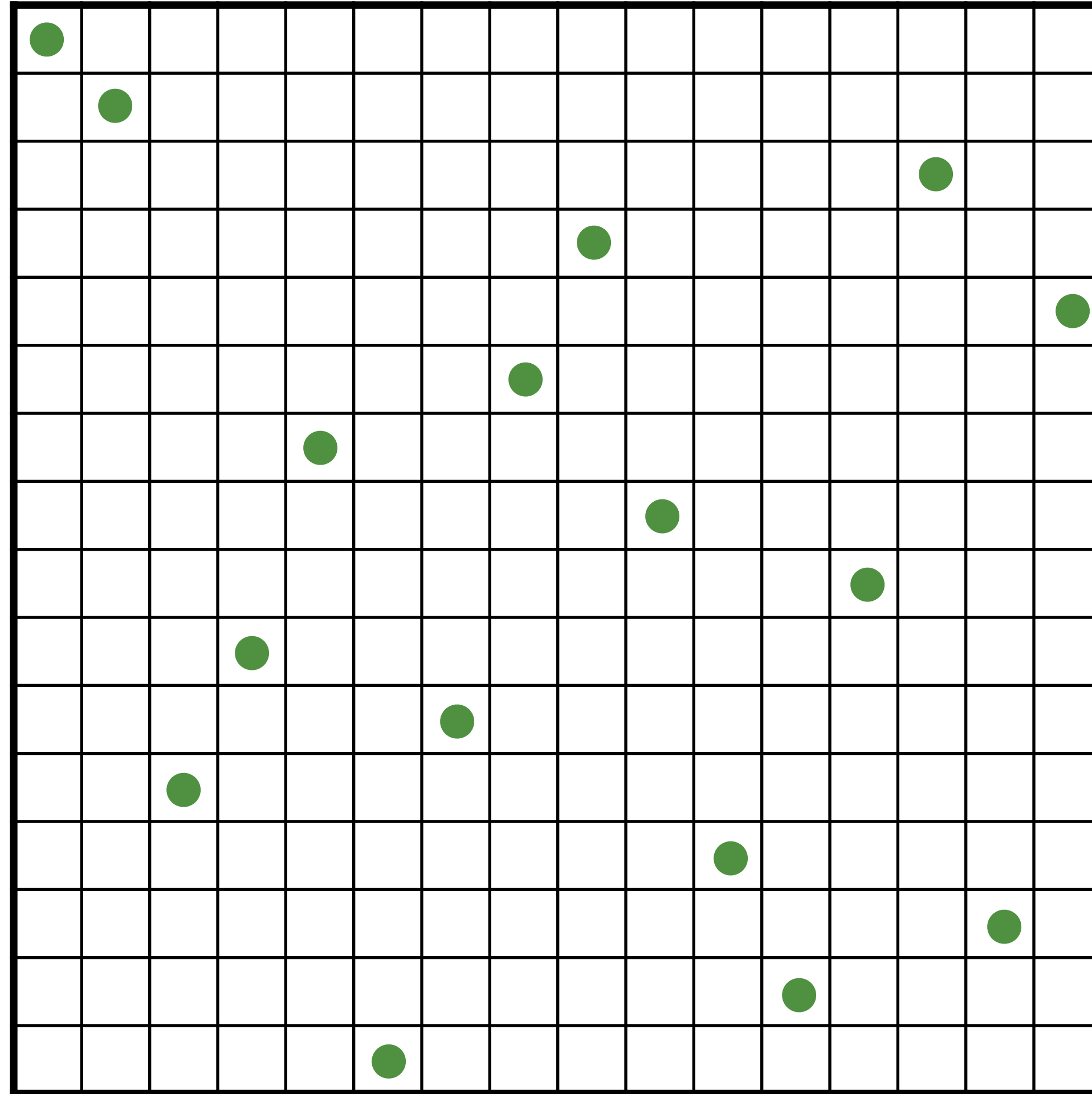
$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \tilde{\mathcal{P}}_{S_N}(\rho) \times \mathcal{P}_f(\rho) d\rho$$

Isotropic Spectrum
Poisson Disk

Samplers	Worst Case	Best Case
Random	$\mathcal{O}(N^{-1})$	$\mathcal{O}(N^{-1})$
Poisson Disk	$\mathcal{O}(N^{-1})$	$\mathcal{O}(N^{-1})$
CCVT	$\mathcal{O}(N^{-1.5})$	$\mathcal{O}(N^{-3})$

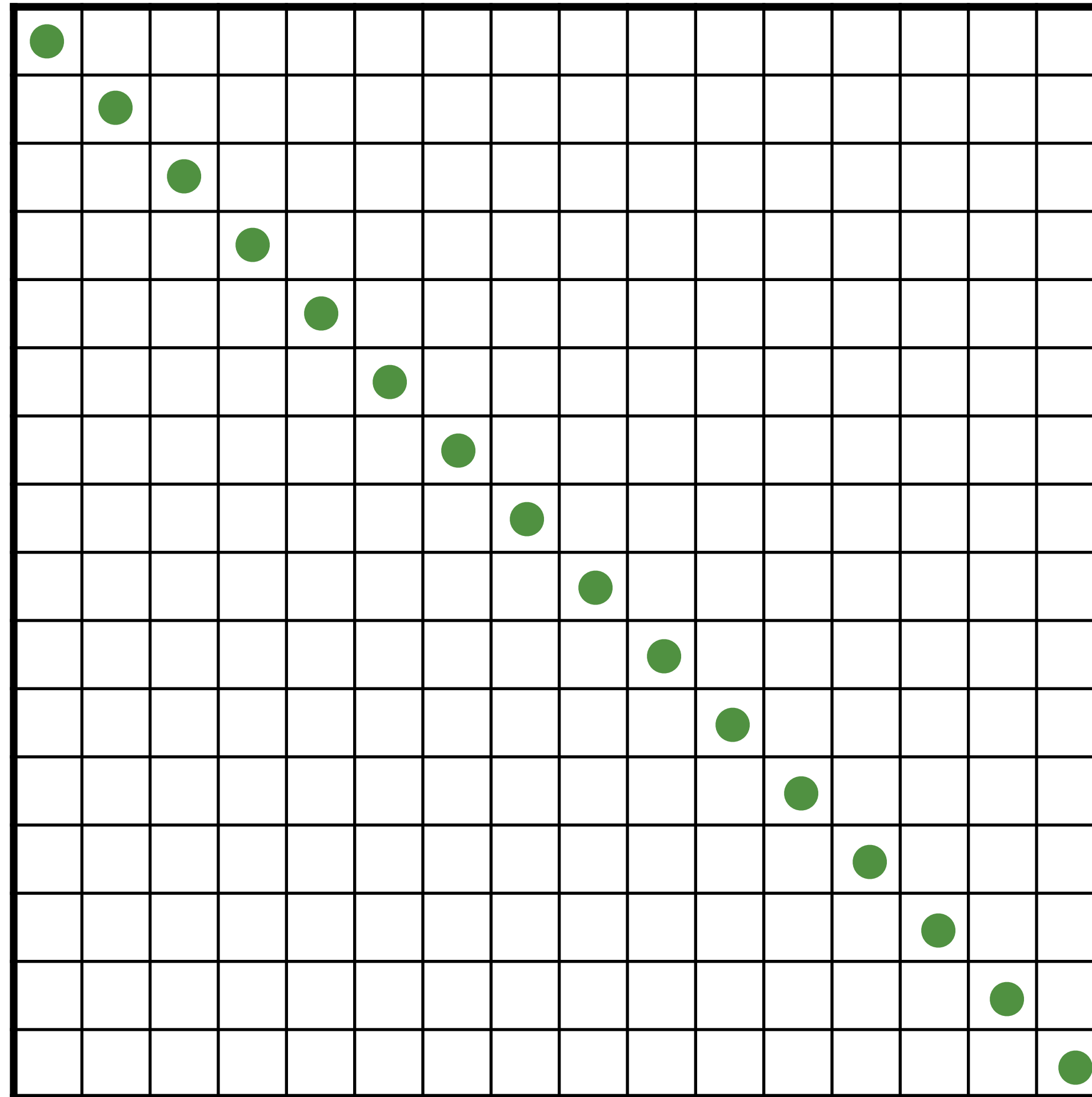


Latin Hypercube Sampler (N-rooks)



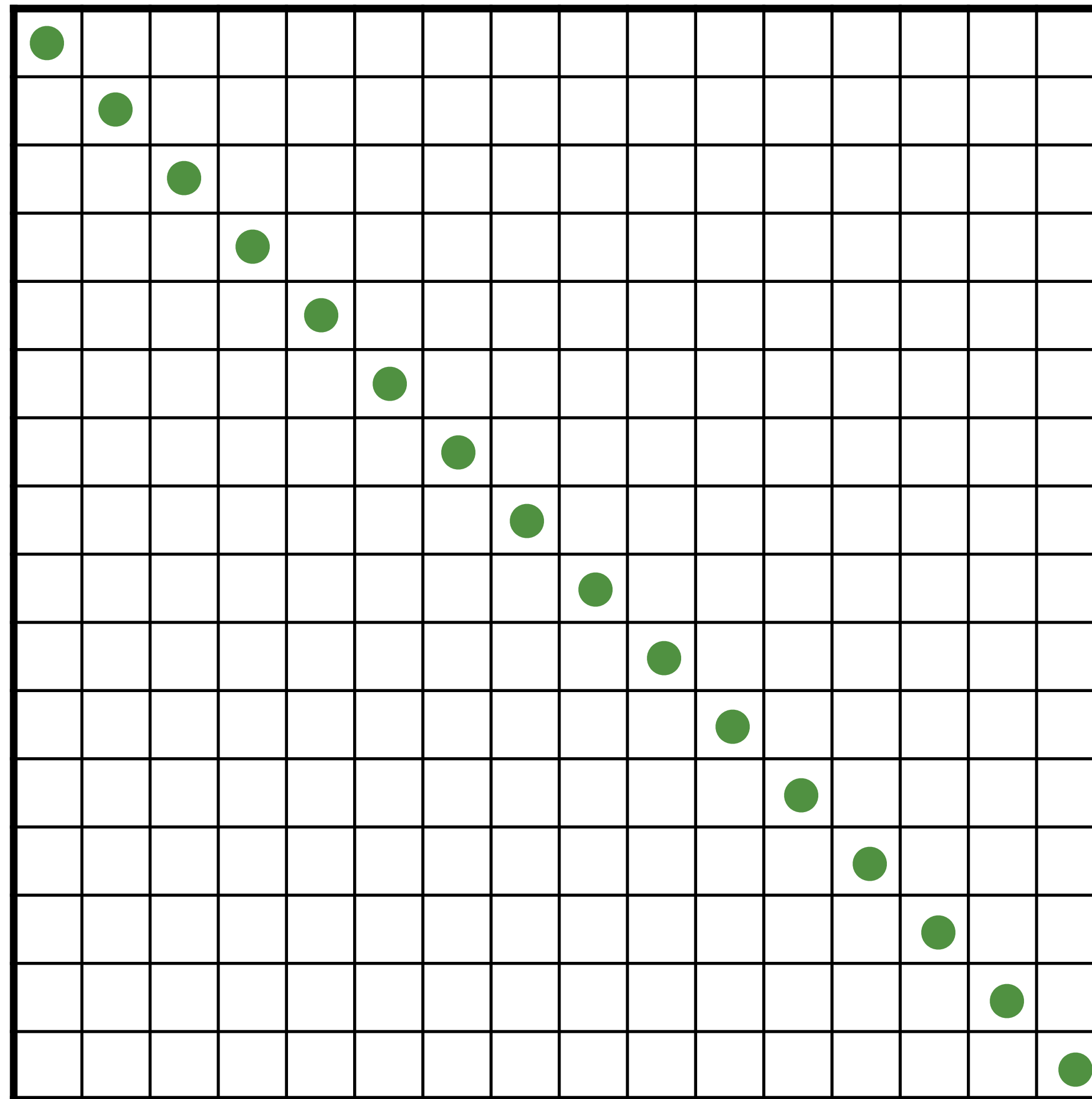
Latin Hypercube Sampler (N-rooks)

Initialize

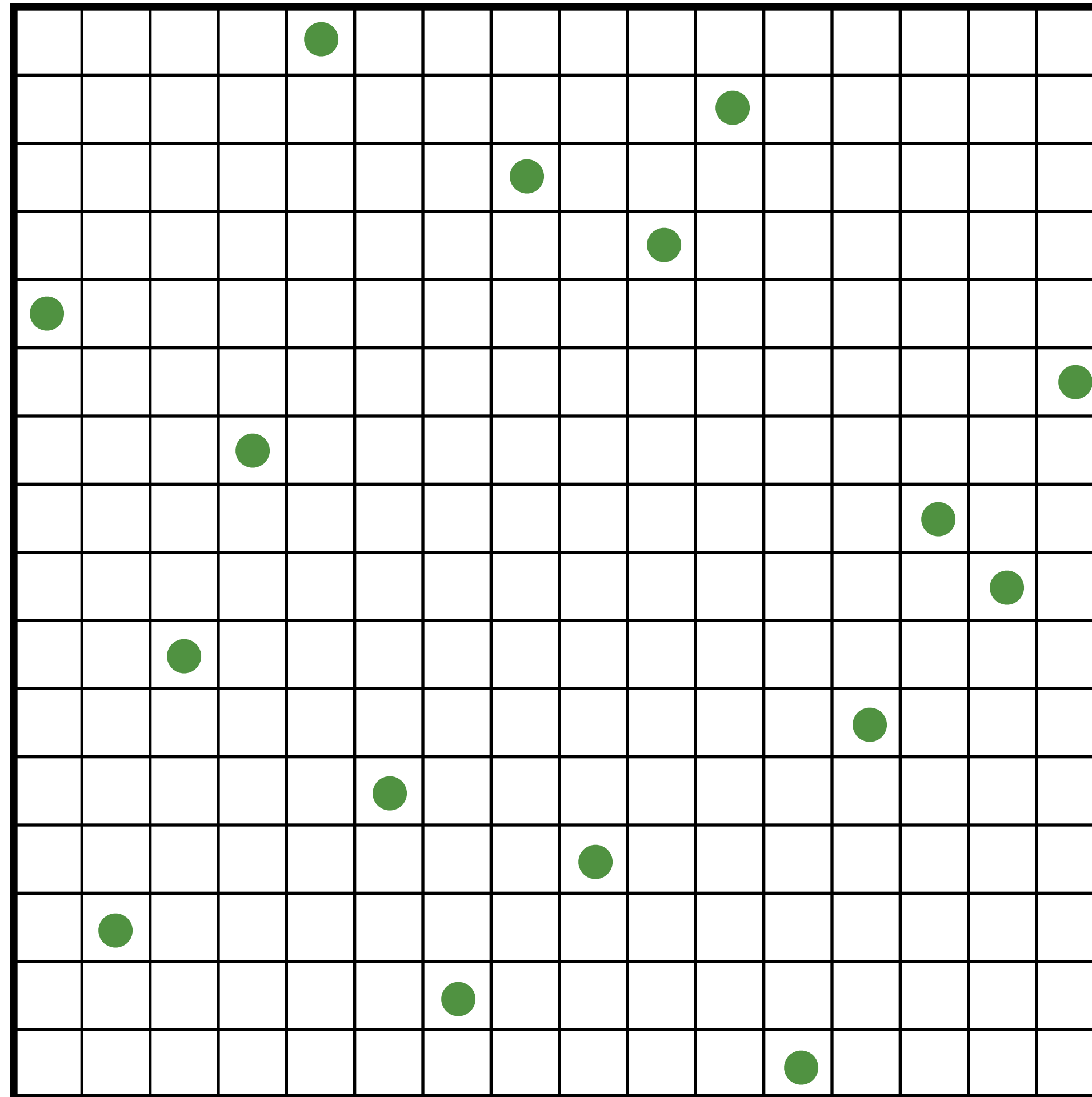


Latin Hypercube Sampler (N-rooks)

Shuffle rows

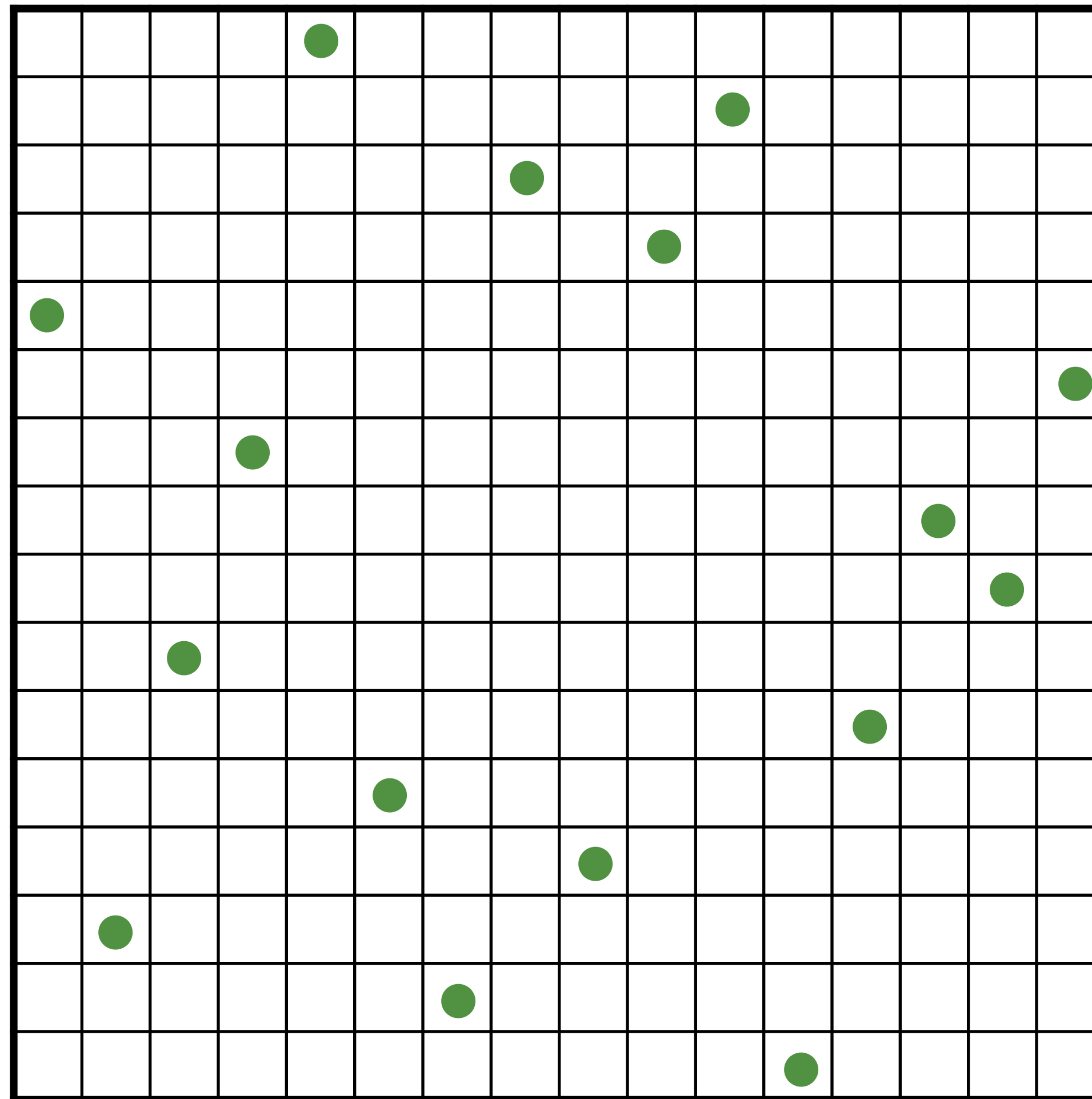


Latin Hypercube Sampler (N-rooks)

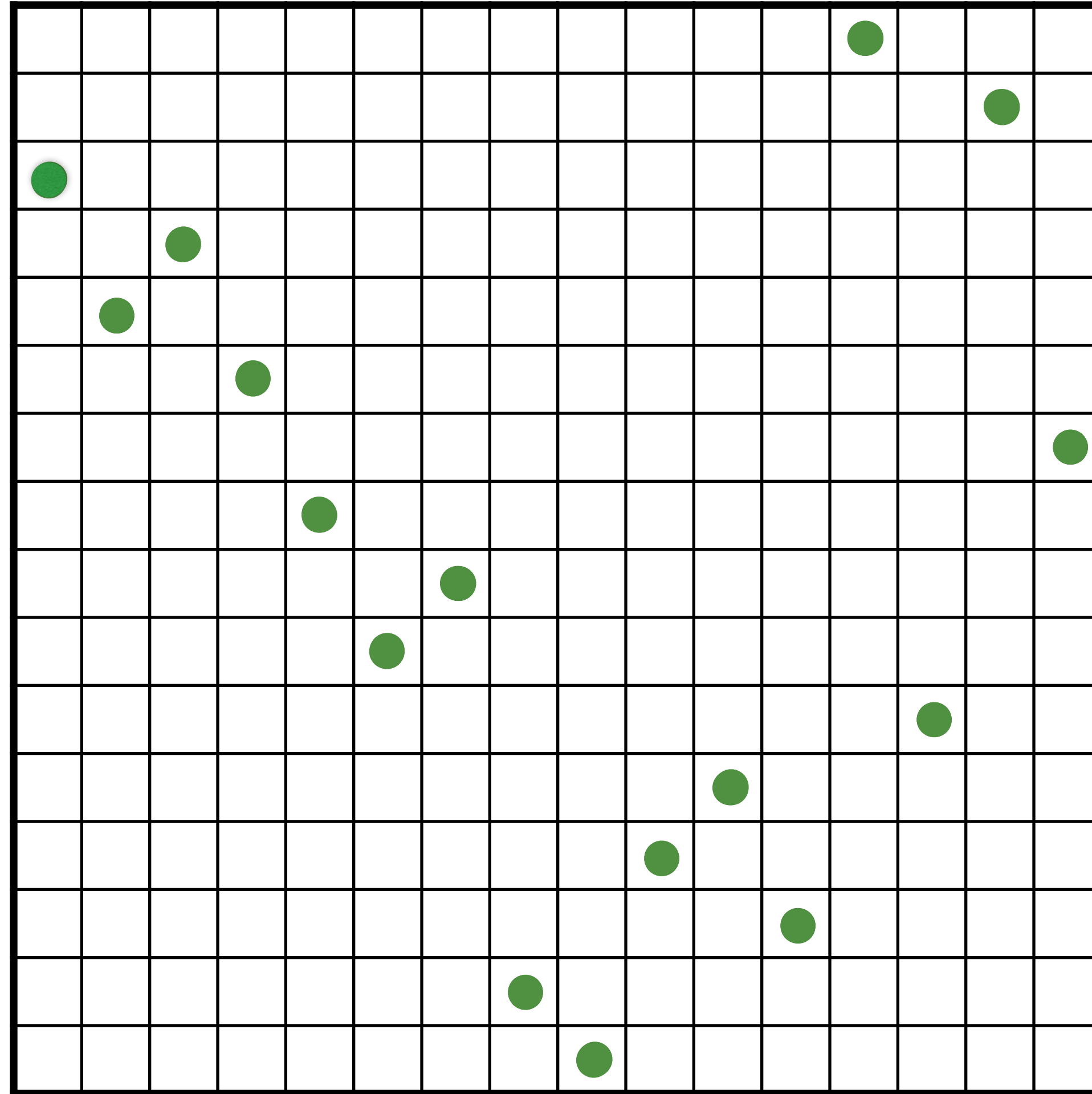


Latin Hypercube Sampler (N-rooks)

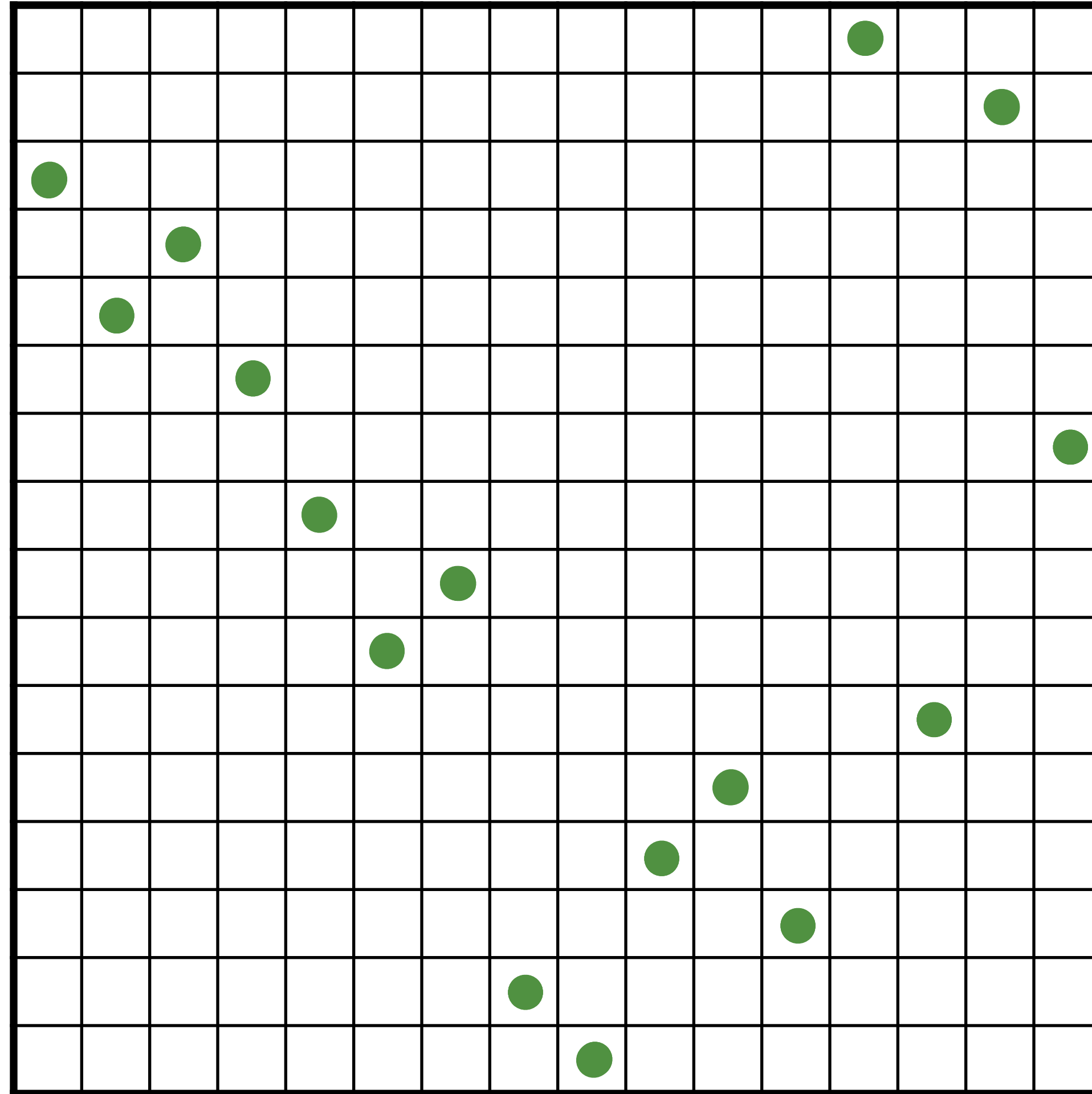
Shuffle columns



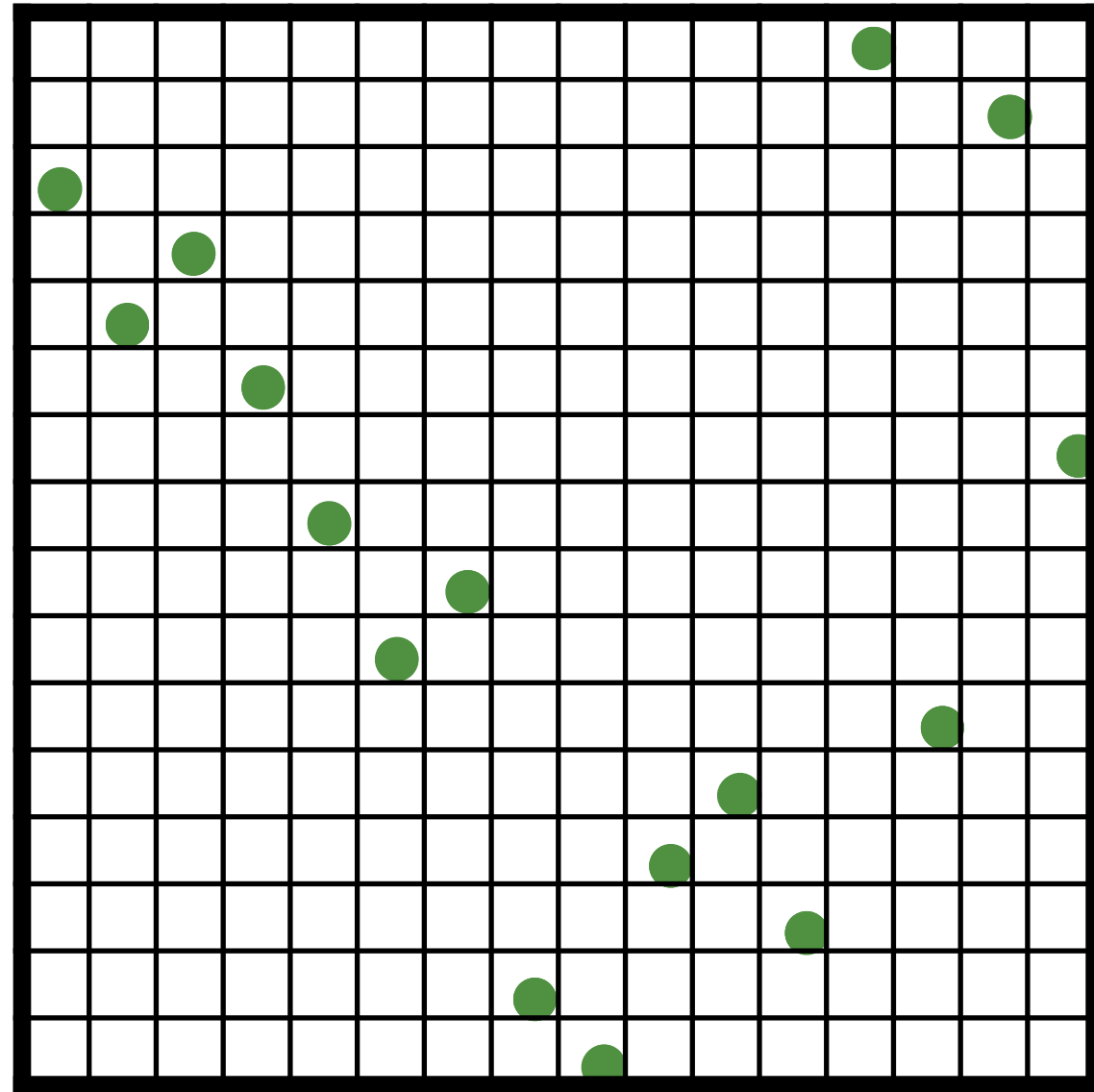
Latin Hypercube Sampler (N-rooks)



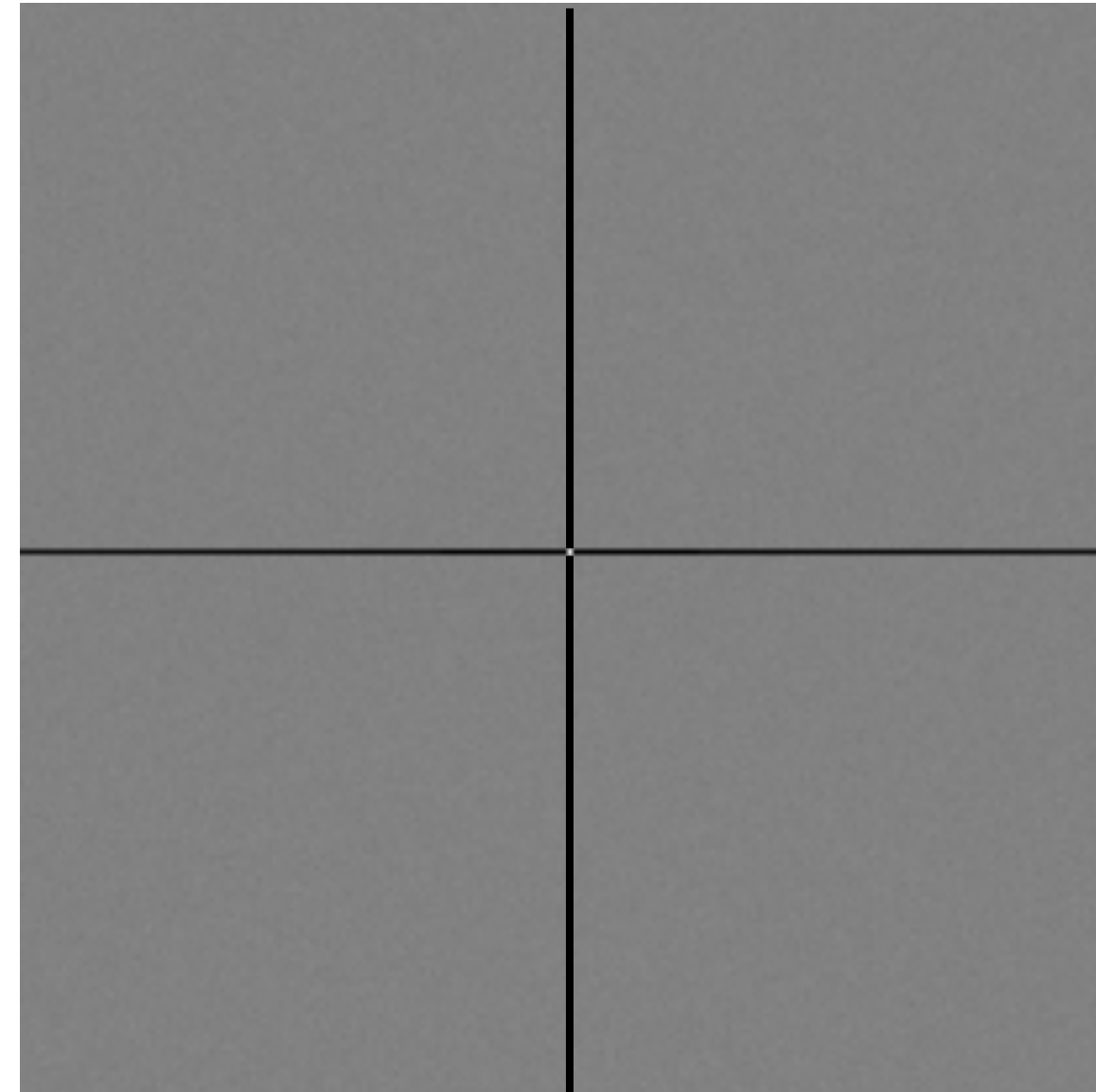
Latin Hypercube Sampler (N-rooks)



Anisotropic Sampling Power Spectra

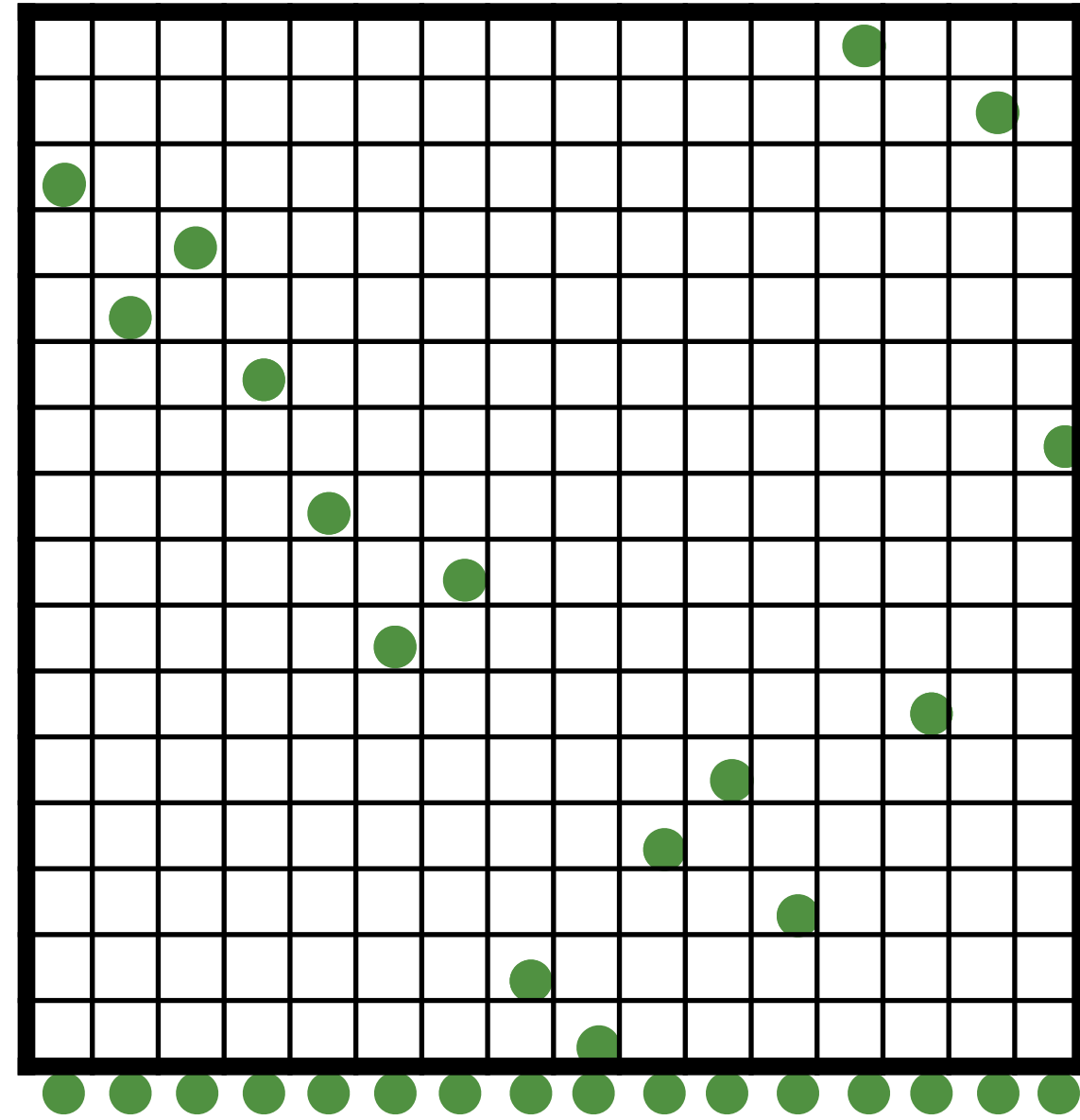


N-rooks /
Latin Hypercube

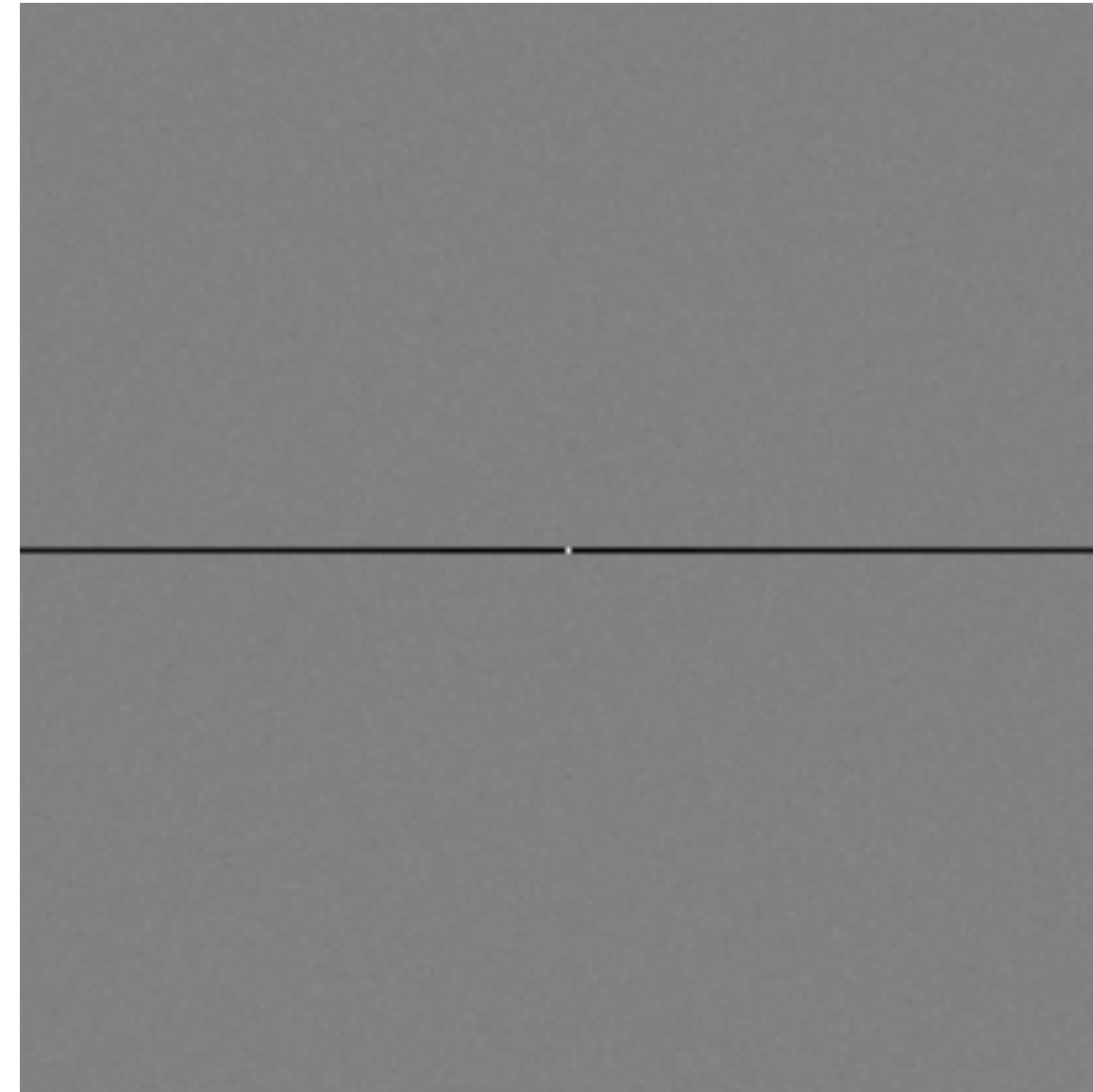


N-rooks
Spectrum

Anisotropic Sampling Power Spectra

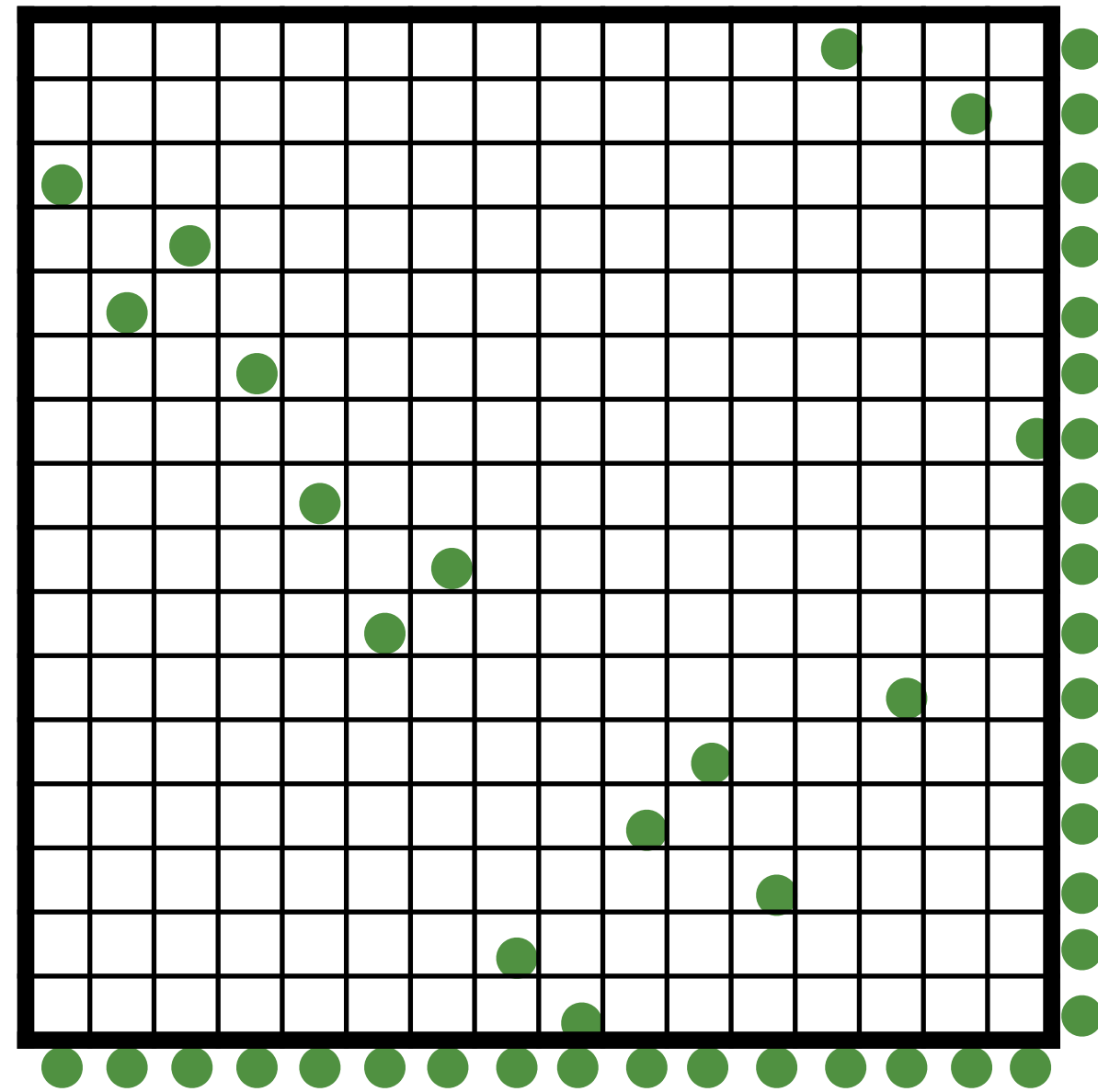


N-rooks /
Latin Hypercube

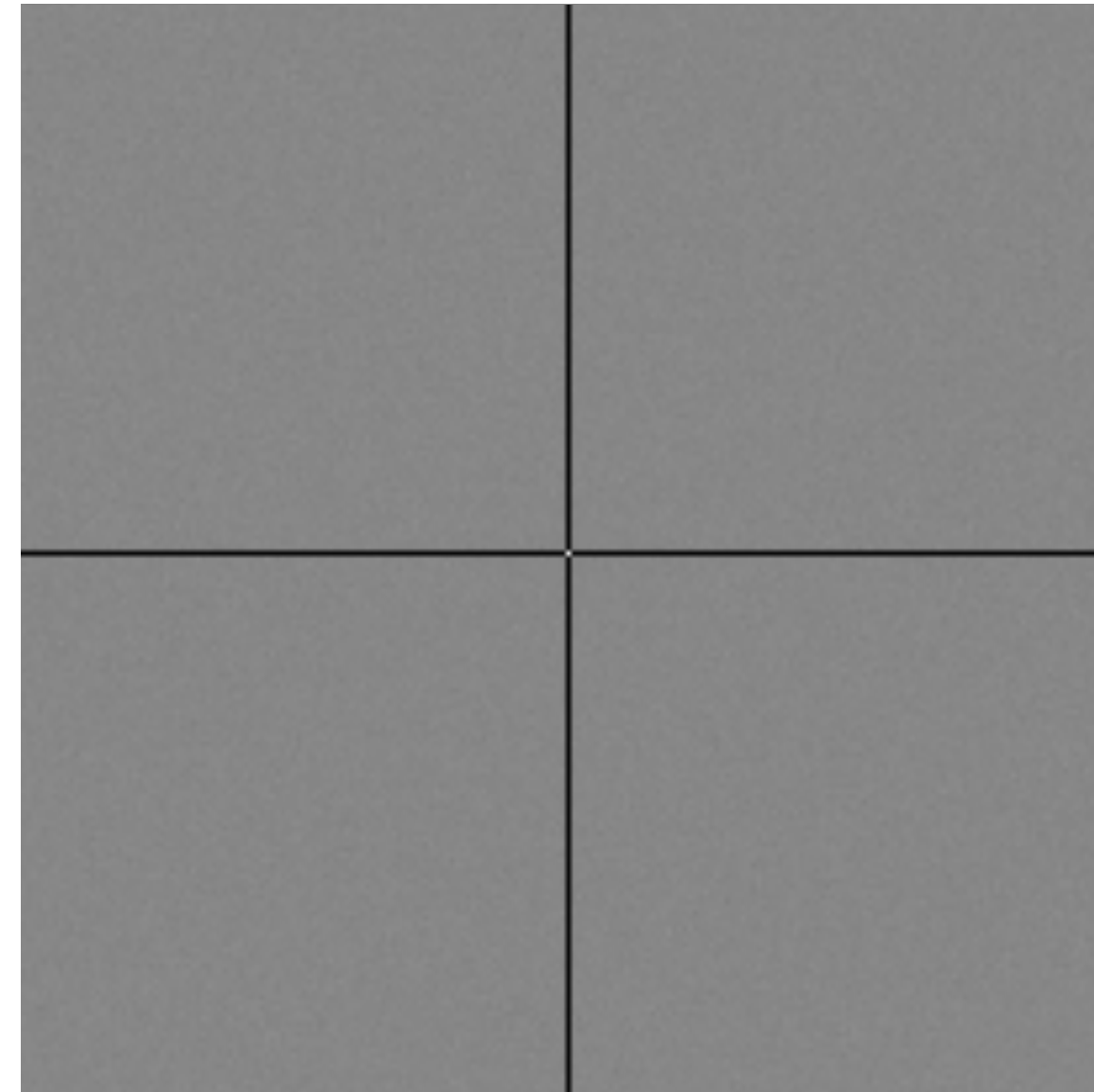


Spectrum

Anisotropic Sampling Power Spectra

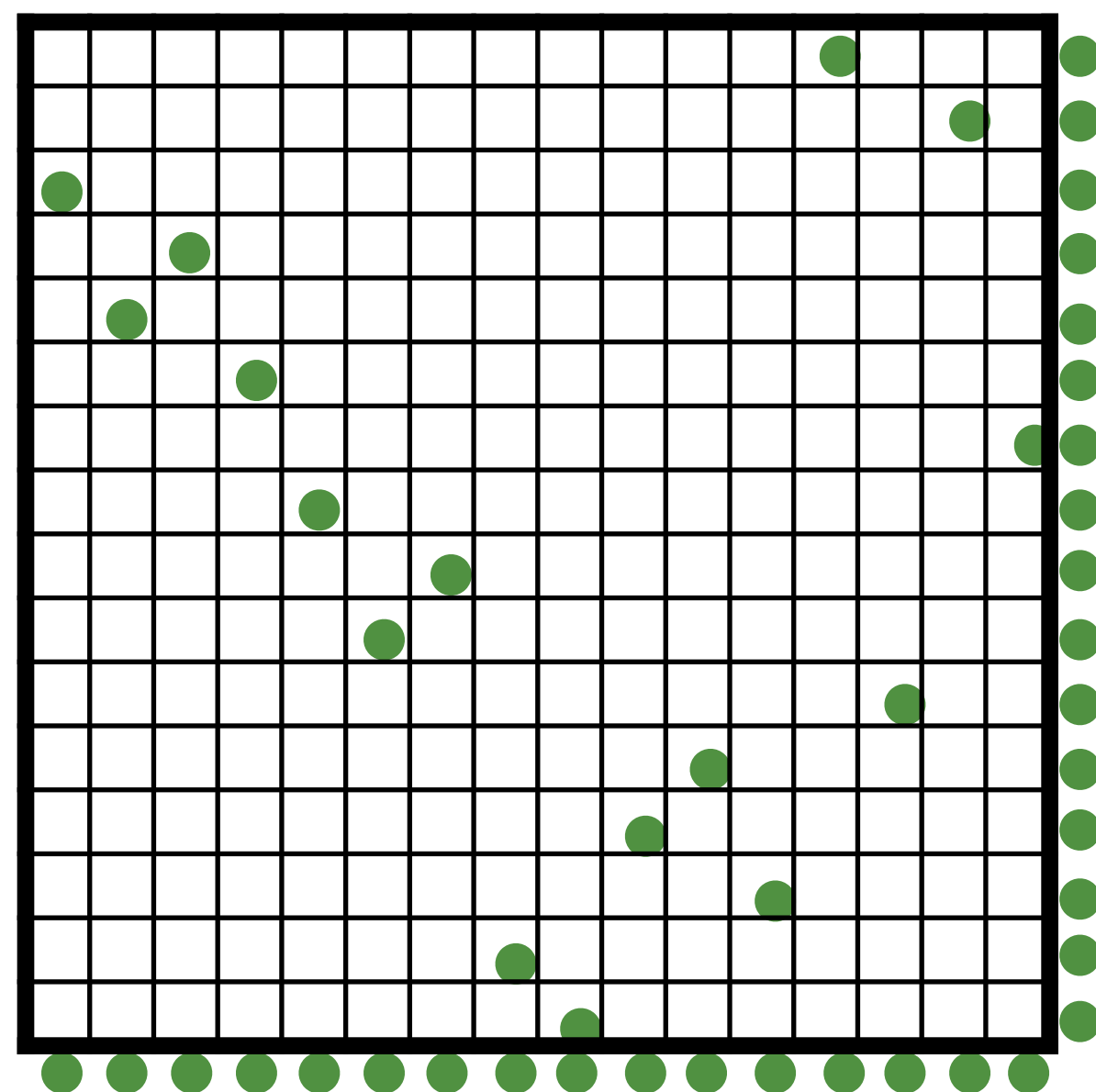


N-rooks /
Latin Hypercube

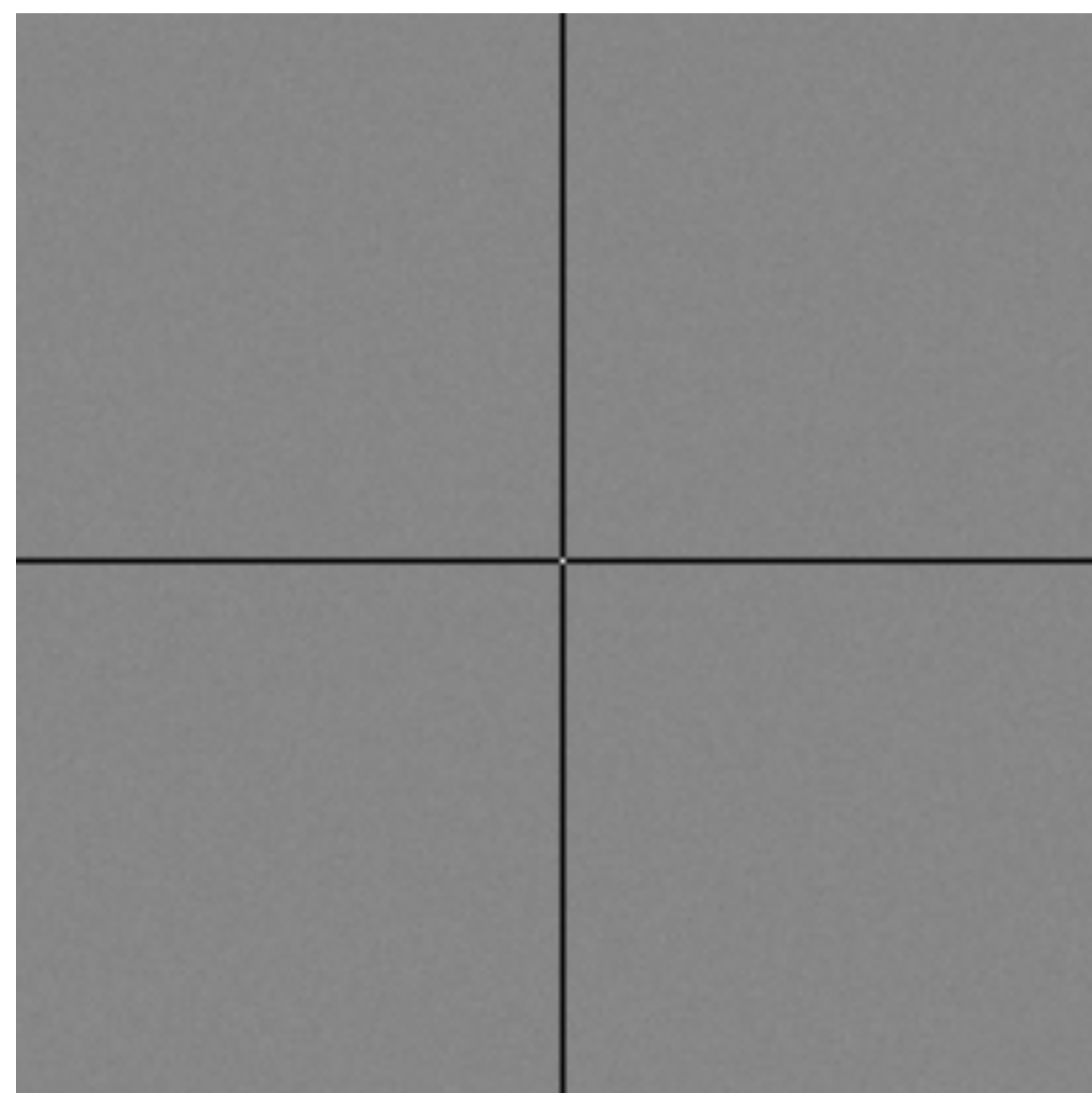


N-rooks
Spectrum

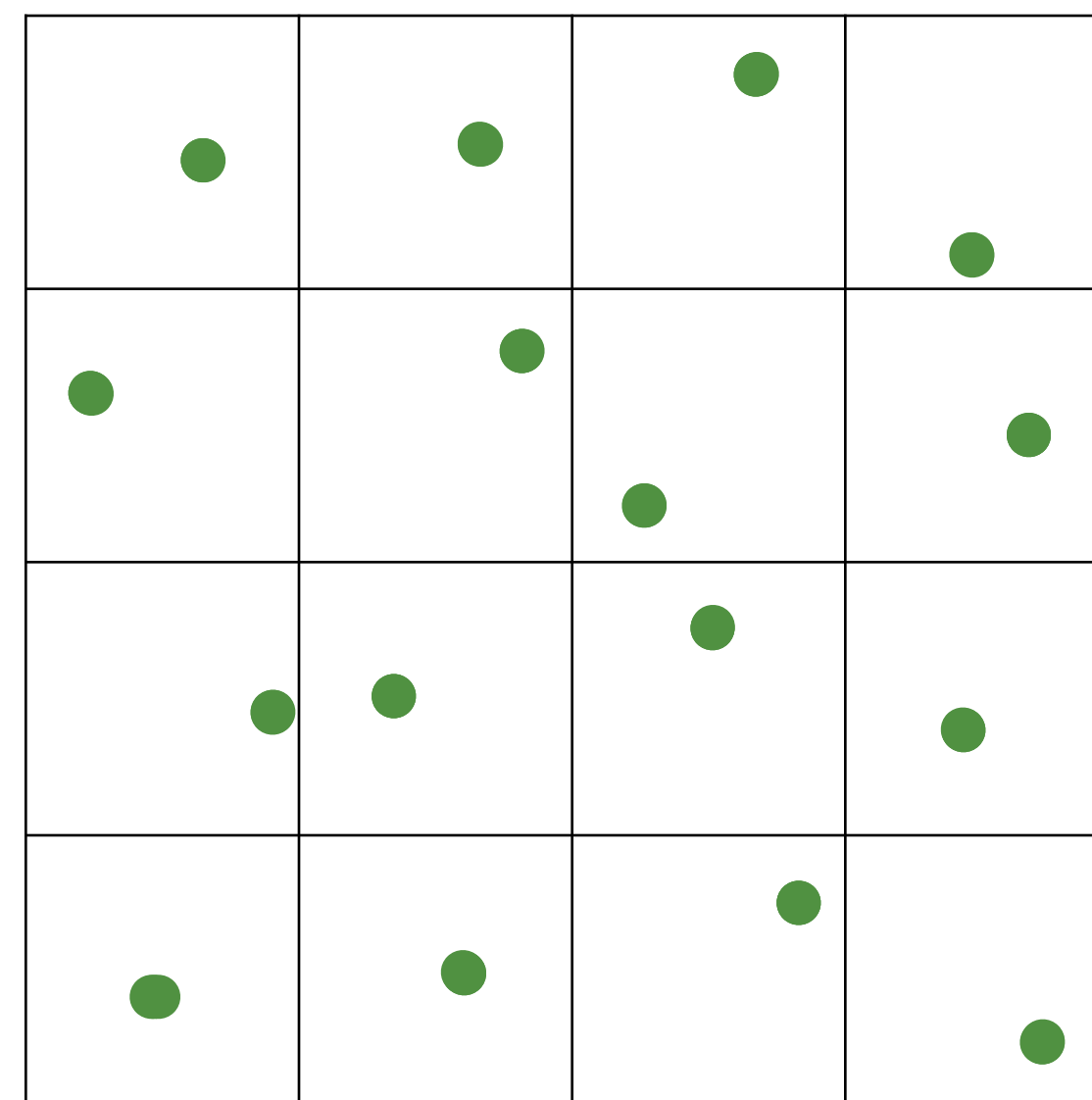
Anisotropic Sampling Power Spectra



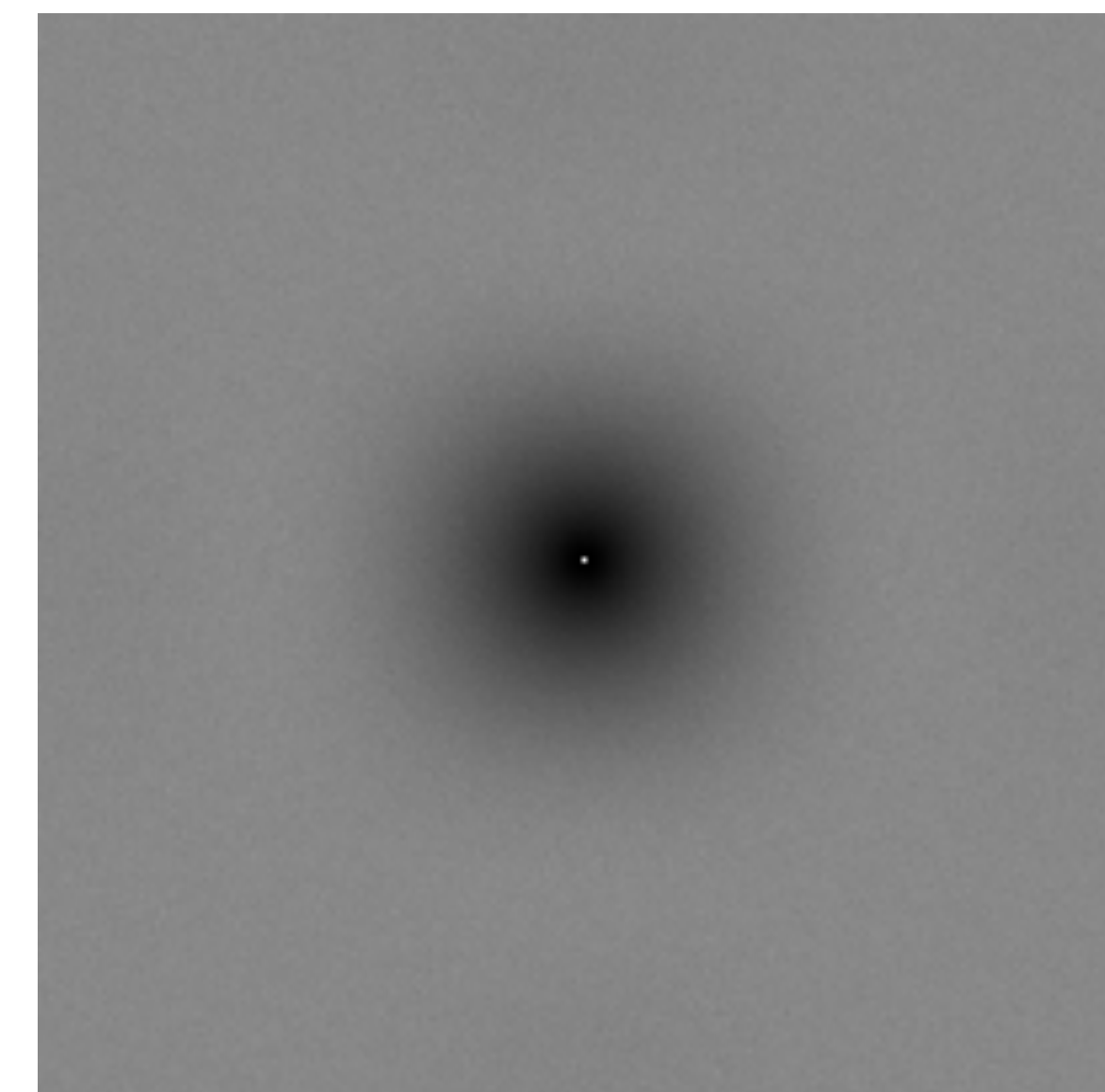
N-rooks /
Latin Hypercube



N-rooks
Spectrum

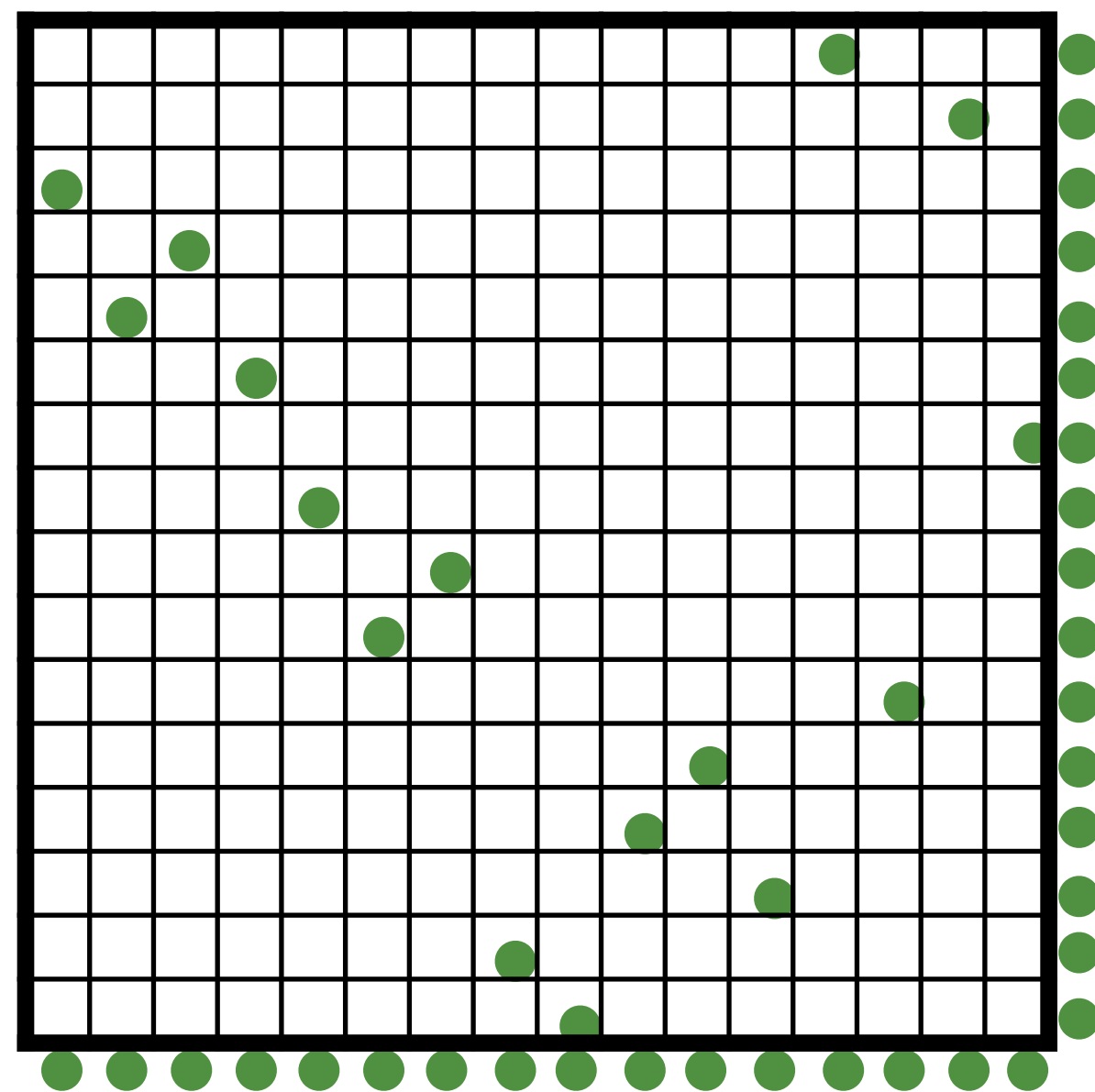


Jitter

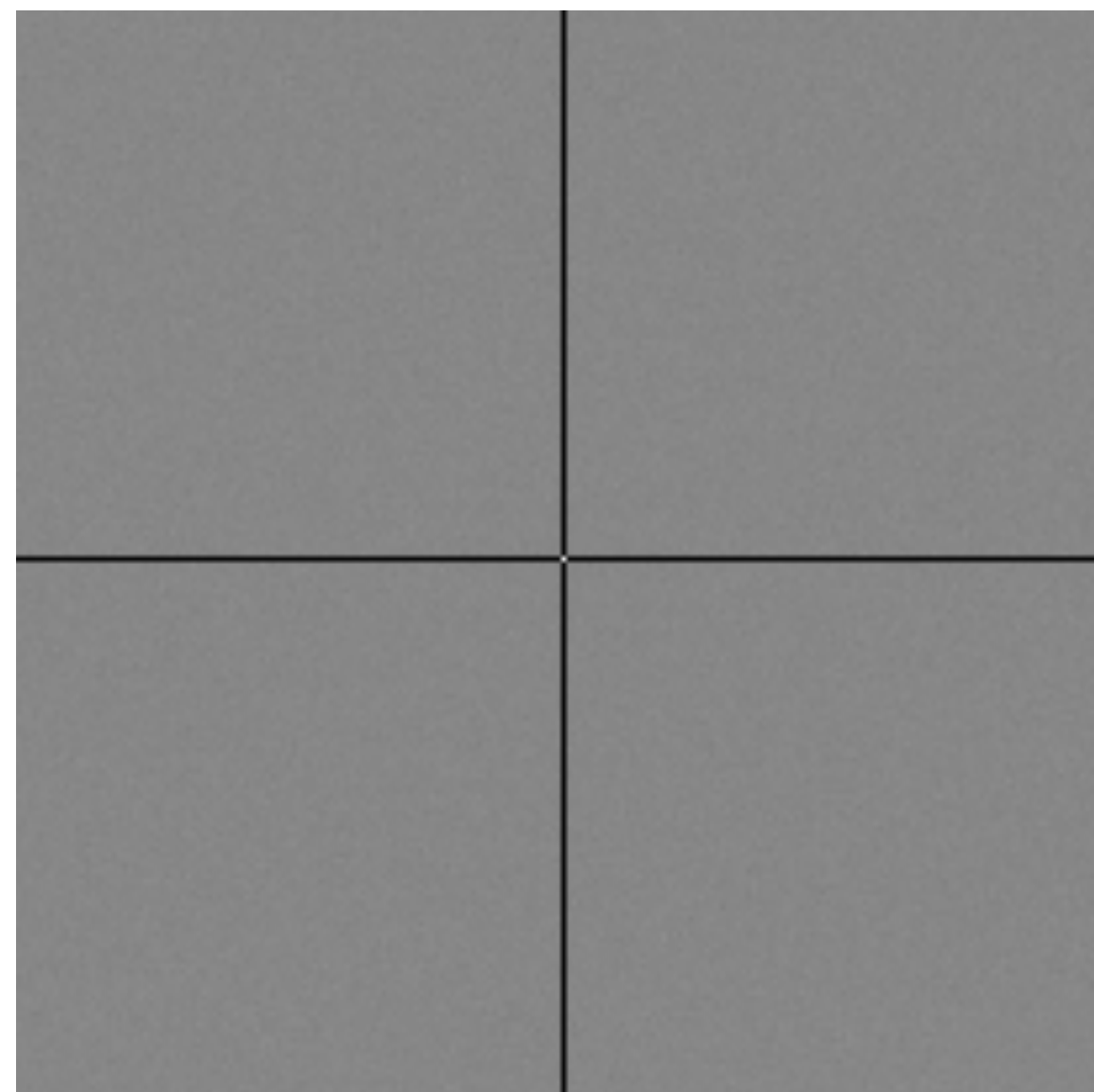


Jitter
Spectrum

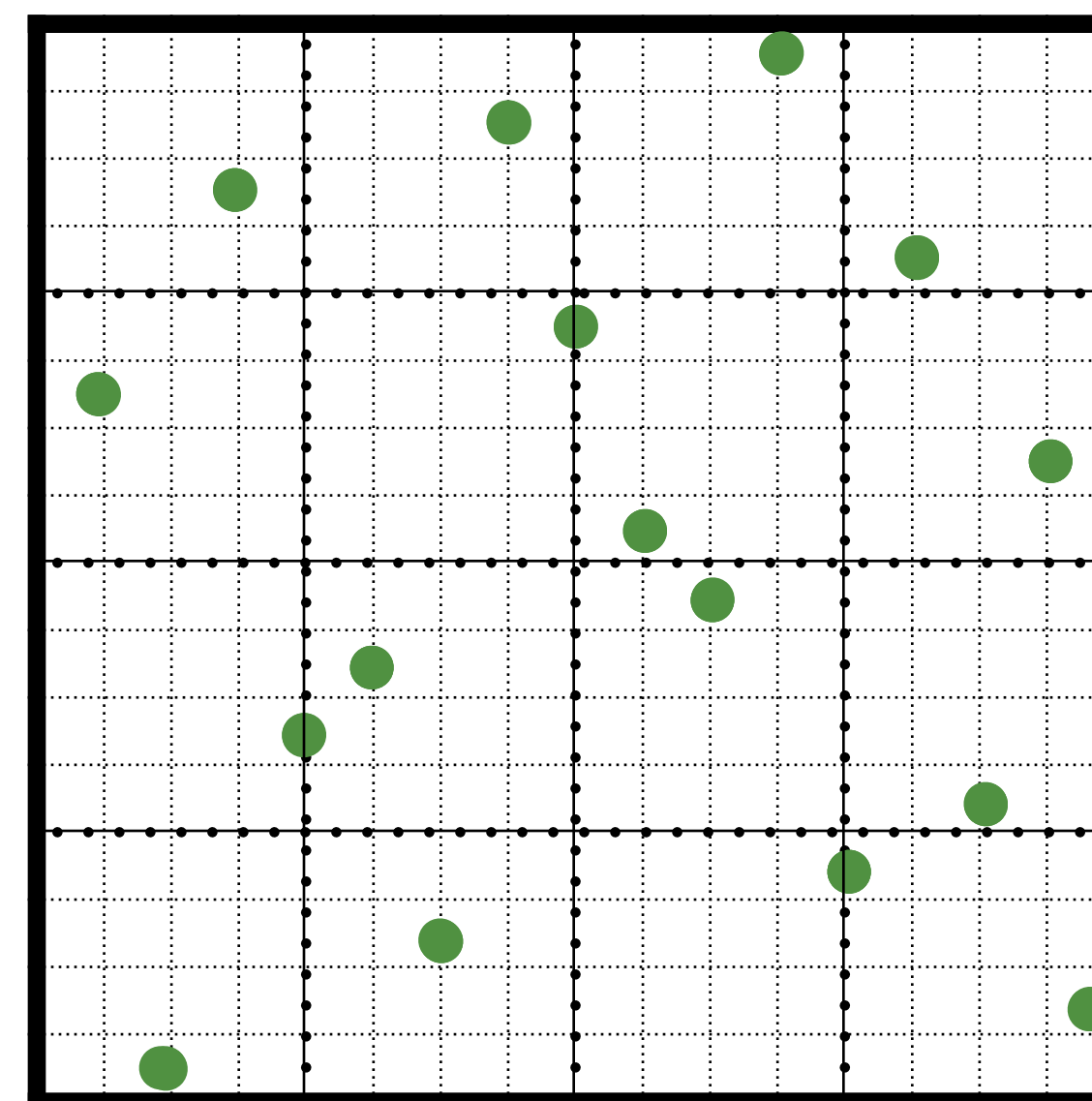
Anisotropic Sampling Power Spectra



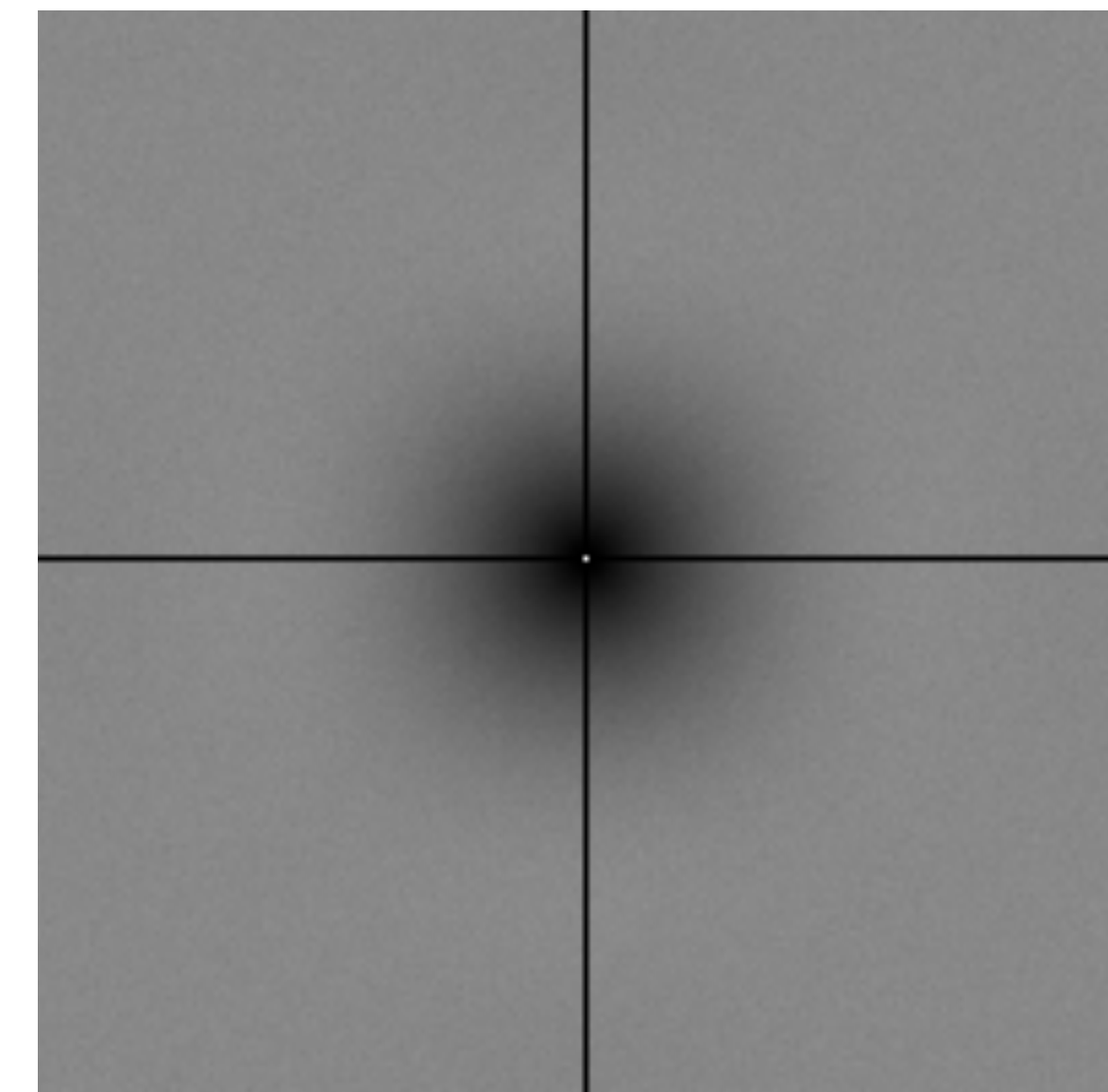
N-rooks /
Latin Hypercube



N-rooks
Spectrum



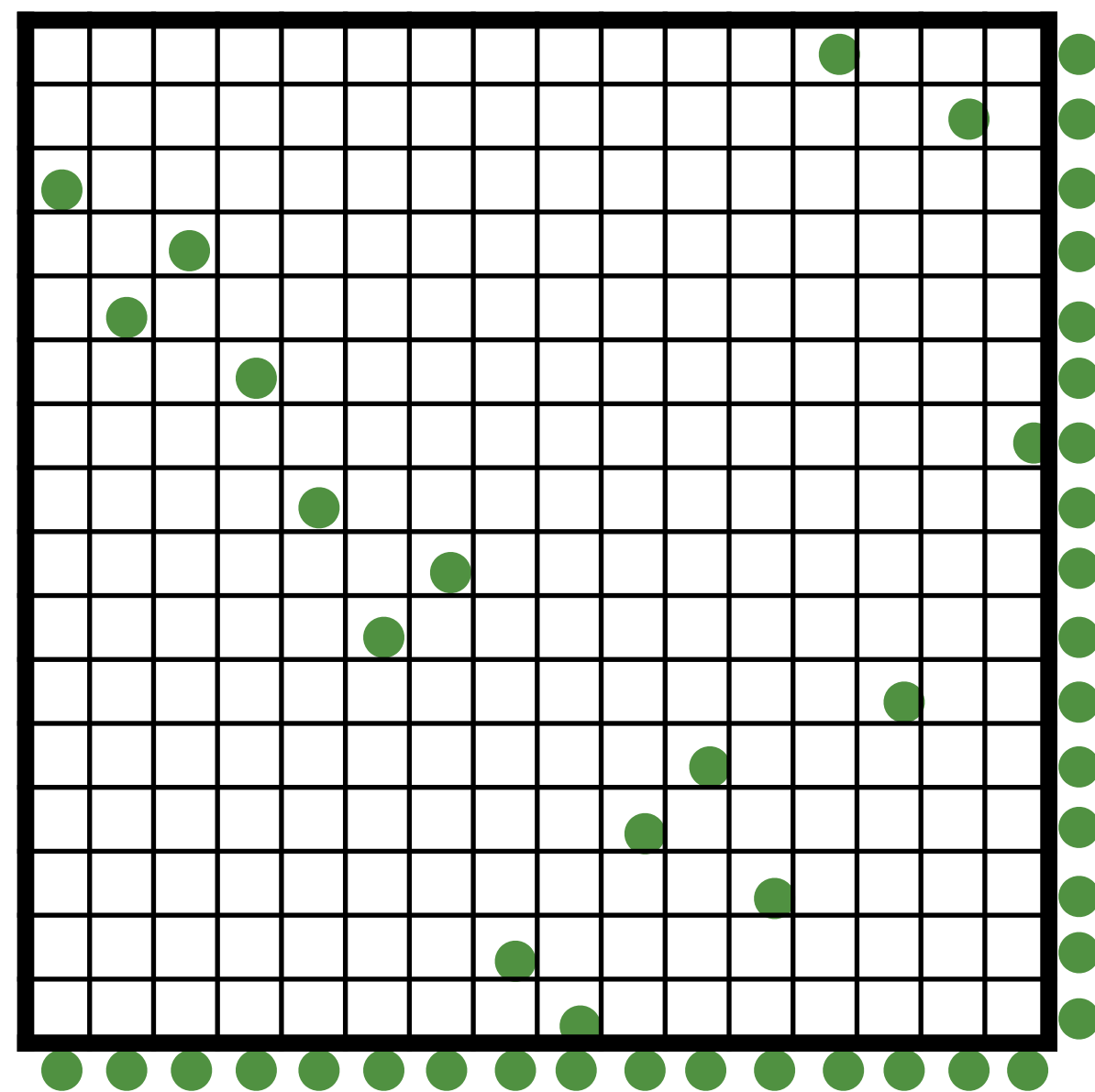
Multi-Jitter



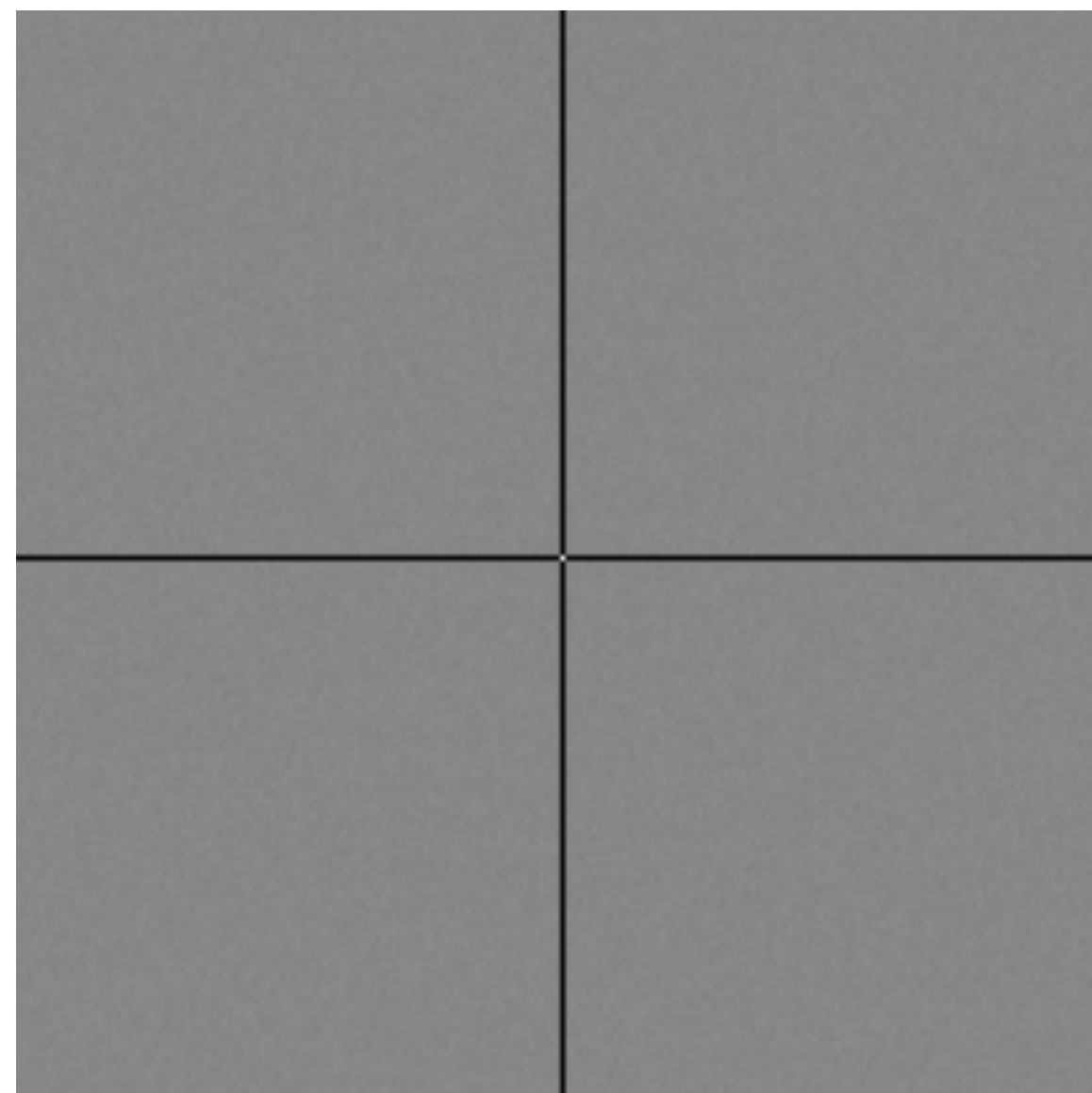
Multi-Jitter
Spectrum

Chiu et al. [1993]

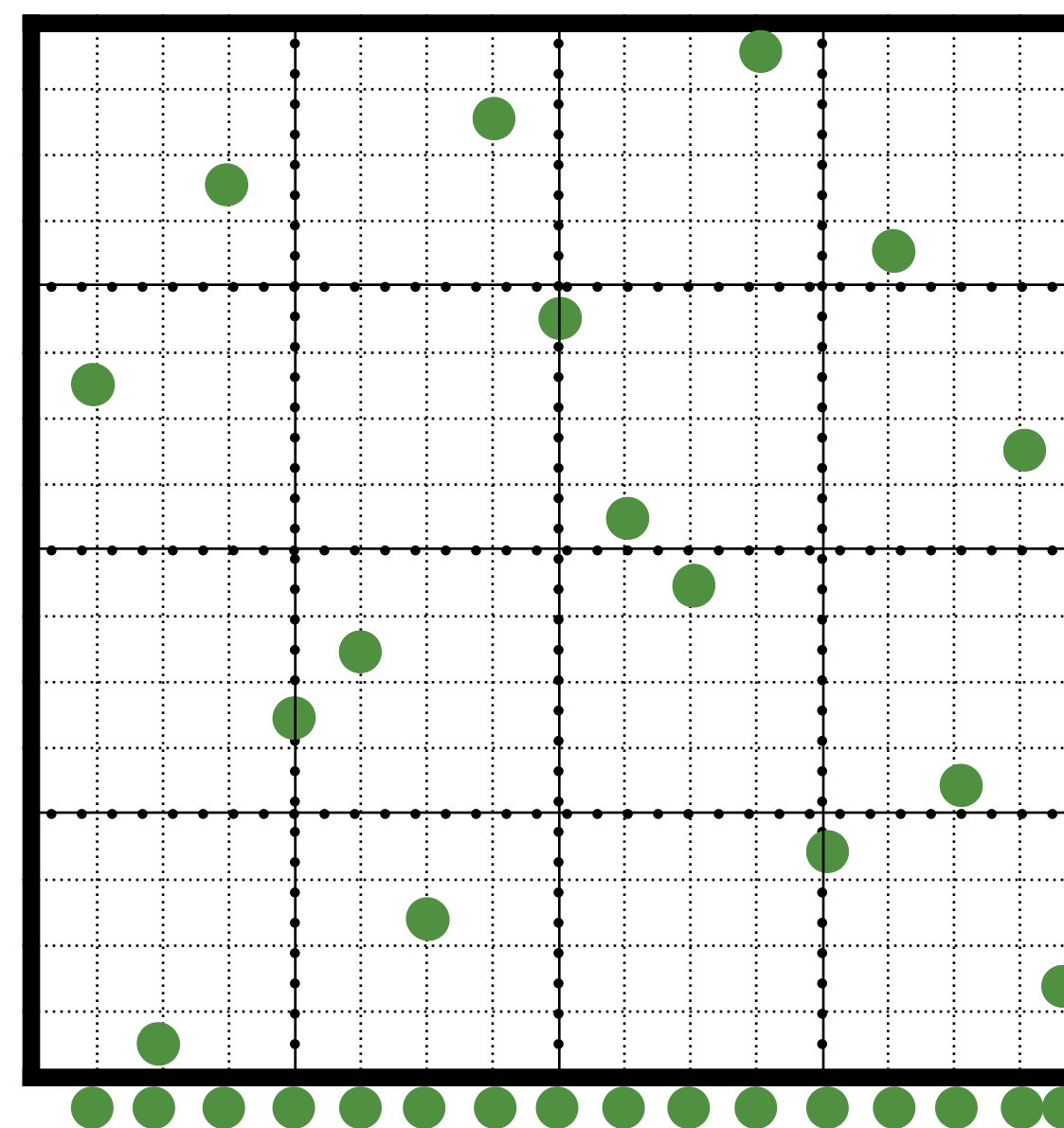
Anisotropic Sampling Power Spectra



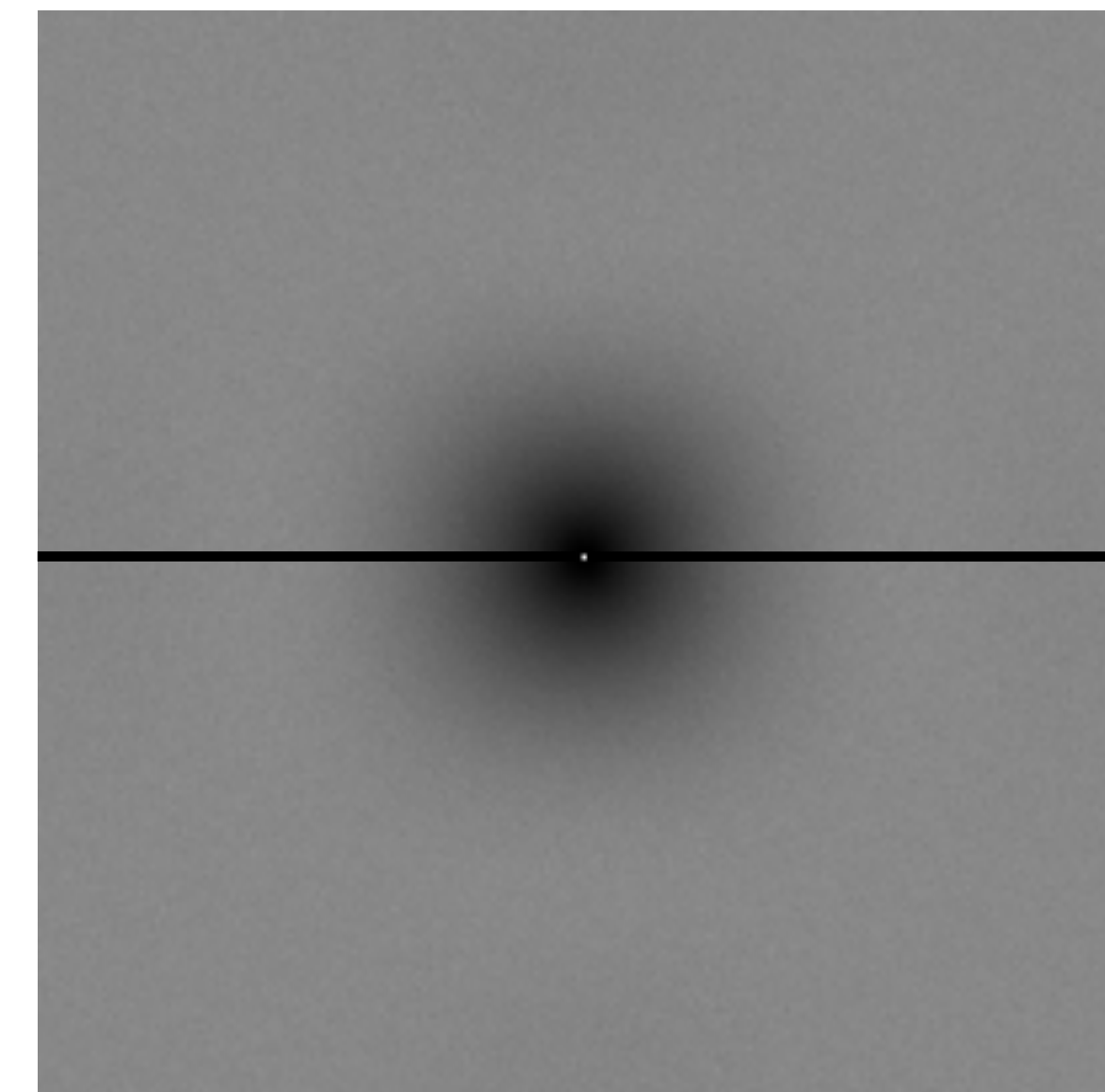
N-rooks /
Latin Hypercube



N-rooks
Spectrum



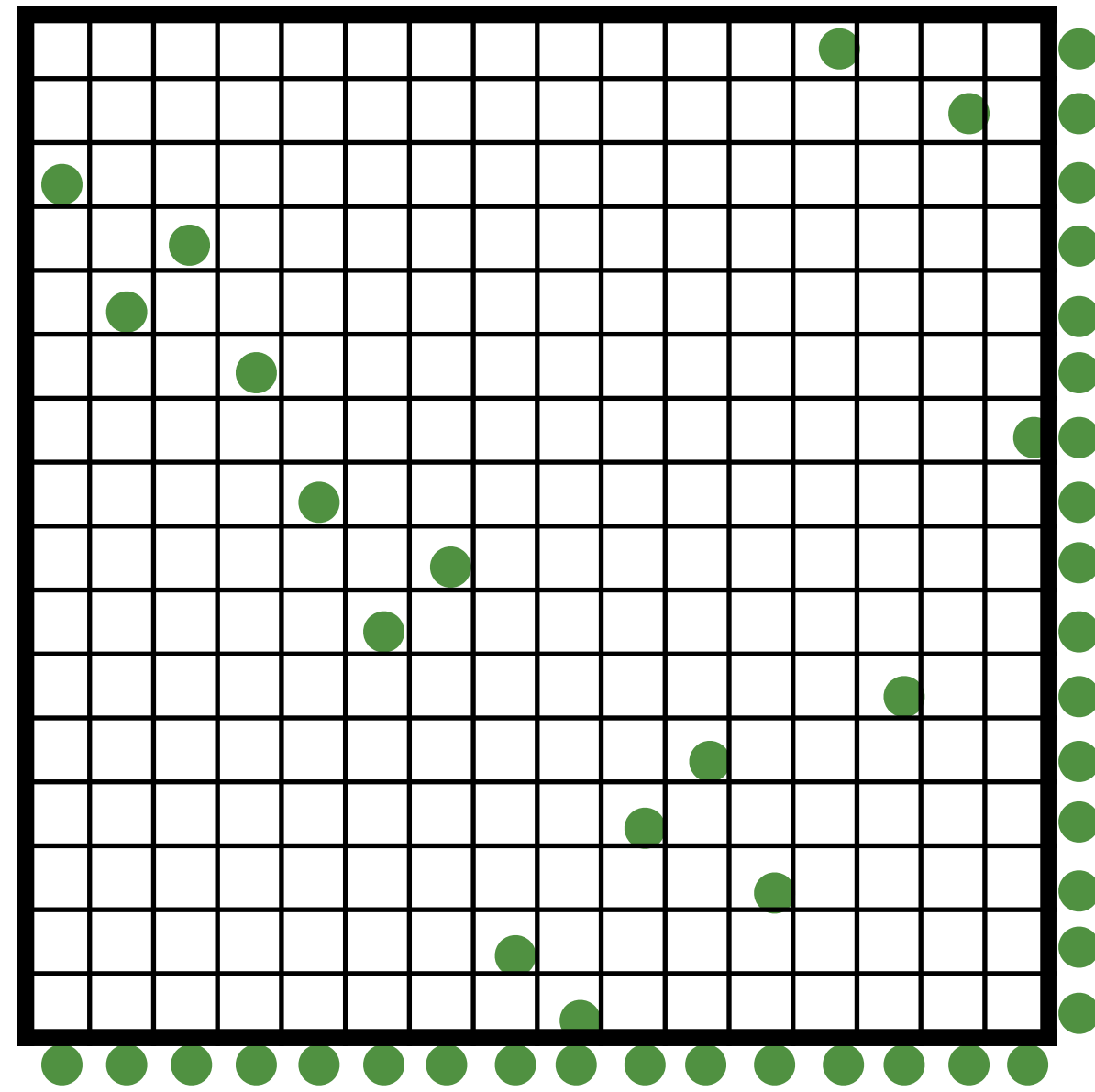
Multi-jitter



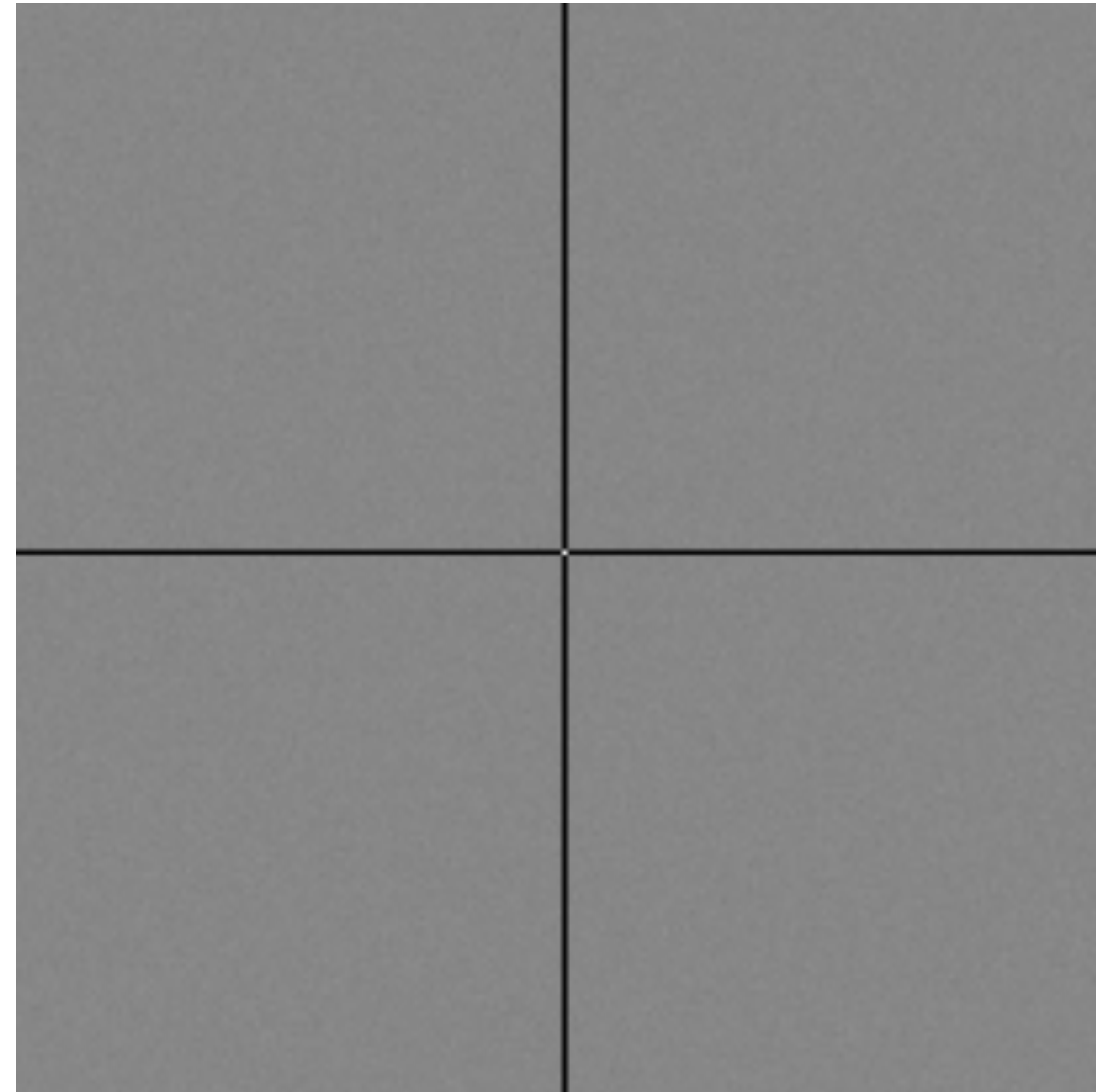
Multi-Jitter
Spectrum

Chiu et al. [1993]

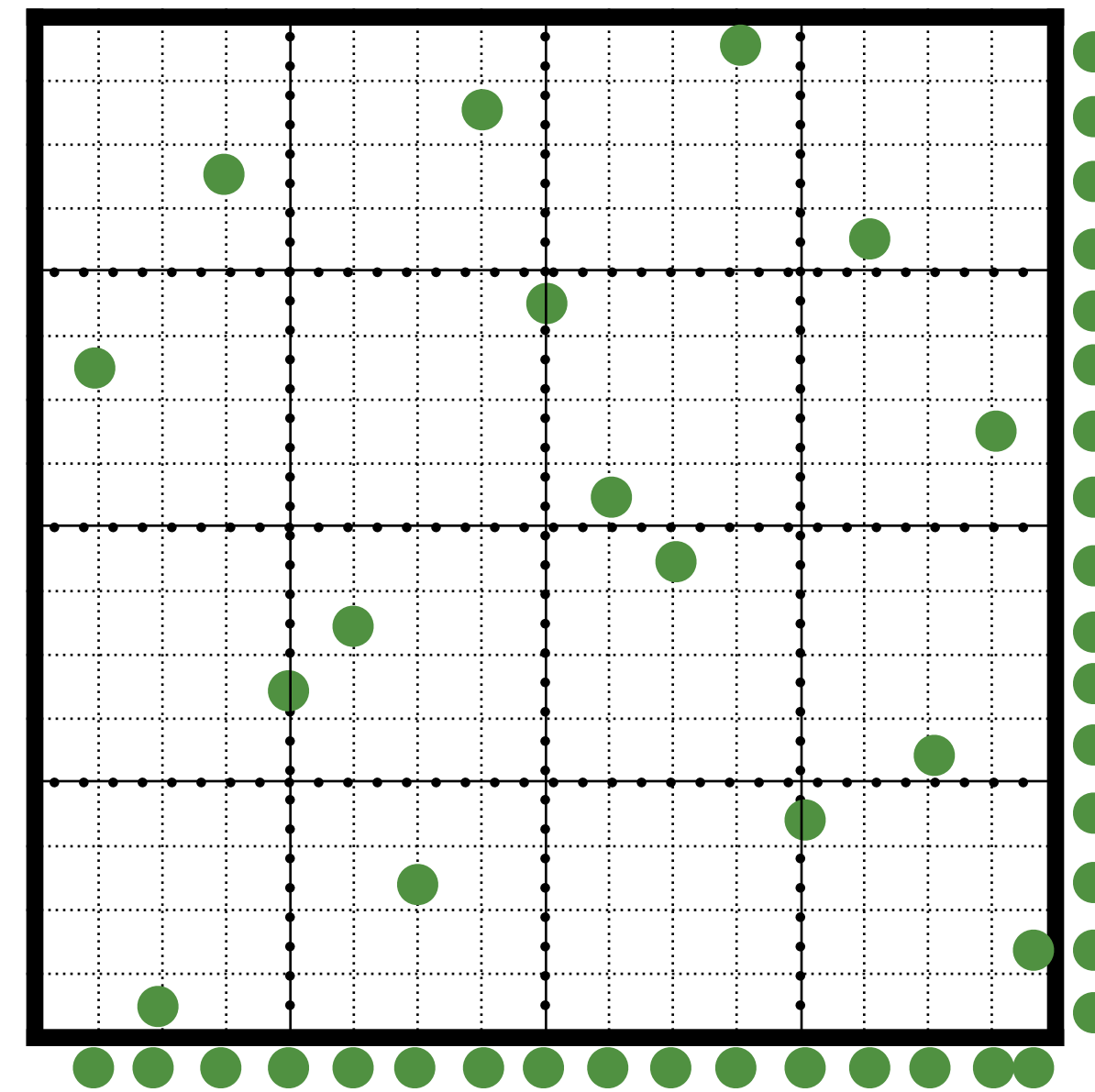
Anisotropic Sampling Power Spectra



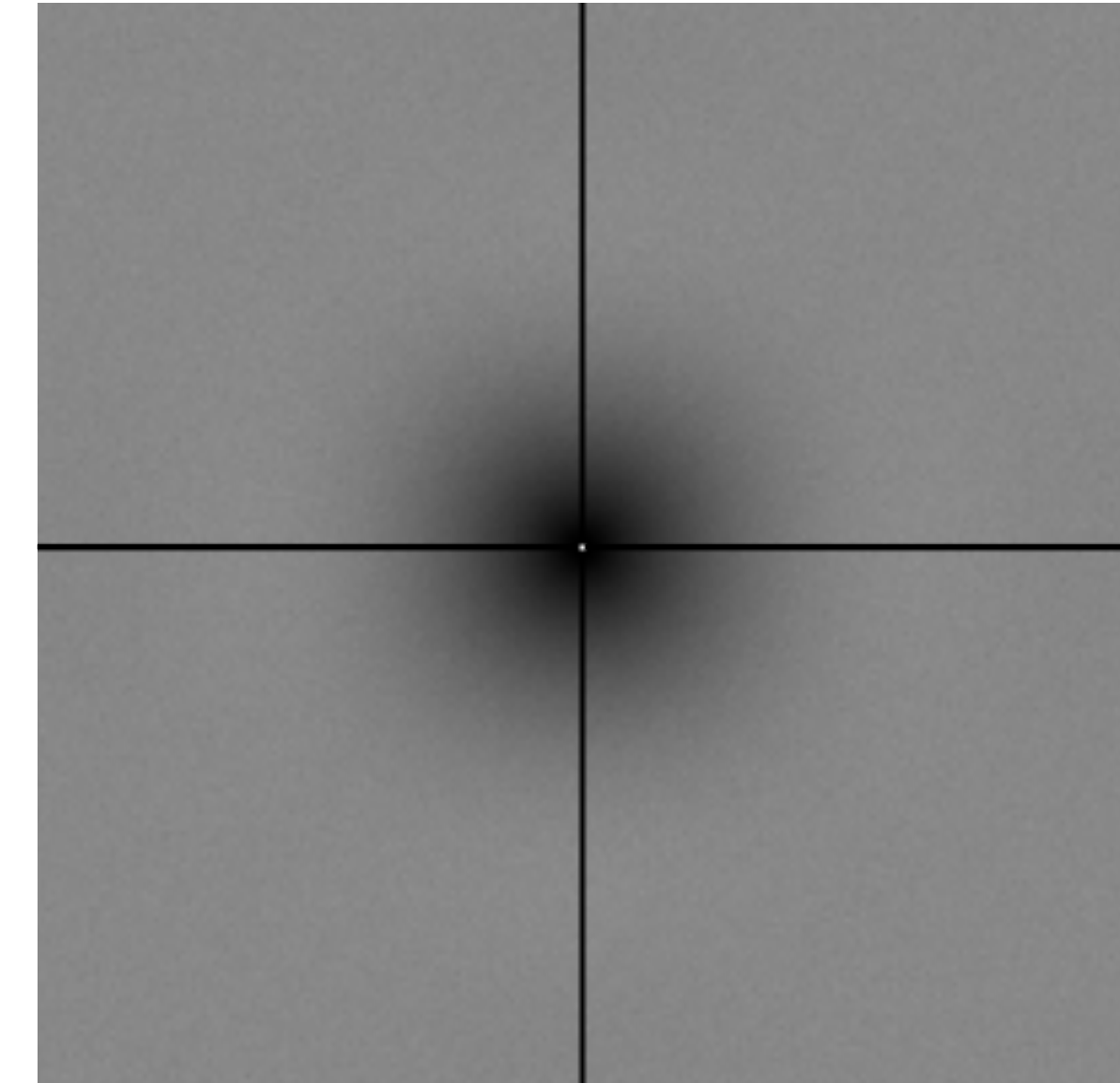
N-rooks /
Latin Hypercube



N-rooks
Spectrum



Multi-jitter

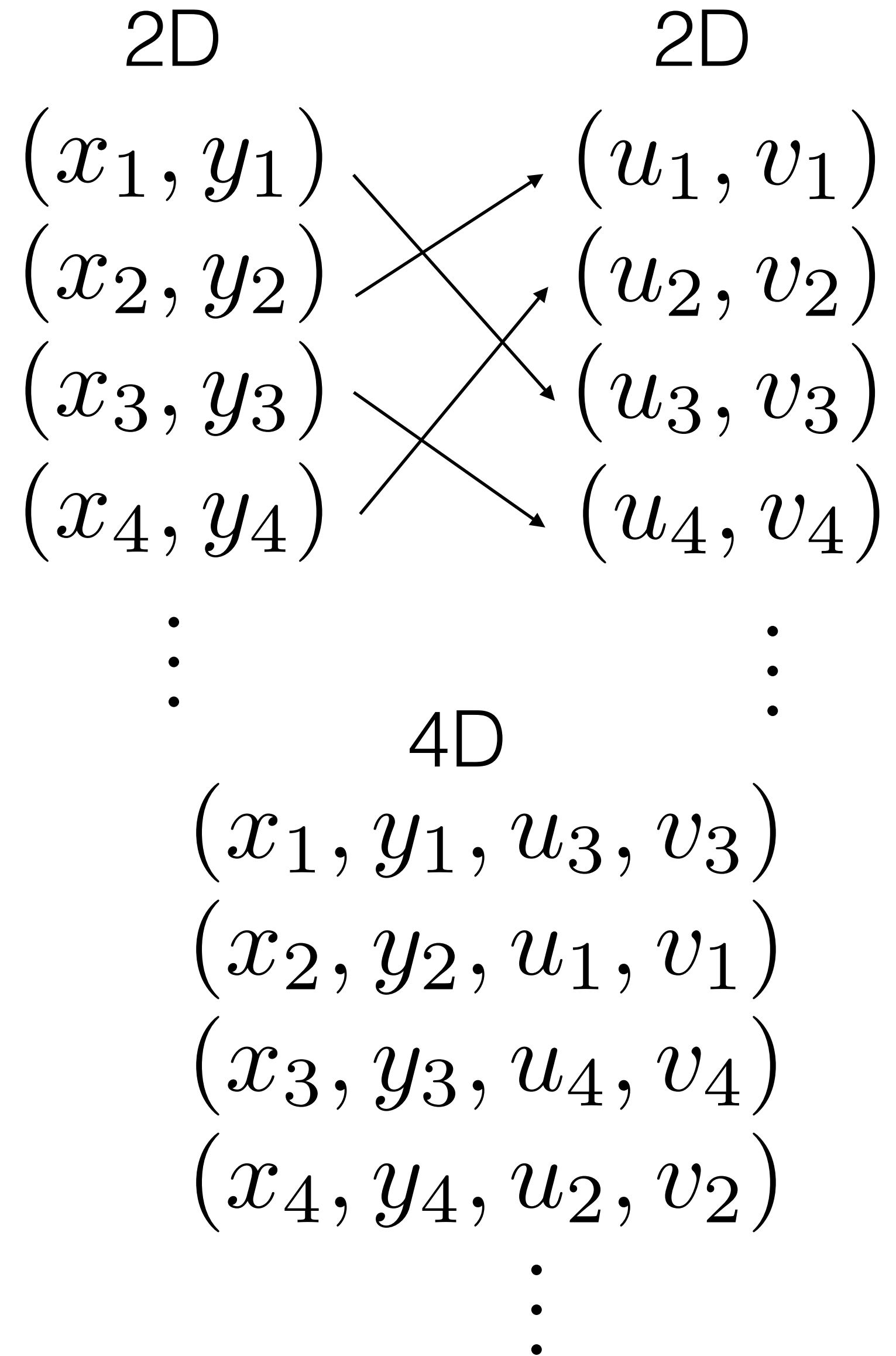
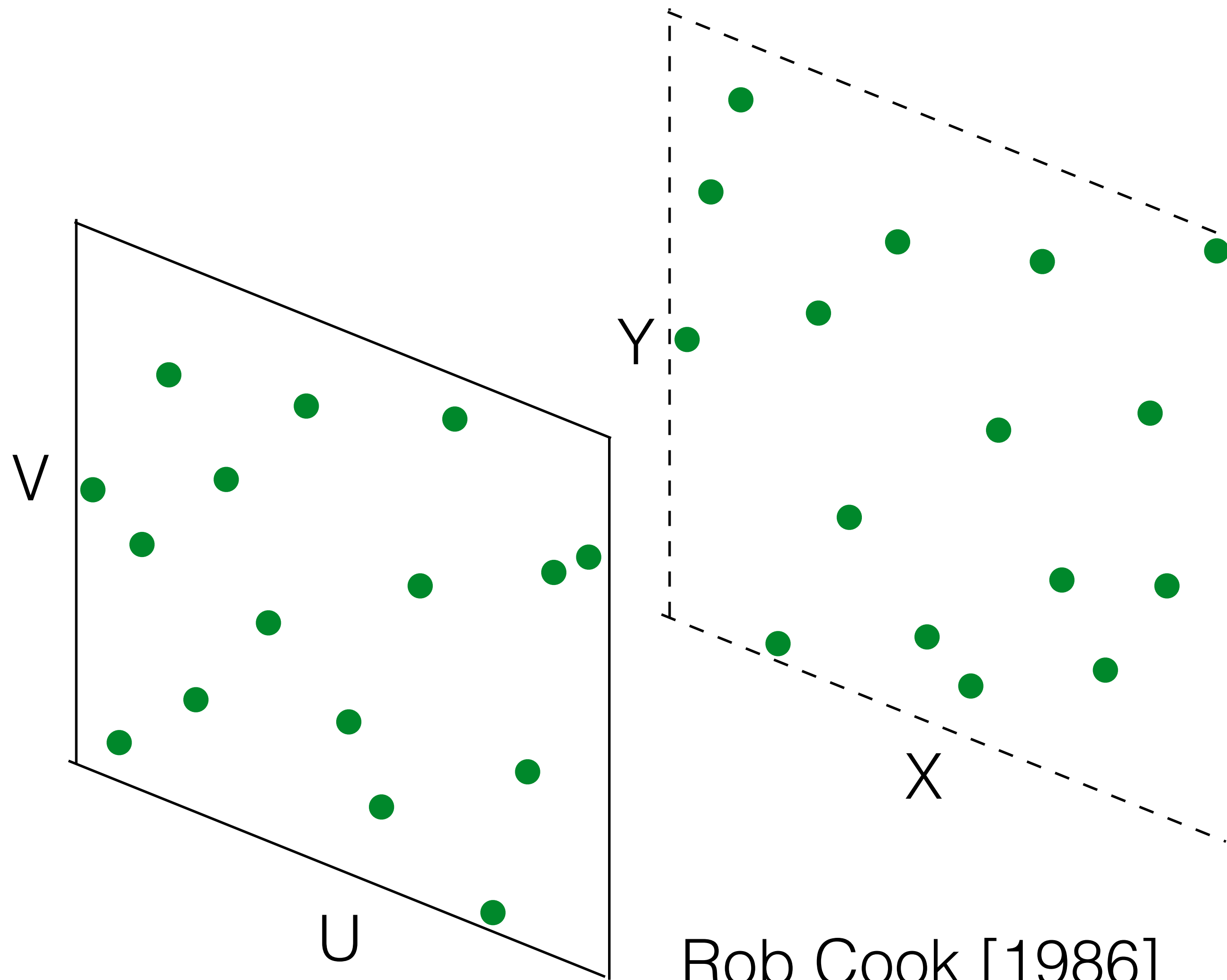


Multi-Jitter
Spectrum

Chiu et al. [1993]

Sampling in Higher Dimensions

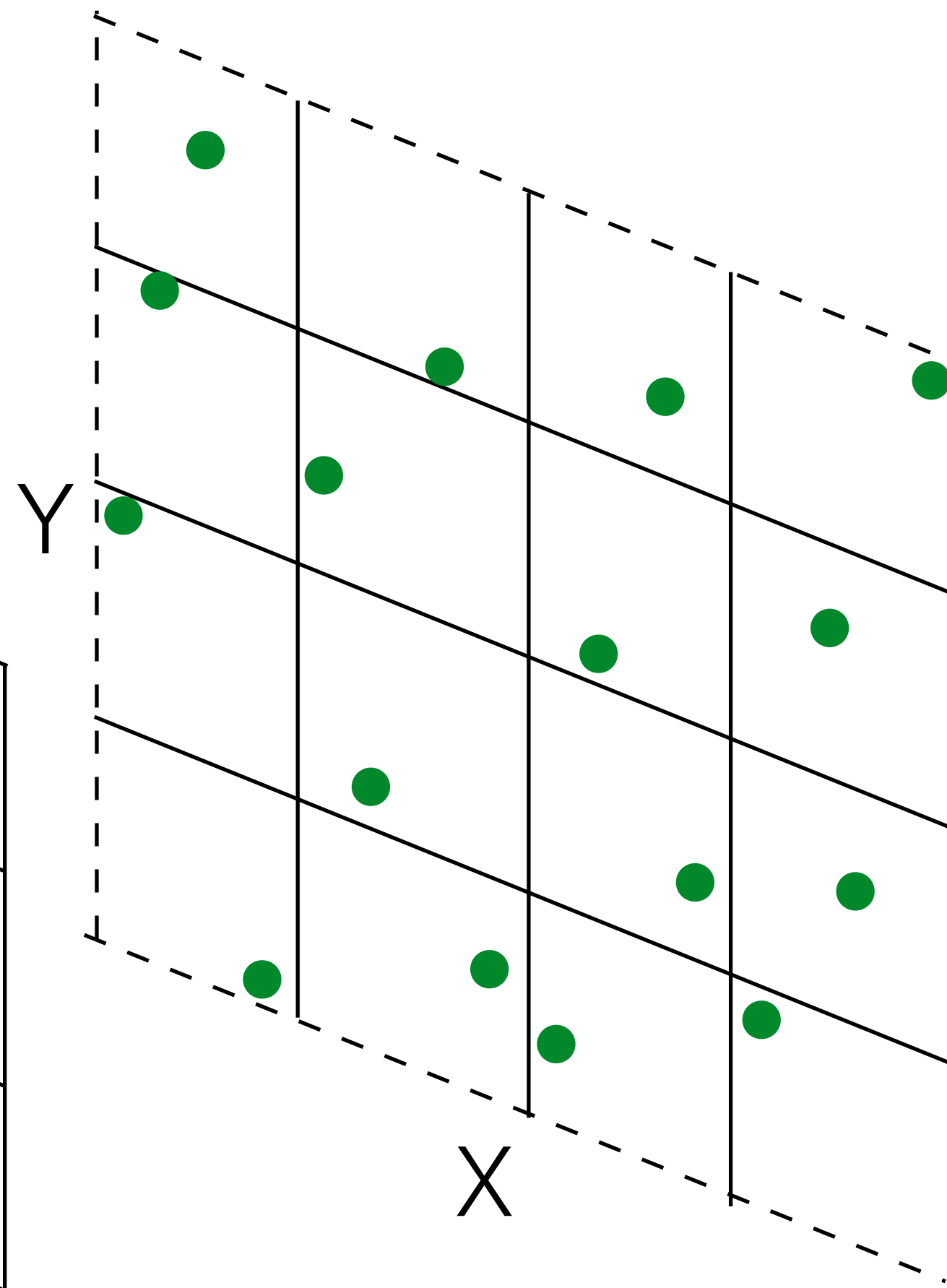
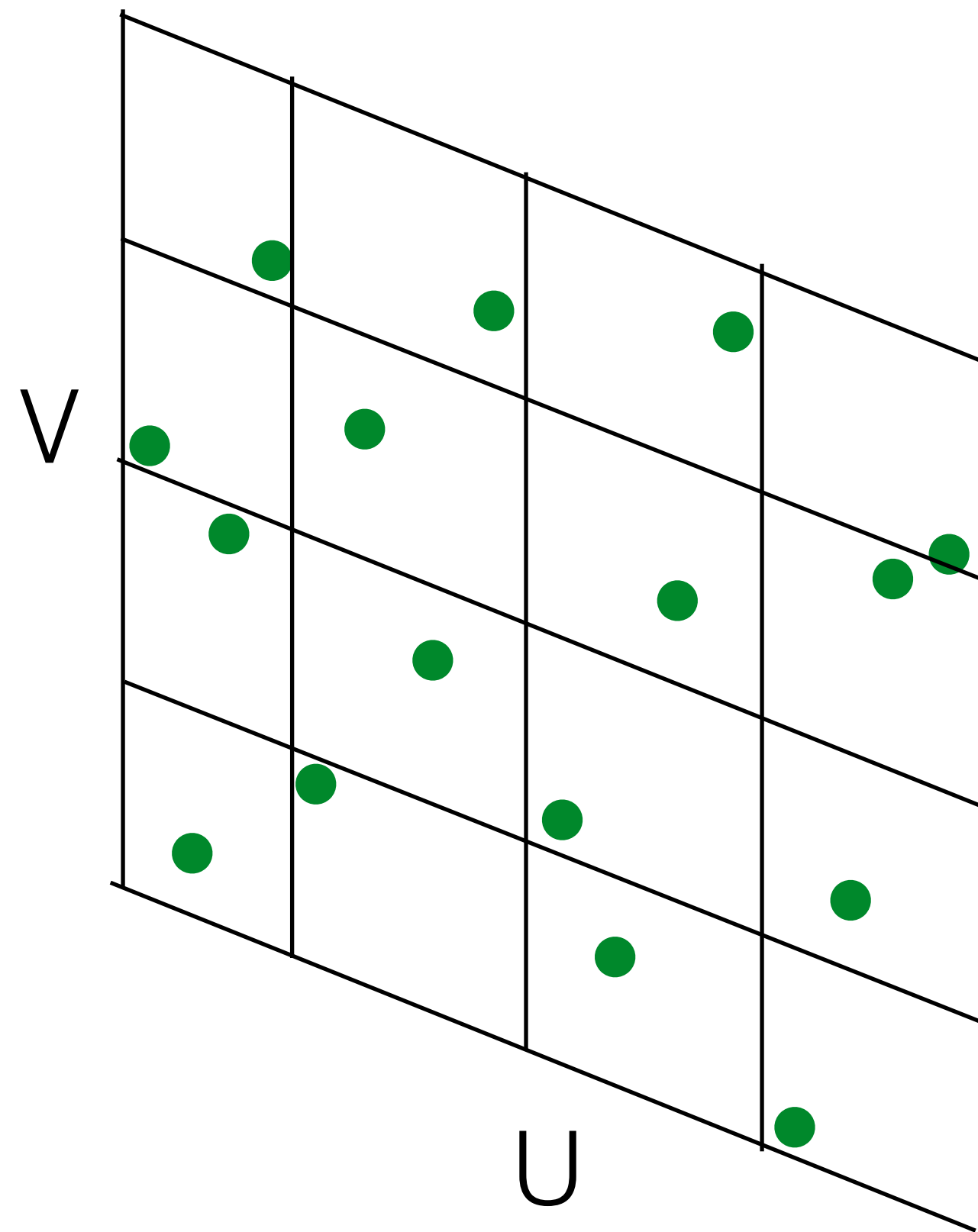
4D Sampling



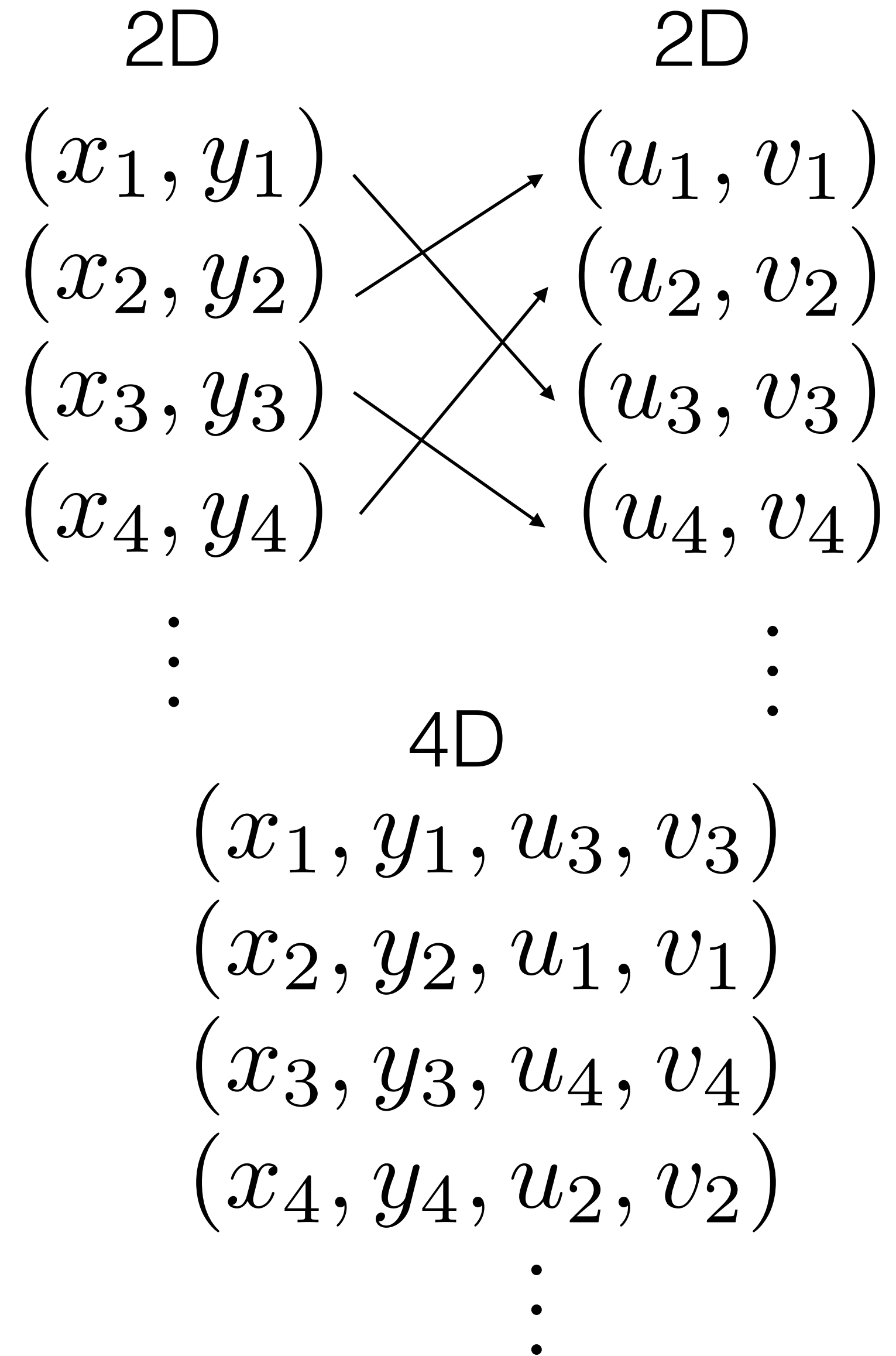
Rob Cook [1986]

4D Sampling

Uncorrelated
Jitter

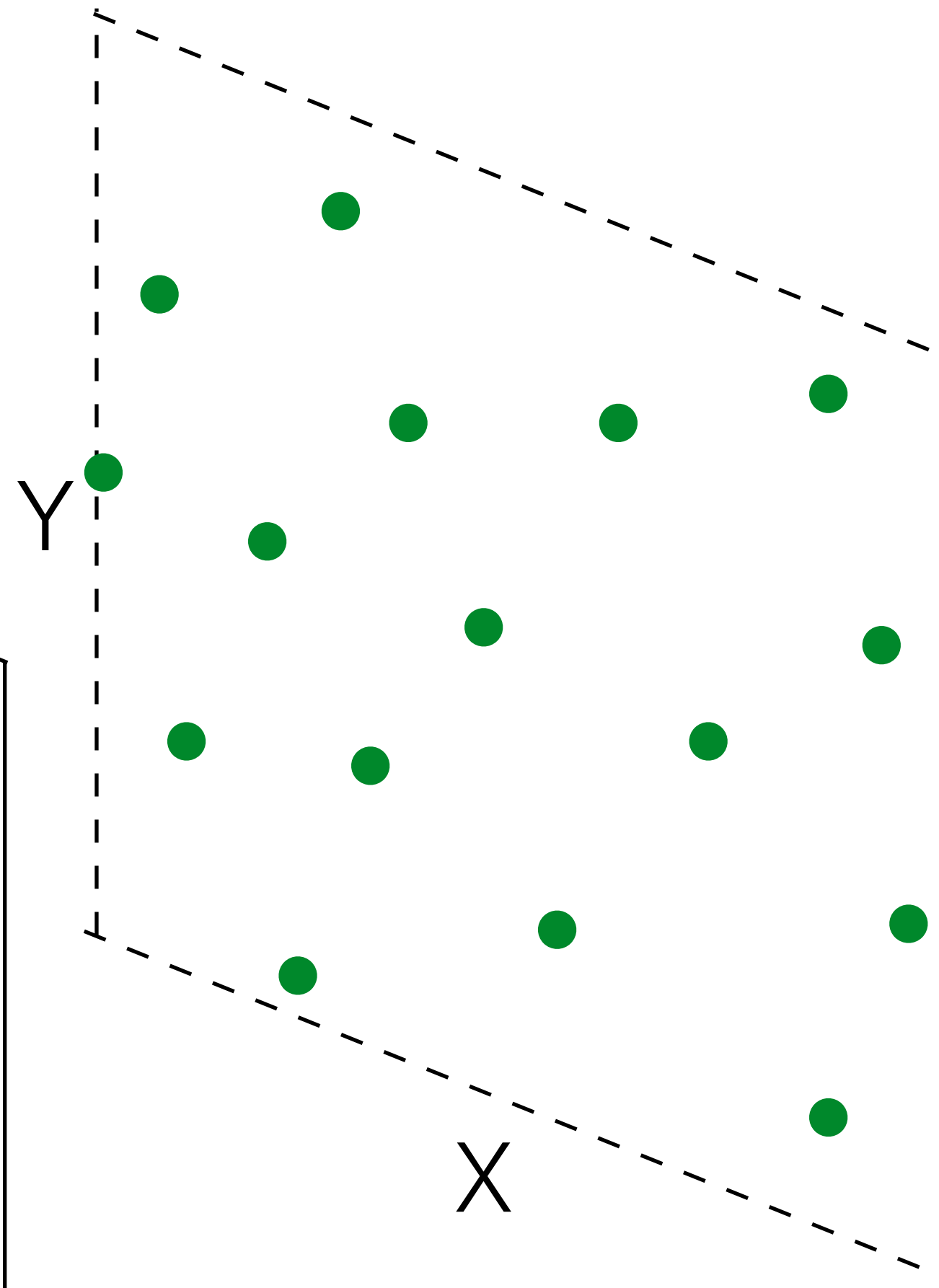
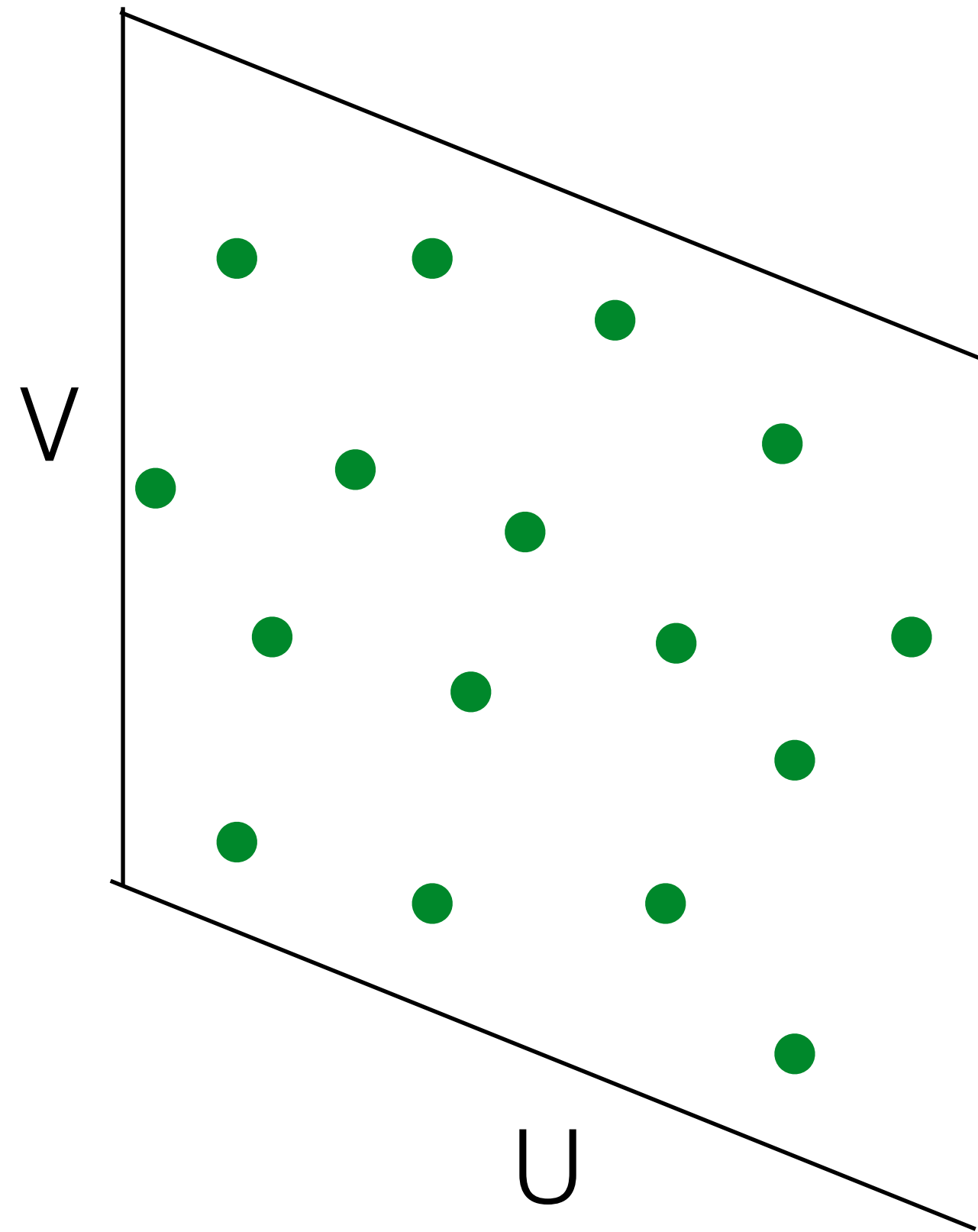


Rob Cook [1986]

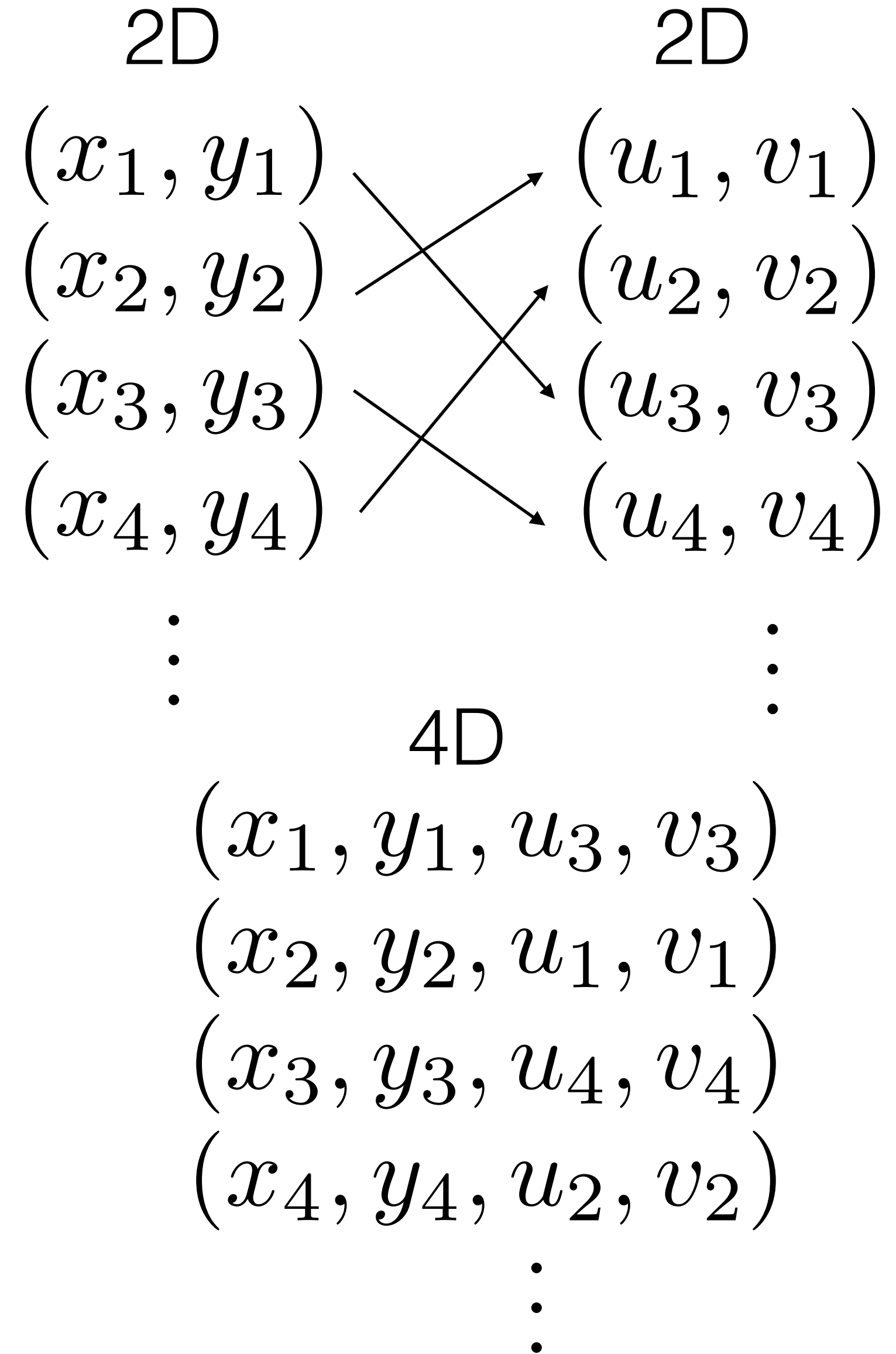


4D Sampling

Uncorrelated
Poisson Disk

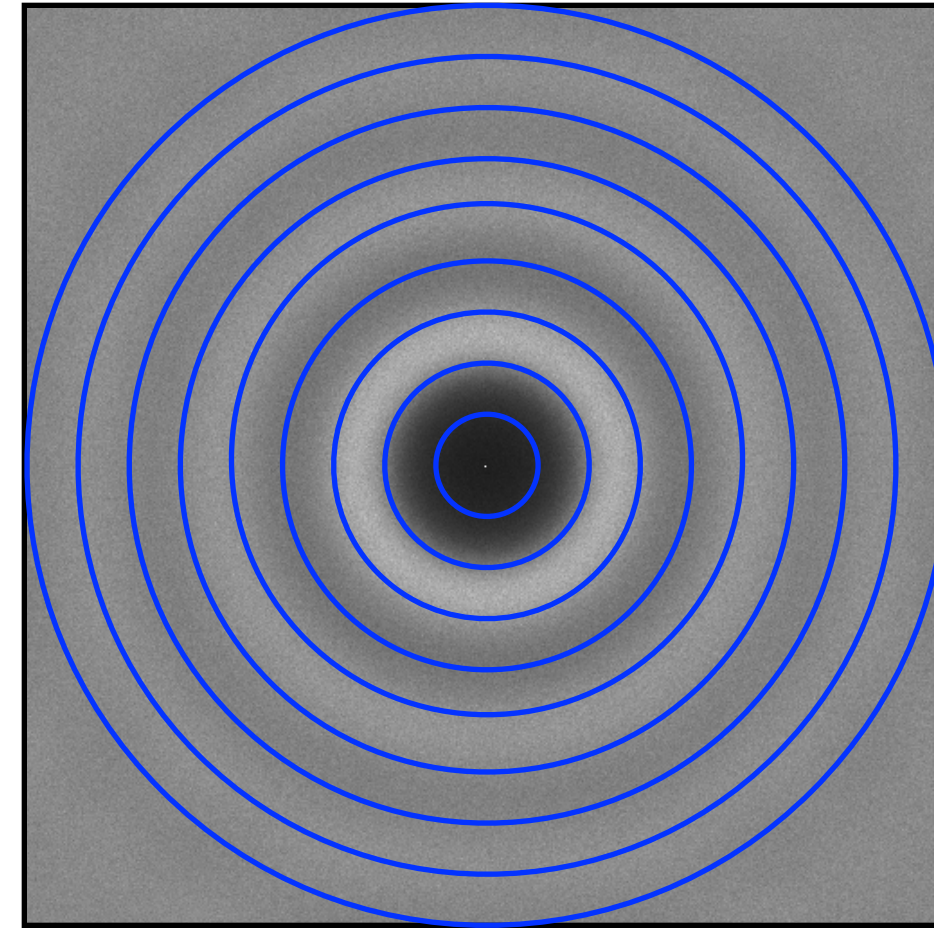
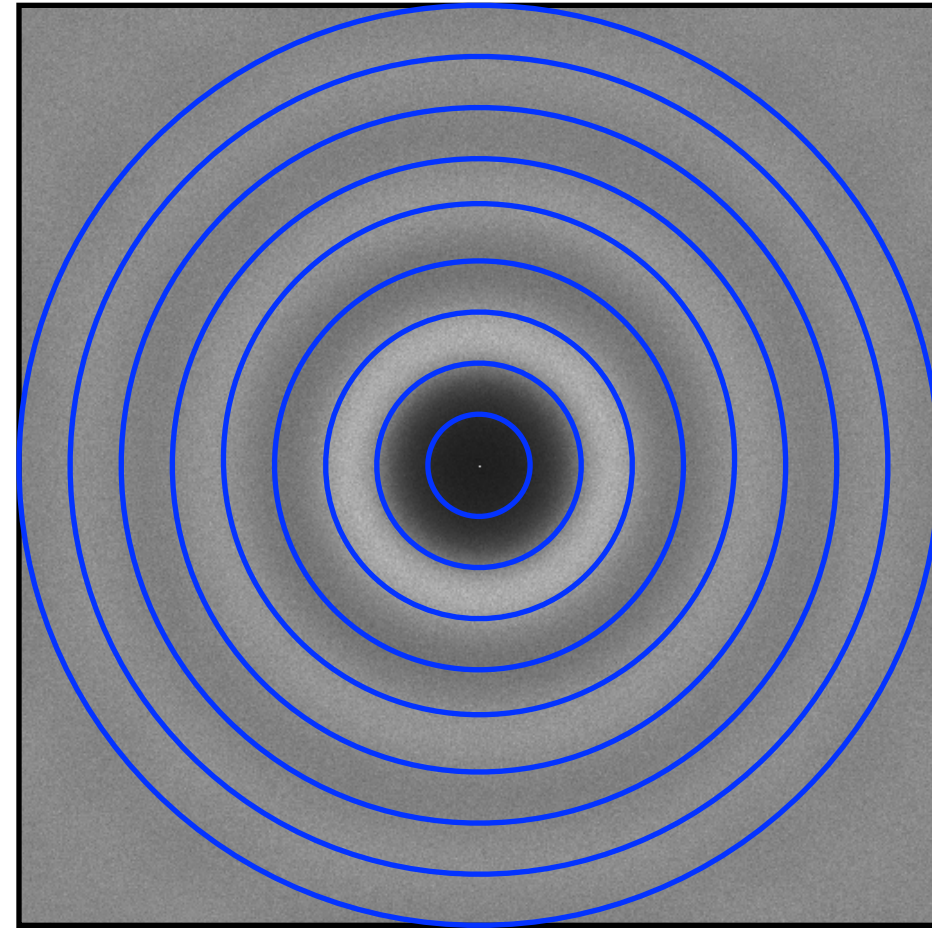


Rob Cook [1986]



4D Sampling Spectra along Projections

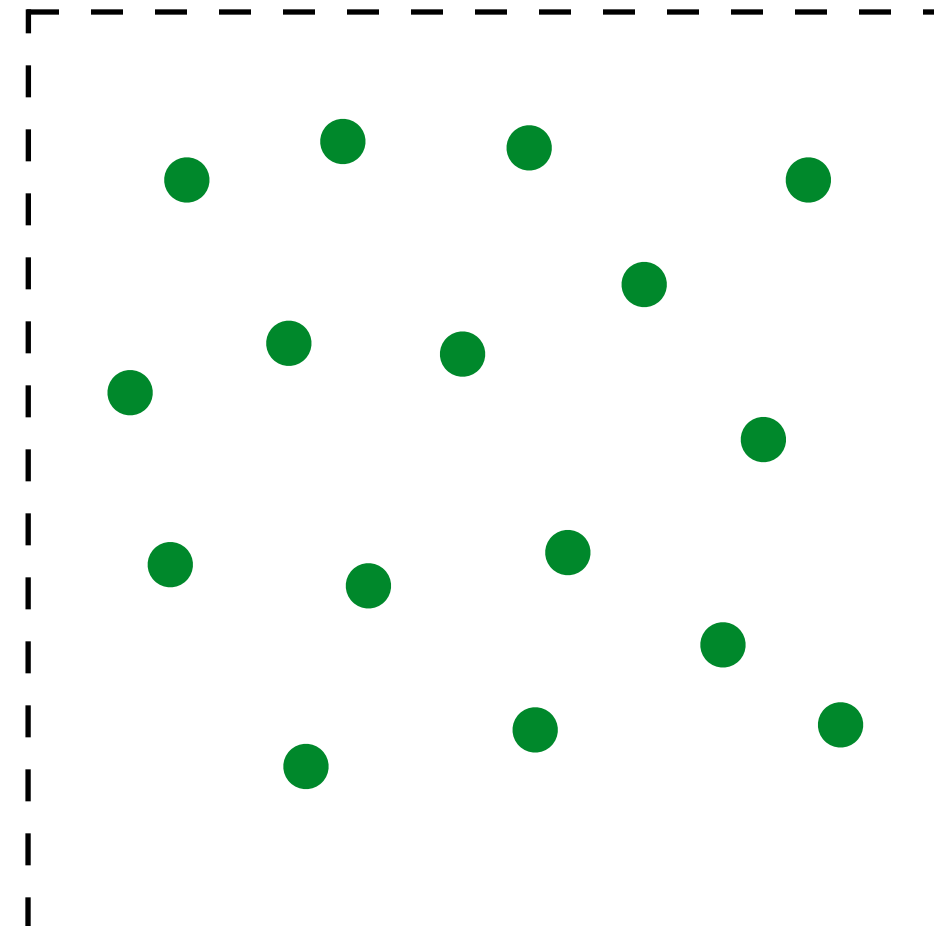
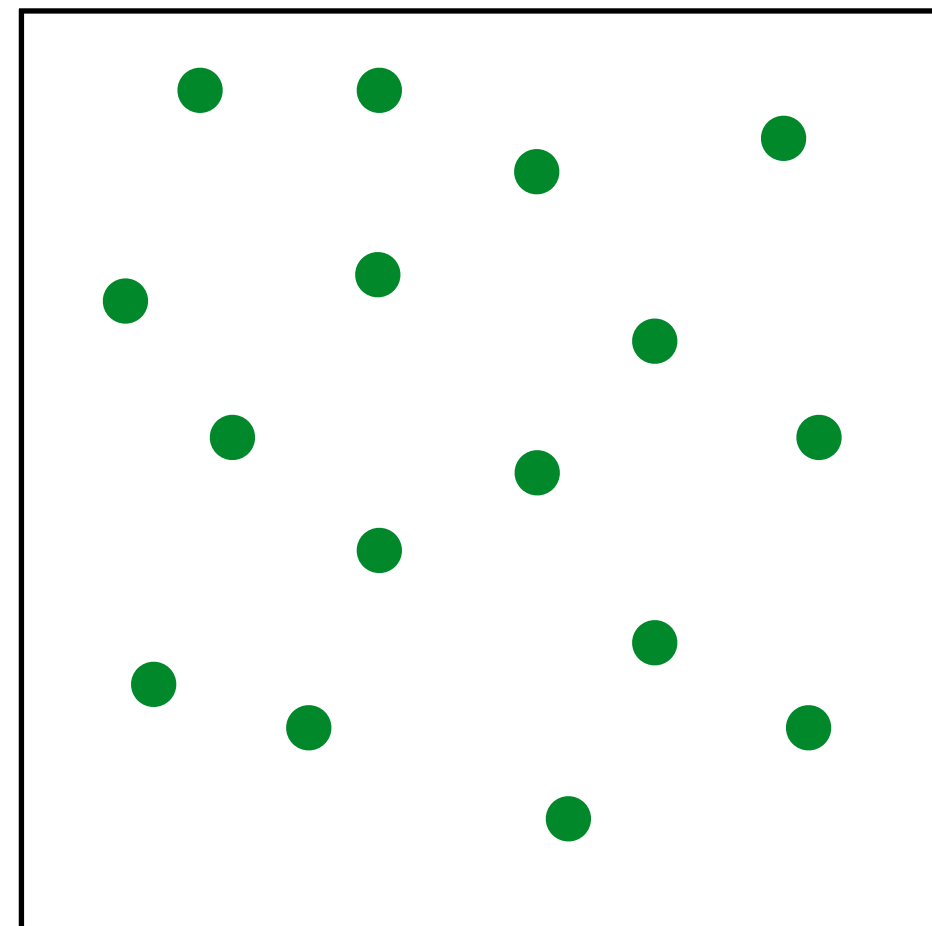
Poisson Disk
Spectra



UV

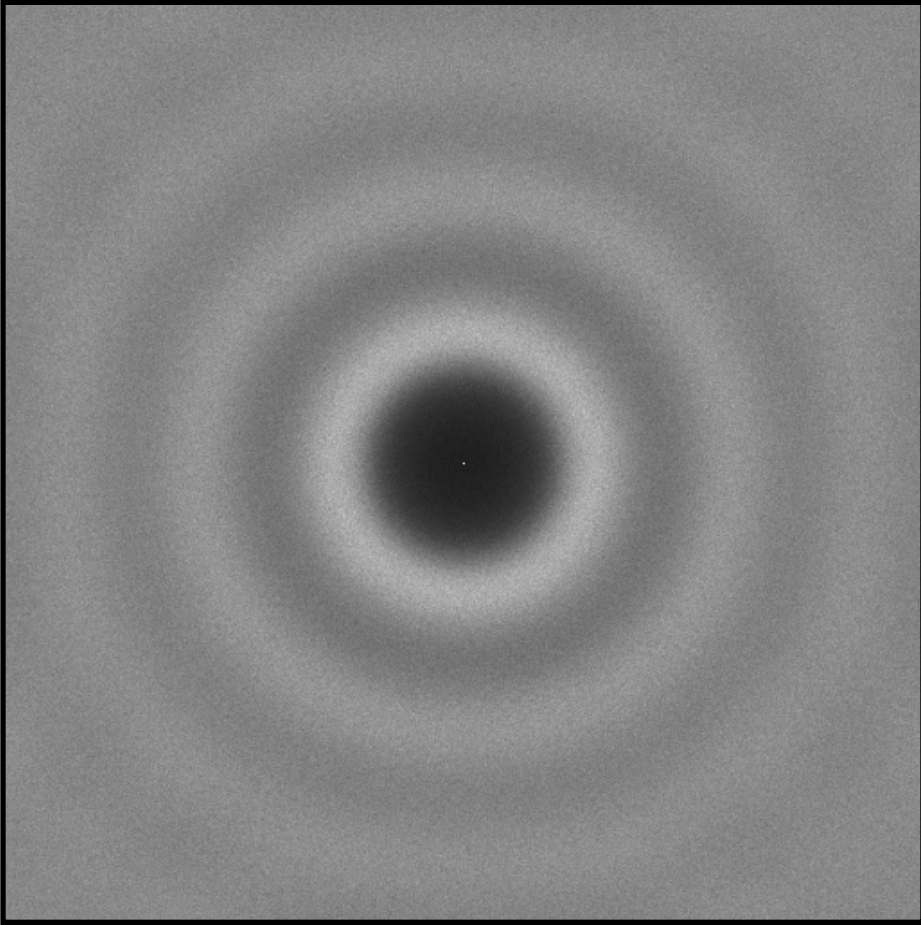
XY

Poisson Disk
Samples

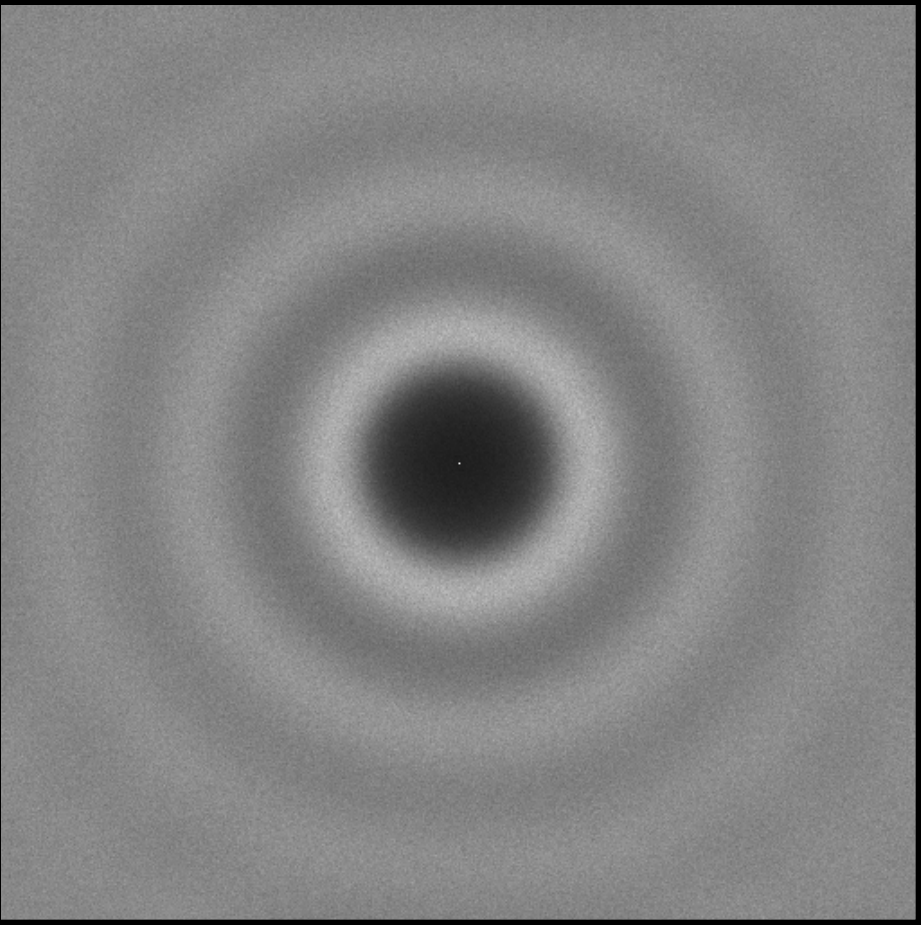


4D Sampling Spectra along Projections

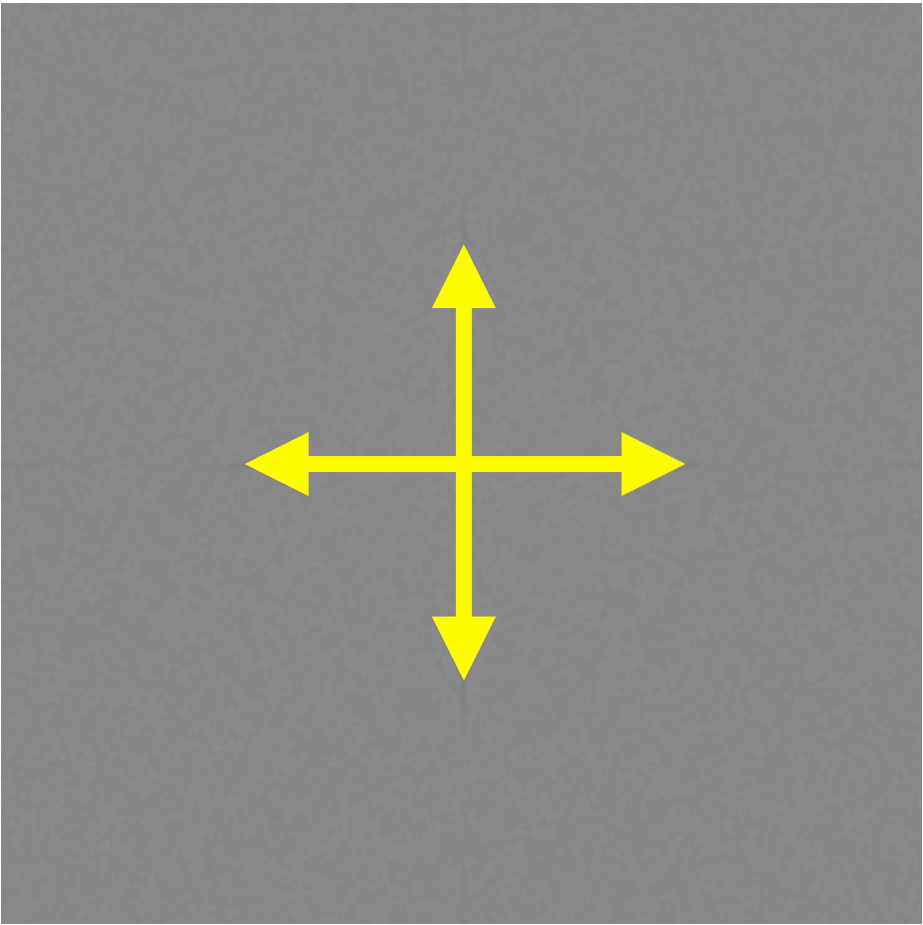
Poisson Disk
Spectra



UV

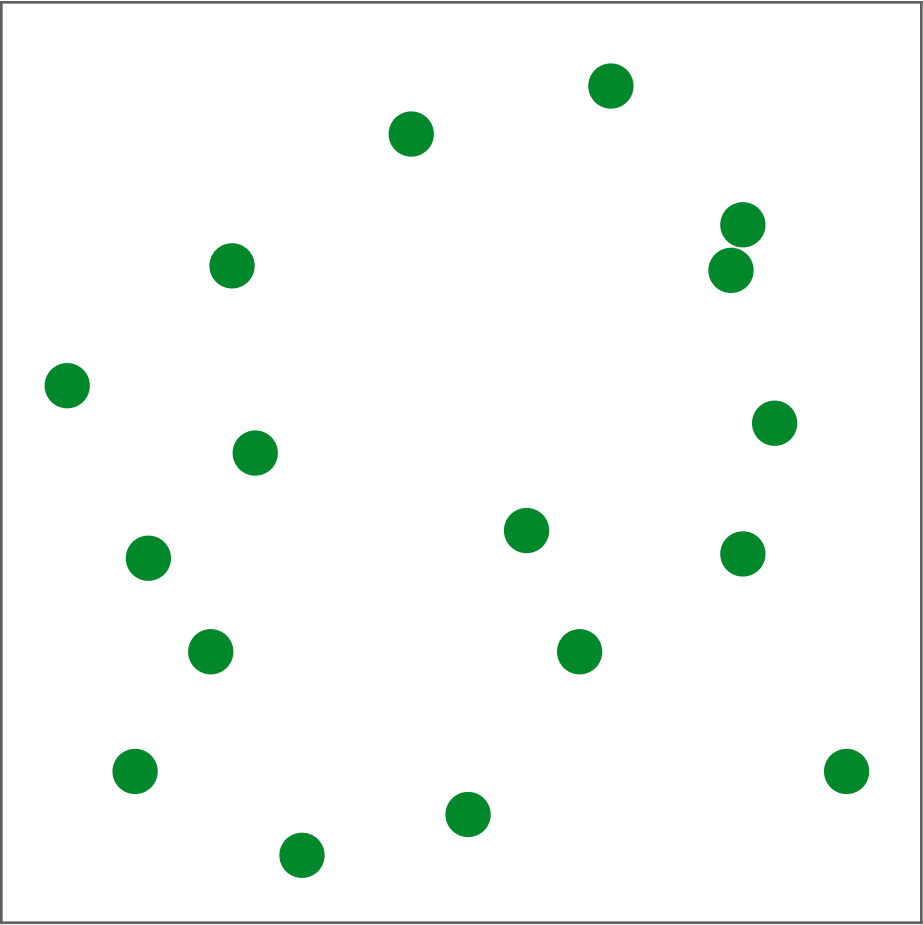
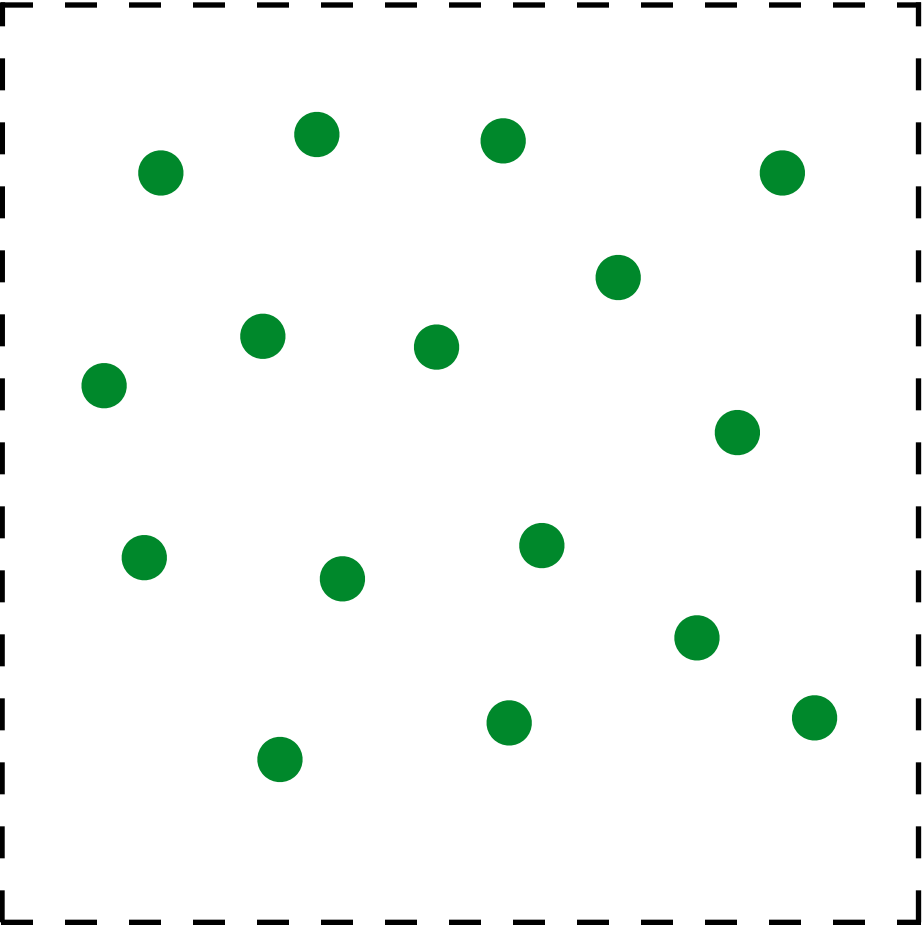
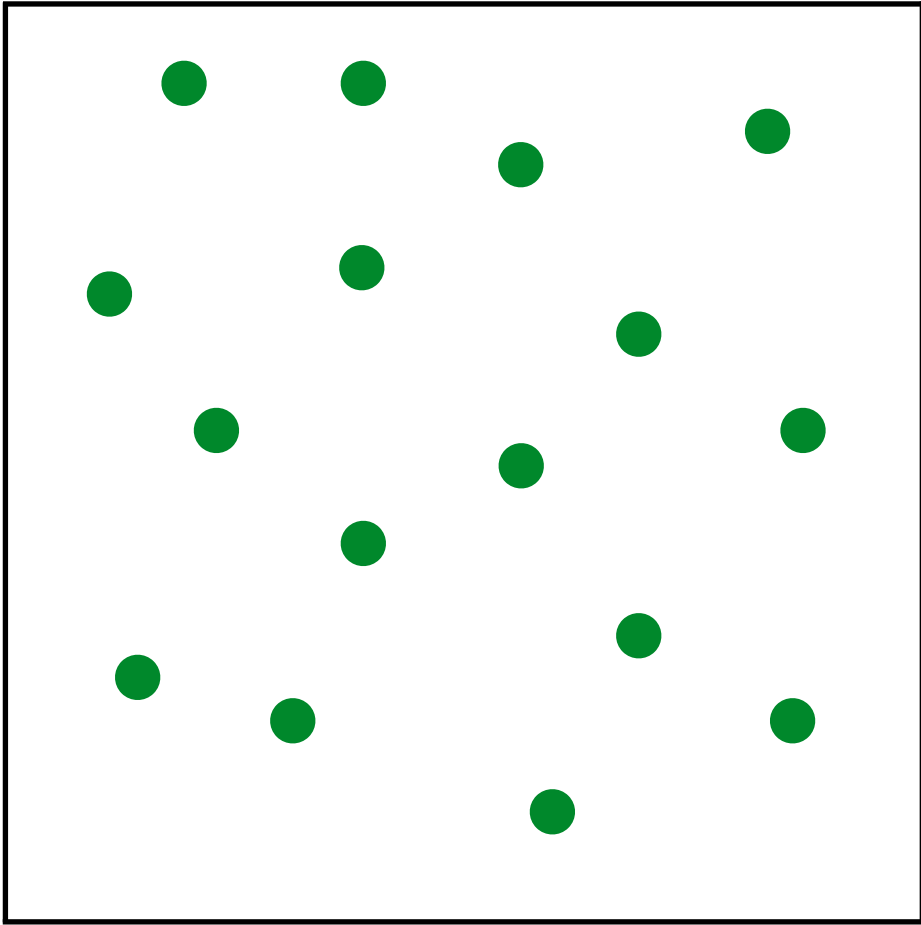


XY

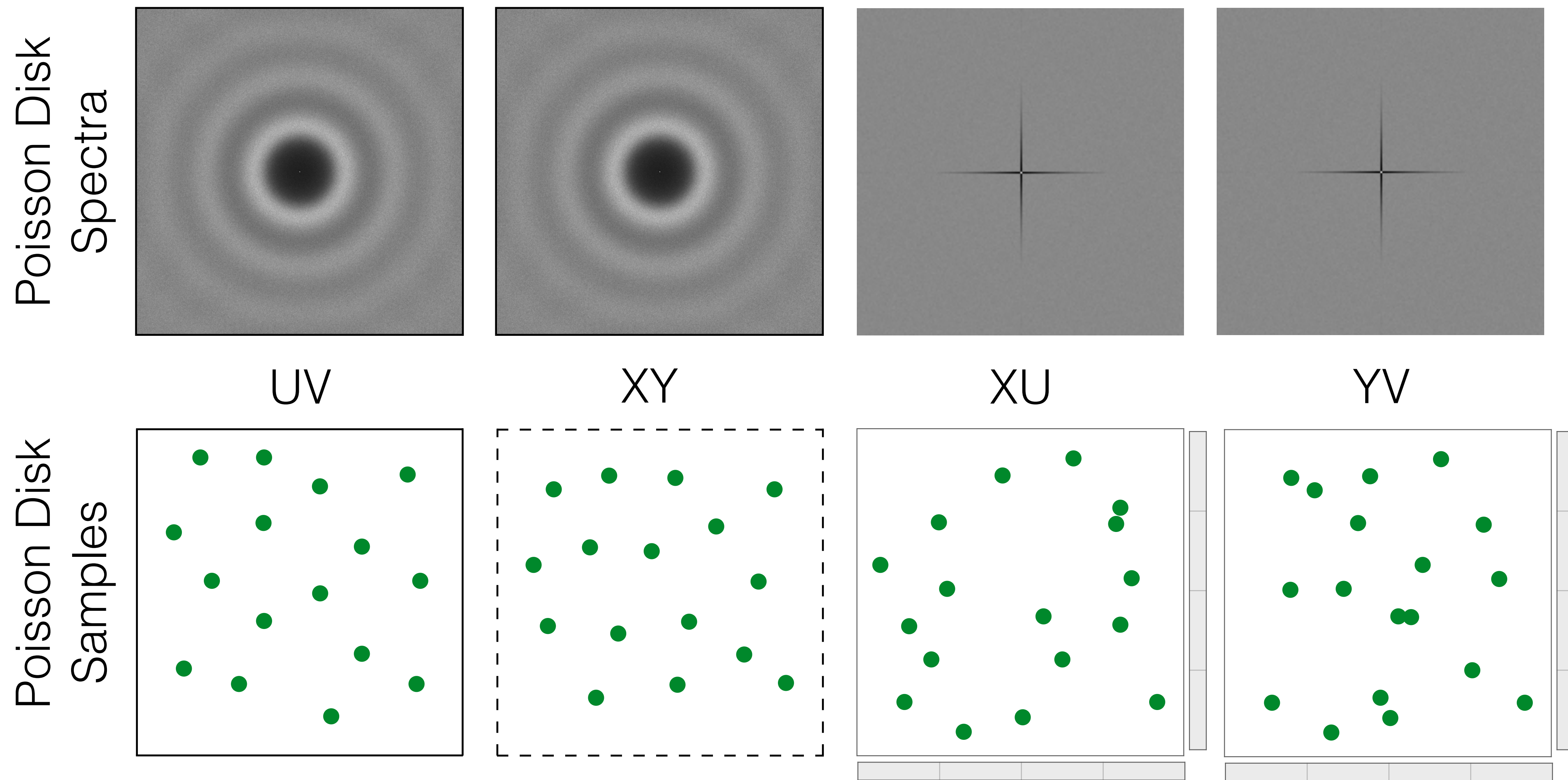


XU

Poisson Disk
Samples



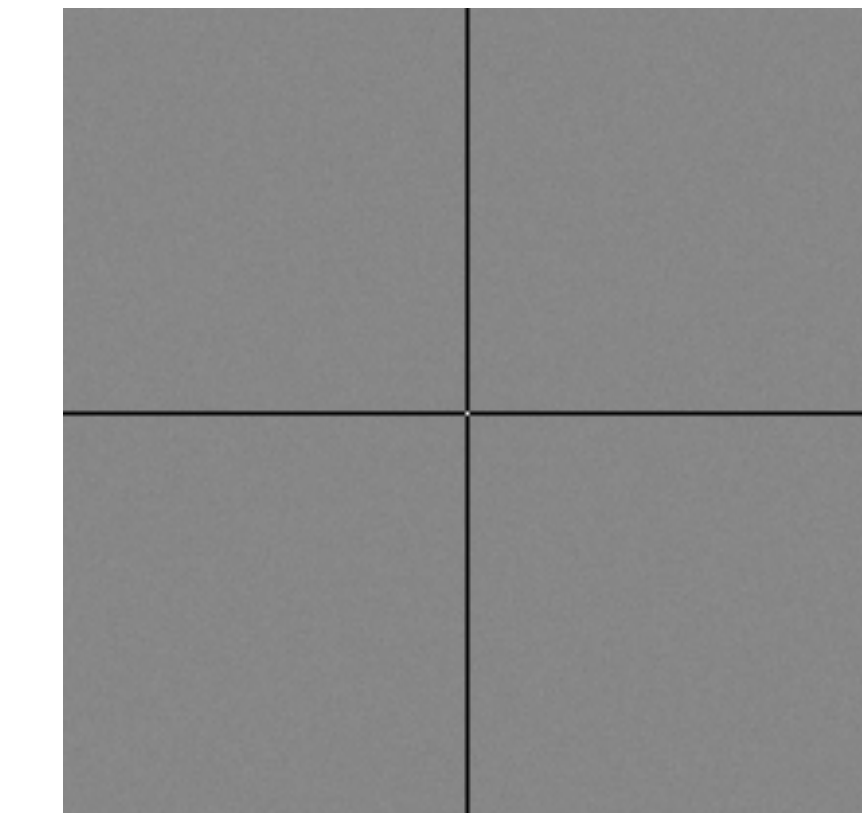
4D Sampling Spectra along Projections



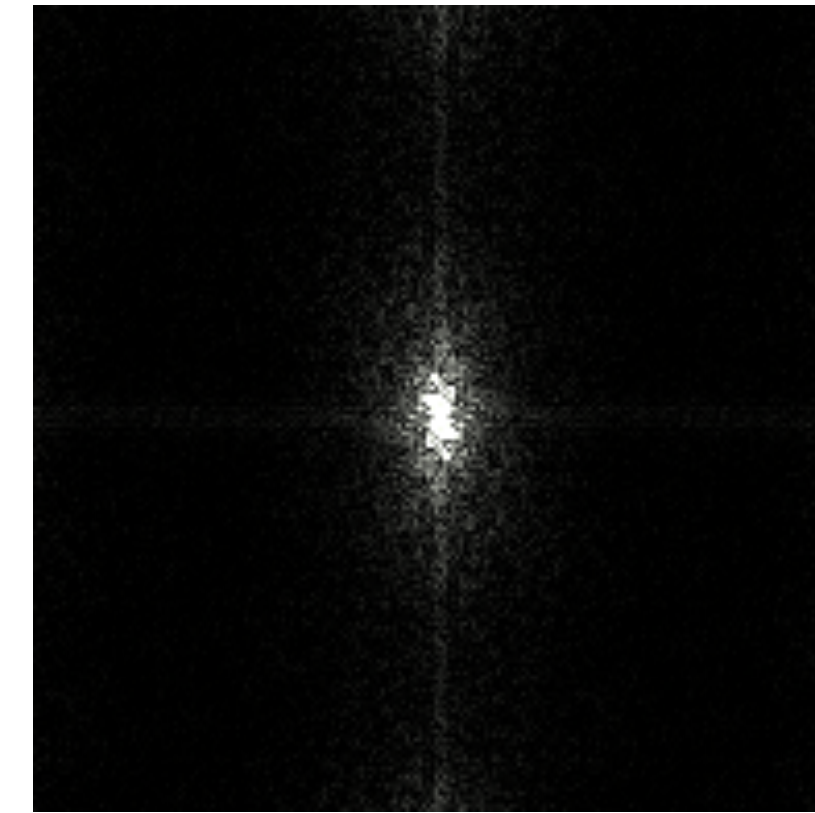
How can we perform Convergence Analysis
for Anisotropic Sampling Spectra ?

Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \int_{\Omega} \langle \mathcal{P}_{S_N}(\nu) \rangle \times \mathcal{P}_f(\nu) d\nu$$



N-rooks spectrum

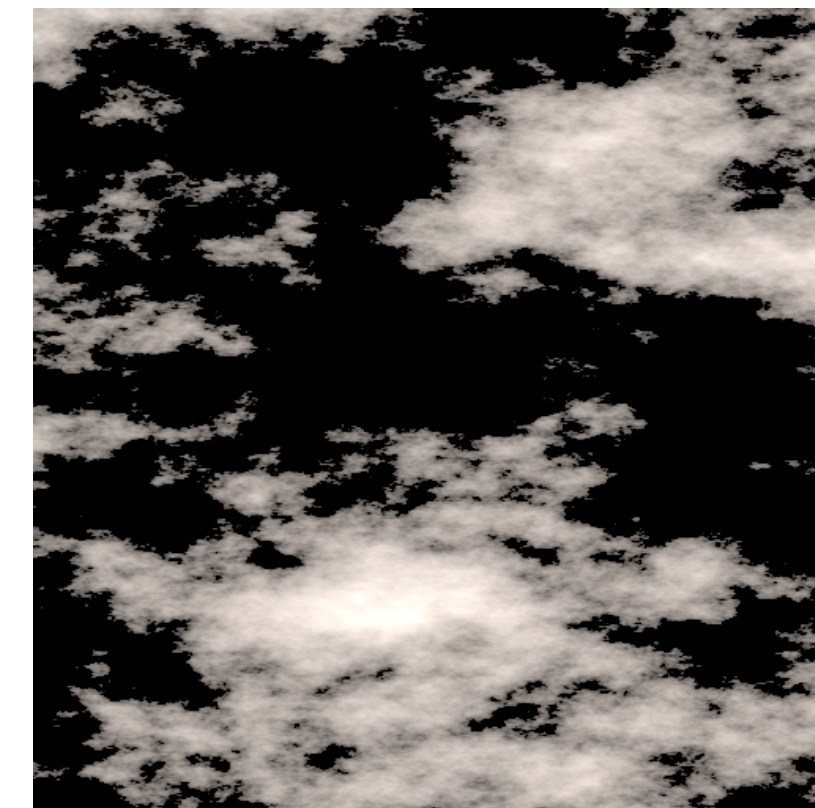
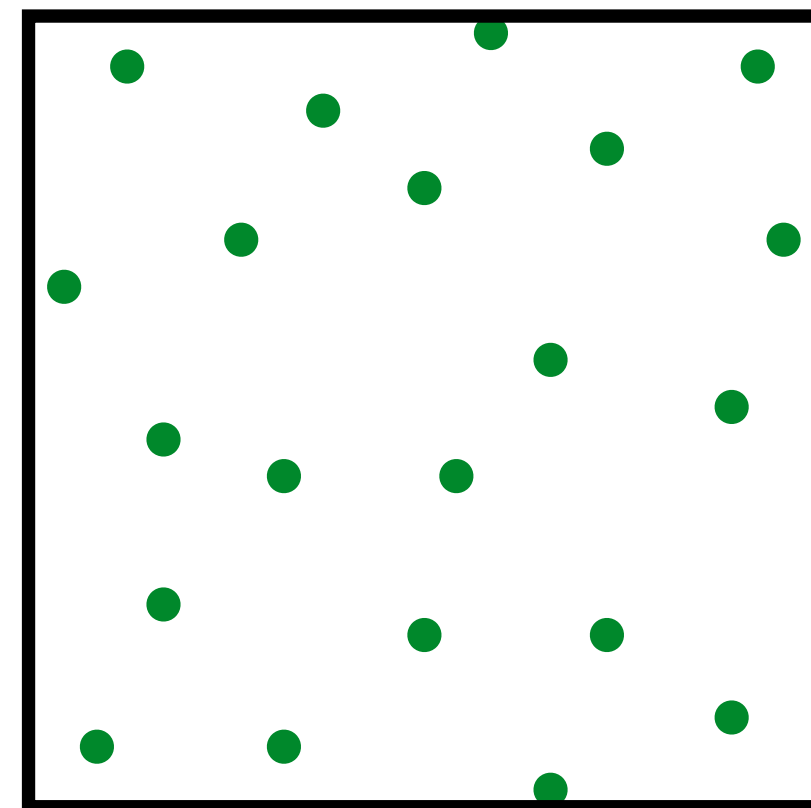


Integrand spectrum

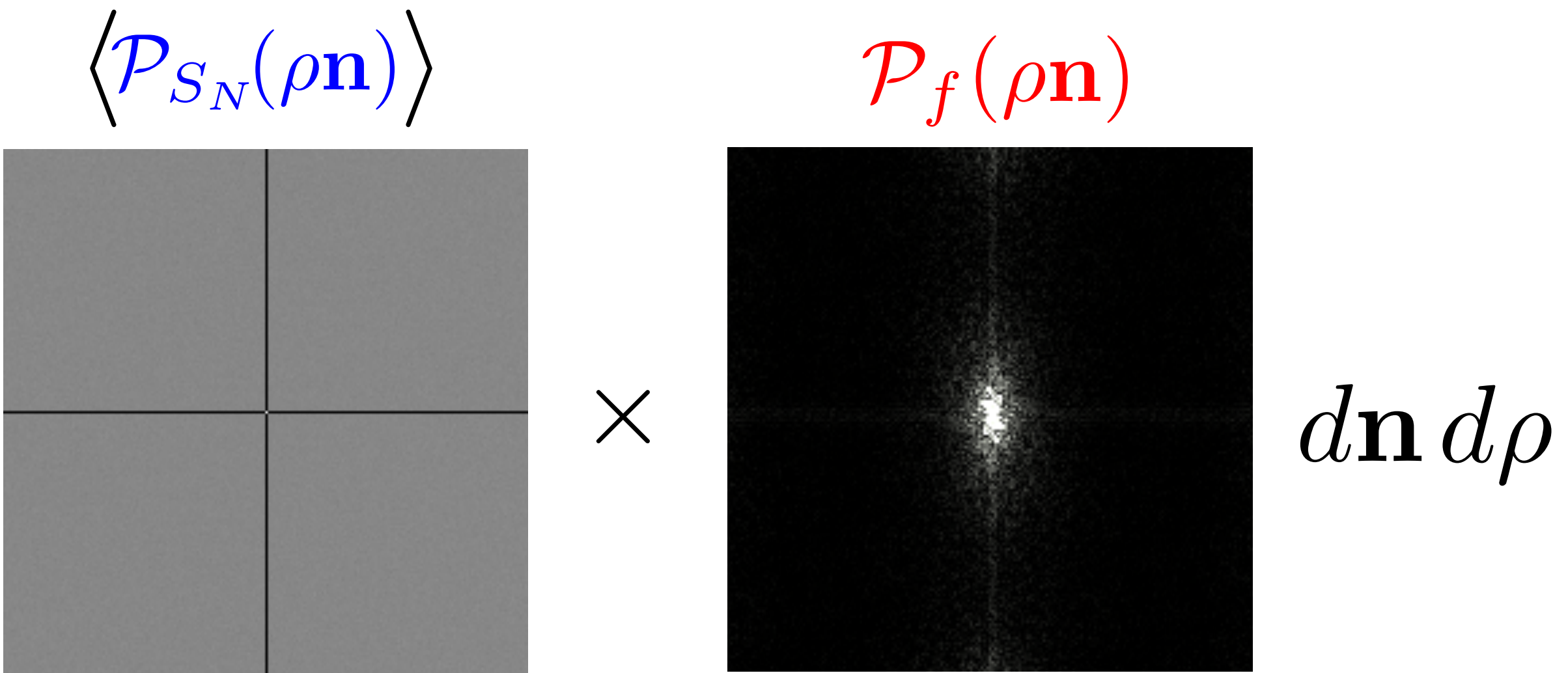
$S_N(\vec{x})$

$f(\vec{x})$

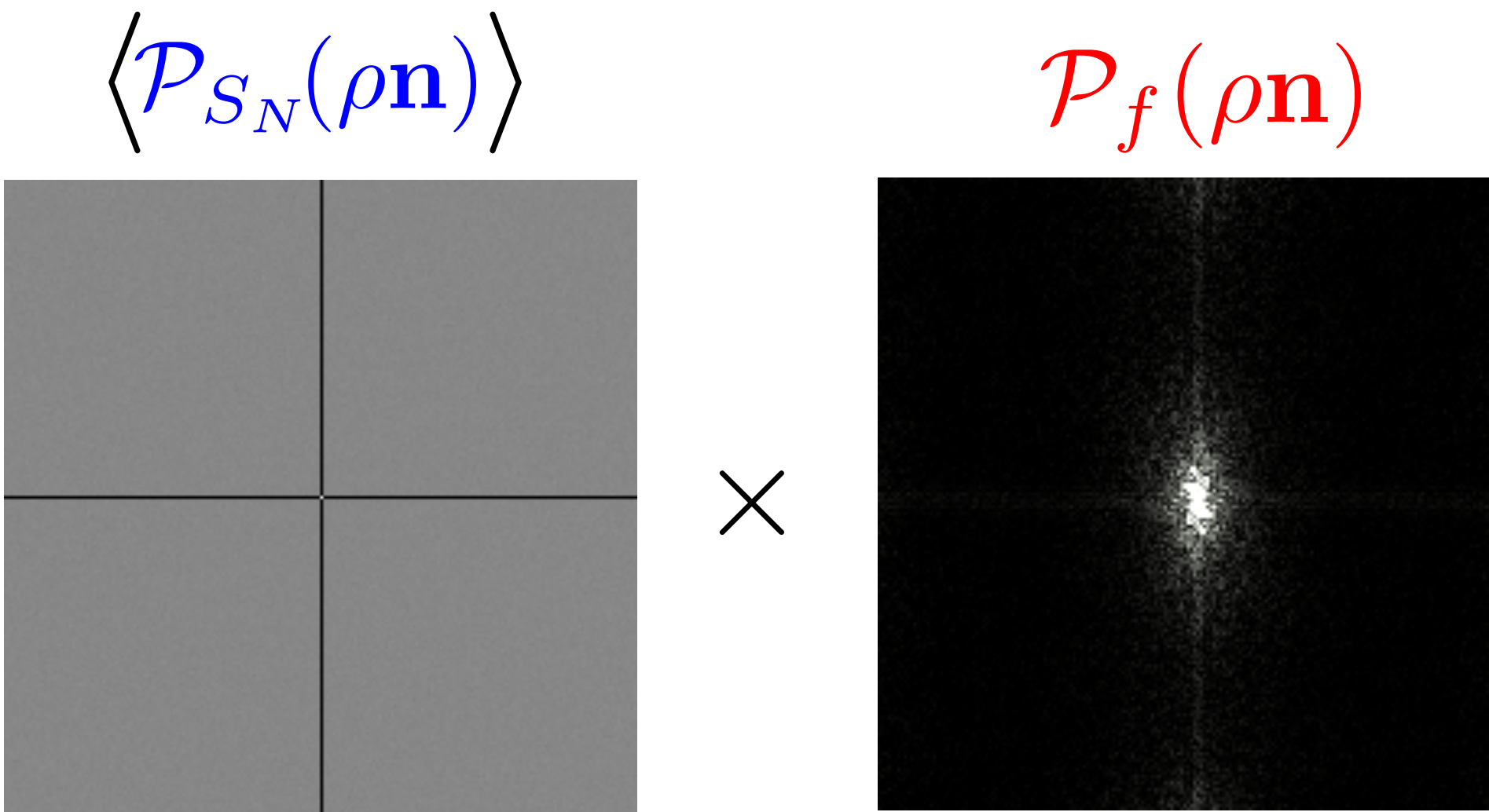
N-rooks



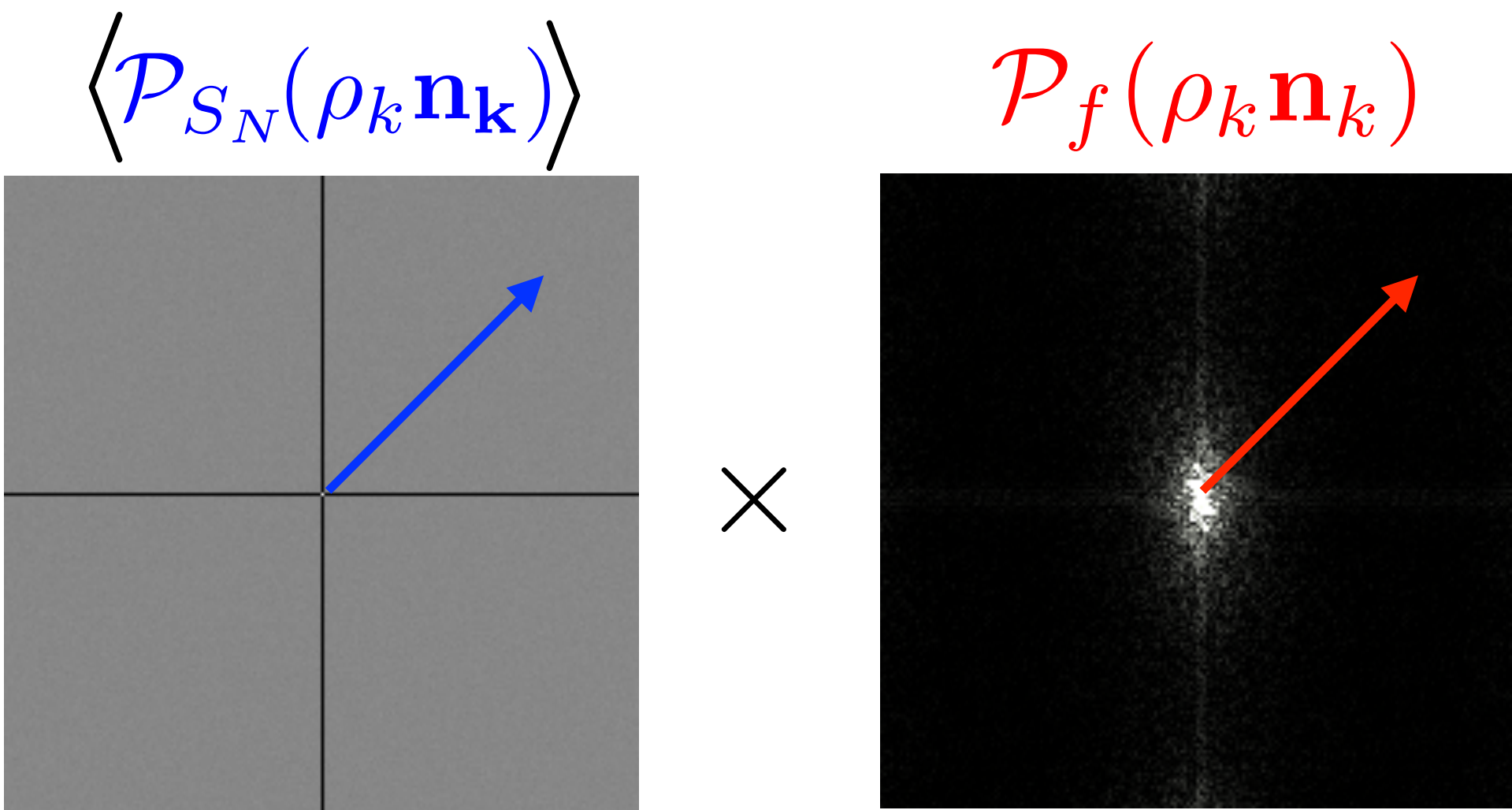
Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \int_0^\infty \rho^{d-1} \int_{\mathcal{S}^{d-1}} \langle \mathcal{P}_{S_N}(\rho \mathbf{n}) \rangle \times \mathcal{P}_f(\rho \mathbf{n}) d\mathbf{n} d\rho$$


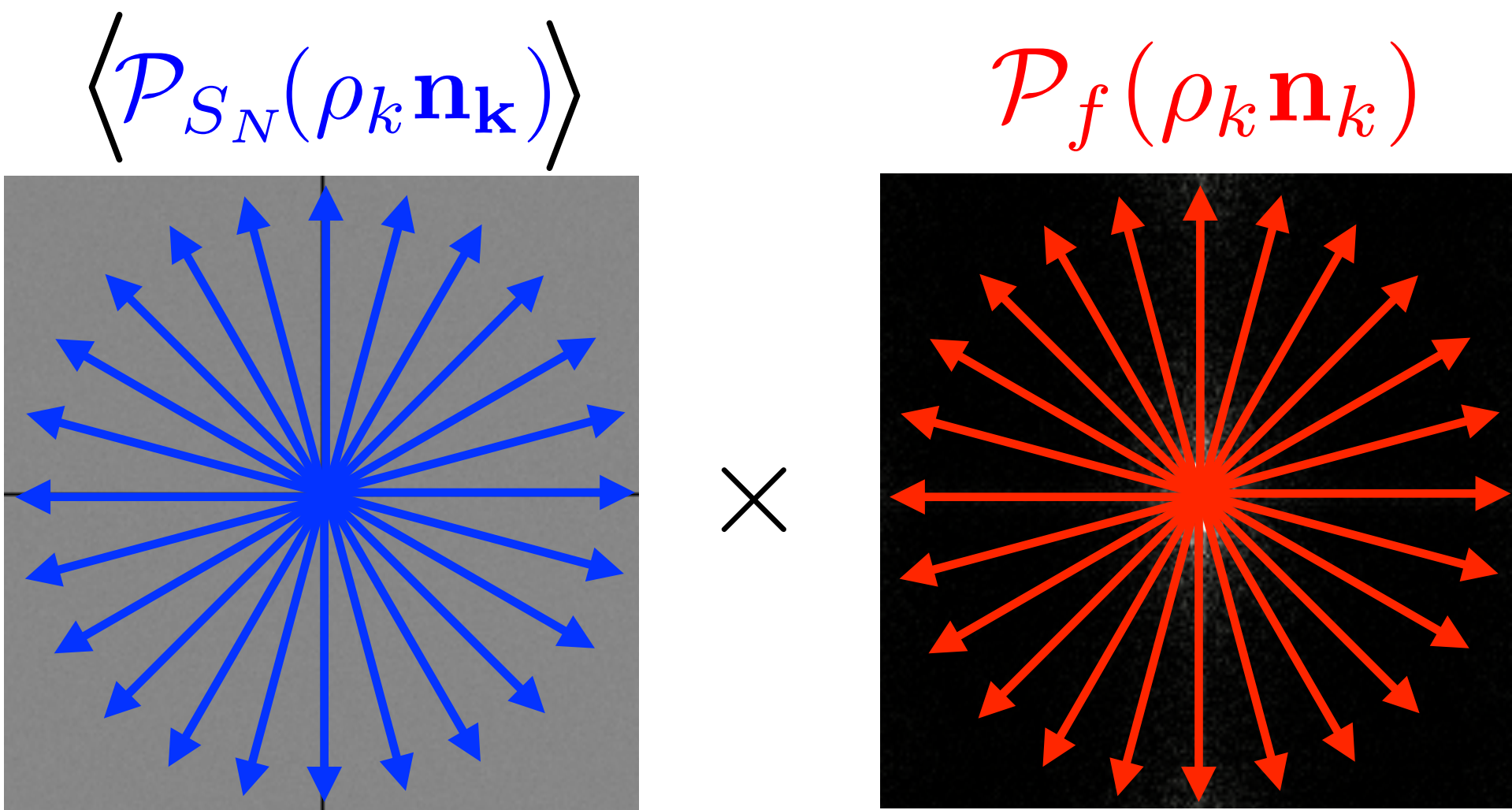
Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \int_{\mathcal{S}^{d-1}} \int_0^\infty \rho^{d-1} \langle \mathcal{P}_{S_N}(\rho \mathbf{n}) \rangle \times \mathcal{P}_f(\rho \mathbf{n}) d\rho d\mathbf{n}$$


Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \int_{\mathcal{S}^{d-1}} \int_0^\infty \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \right\rangle \times \mathcal{P}_f(\rho_k \mathbf{n}_k) d\rho d\mathbf{n}$$


Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \int_{\mathcal{S}^{d-1}} \int_0^\infty \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \right\rangle \times \mathcal{P}_f(\rho_k \mathbf{n}_k) d\rho d\mathbf{n}$$


The diagram illustrates the variance formulation for anisotropic sampling spectra. It shows the variance of the intensity I_N as a function of the sampling probability $\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \rangle$ and the function $\mathcal{P}_f(\rho_k \mathbf{n}_k)$. The integral is over the solid angle \mathcal{S}^{d-1} and the radial distance ρ .

The sampling probability $\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \rangle$ is visualized as a gray square with blue arrows radiating from the center, representing the sampling distribution. The function $\mathcal{P}_f(\rho_k \mathbf{n}_k)$ is visualized as a black square with red arrows radiating from the center, representing the function being sampled.

Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \lim_{m \rightarrow \infty} \sum_{k=1}^m \int_0^{\infty} \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \right\rangle \times \mathcal{P}_f(\rho_k \mathbf{n}_k) d\rho \Delta \mathbf{n}_k$$

Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \lim_{m \rightarrow \infty} \sum_{k=1}^m \int_0^{\infty} \rho^{d-1} \langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \rangle \times \mathcal{P}_f(\rho_k \mathbf{n}_k) d\rho \Delta \mathbf{n}_k$$

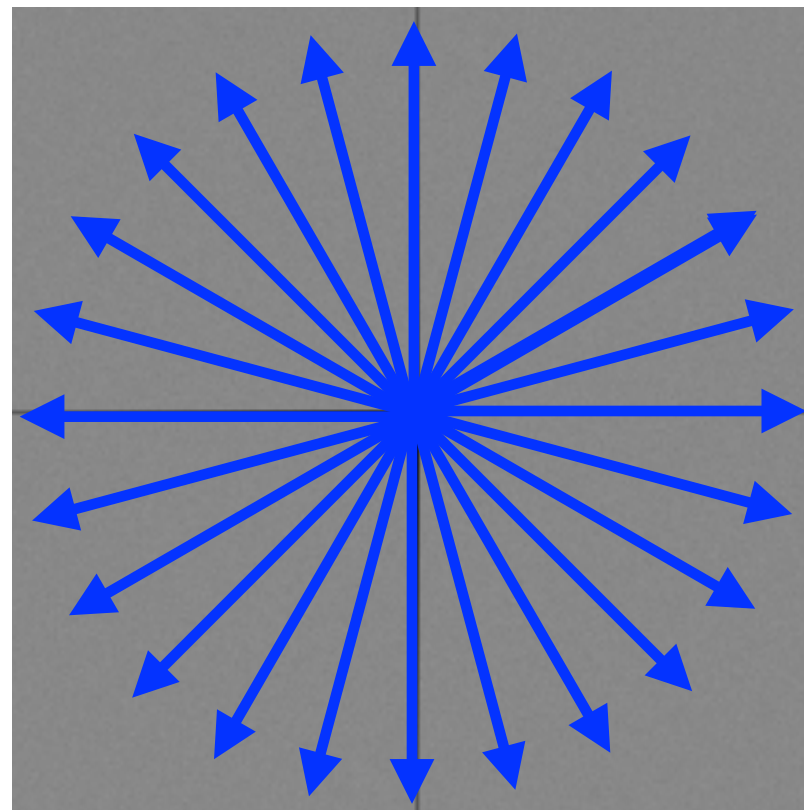
Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \lim_{m \rightarrow \infty} \sum_{k=1}^m \int_0^{\infty} \rho^{d-1} \langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \rangle \mathcal{P}_f(\rho_k \mathbf{n}_k) d\rho \Delta \mathbf{n}_k$$

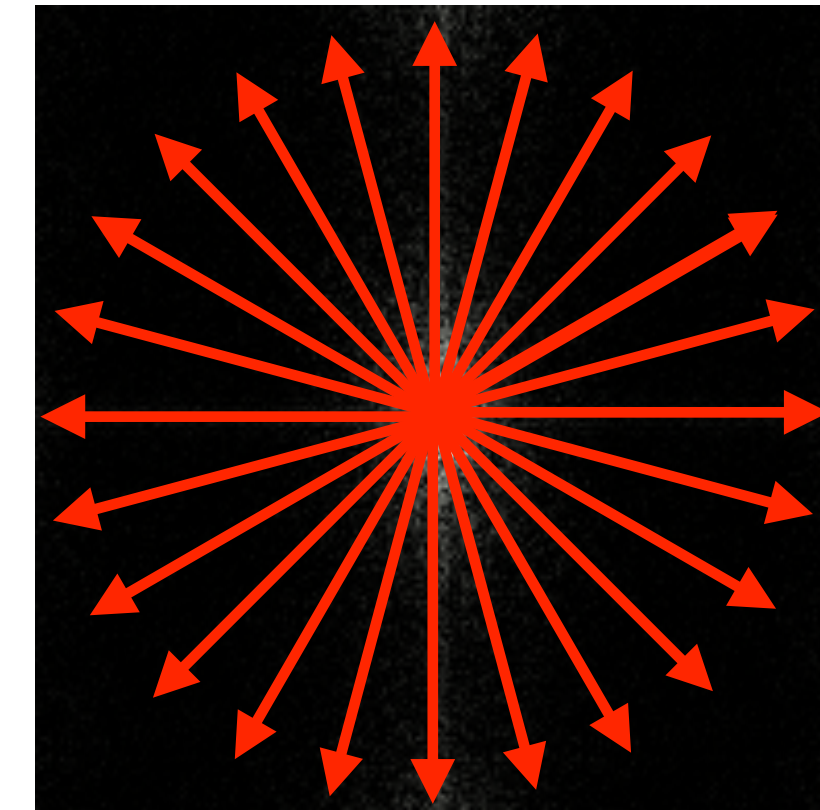
Variance Formulation for Anisotropic Sampling Spectra

$$\text{Var}(I_N) = \lim_{m \rightarrow \infty} \sum_{k=1}^m \int_0^{\infty} \rho^{d-1} \langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \rangle \mathcal{P}_f(\rho_k \mathbf{n}_k) d\rho \Delta \mathbf{n}_k$$

$\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_k) \rangle$

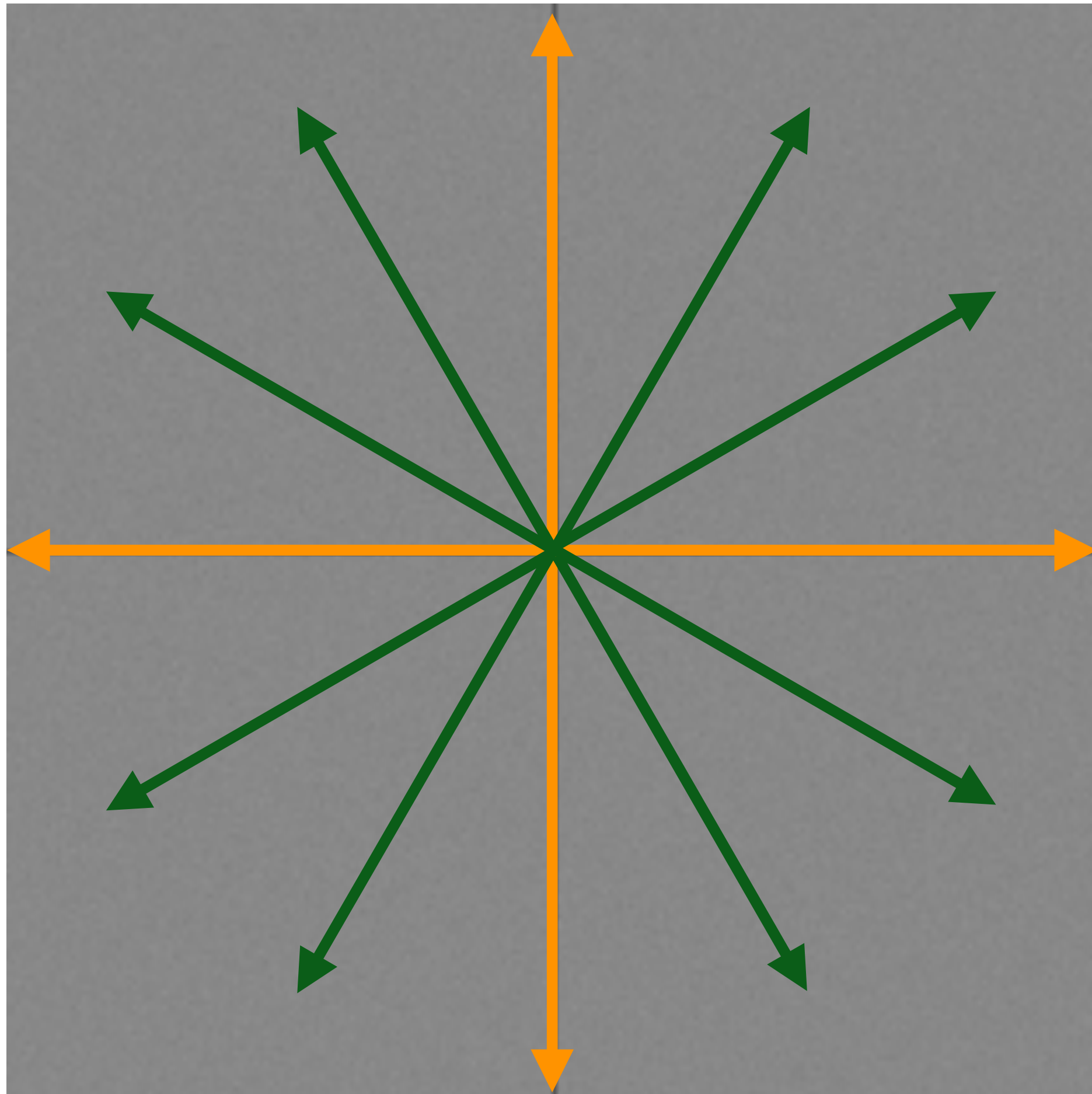


$\mathcal{P}_f(\rho_k \mathbf{n}_k)$

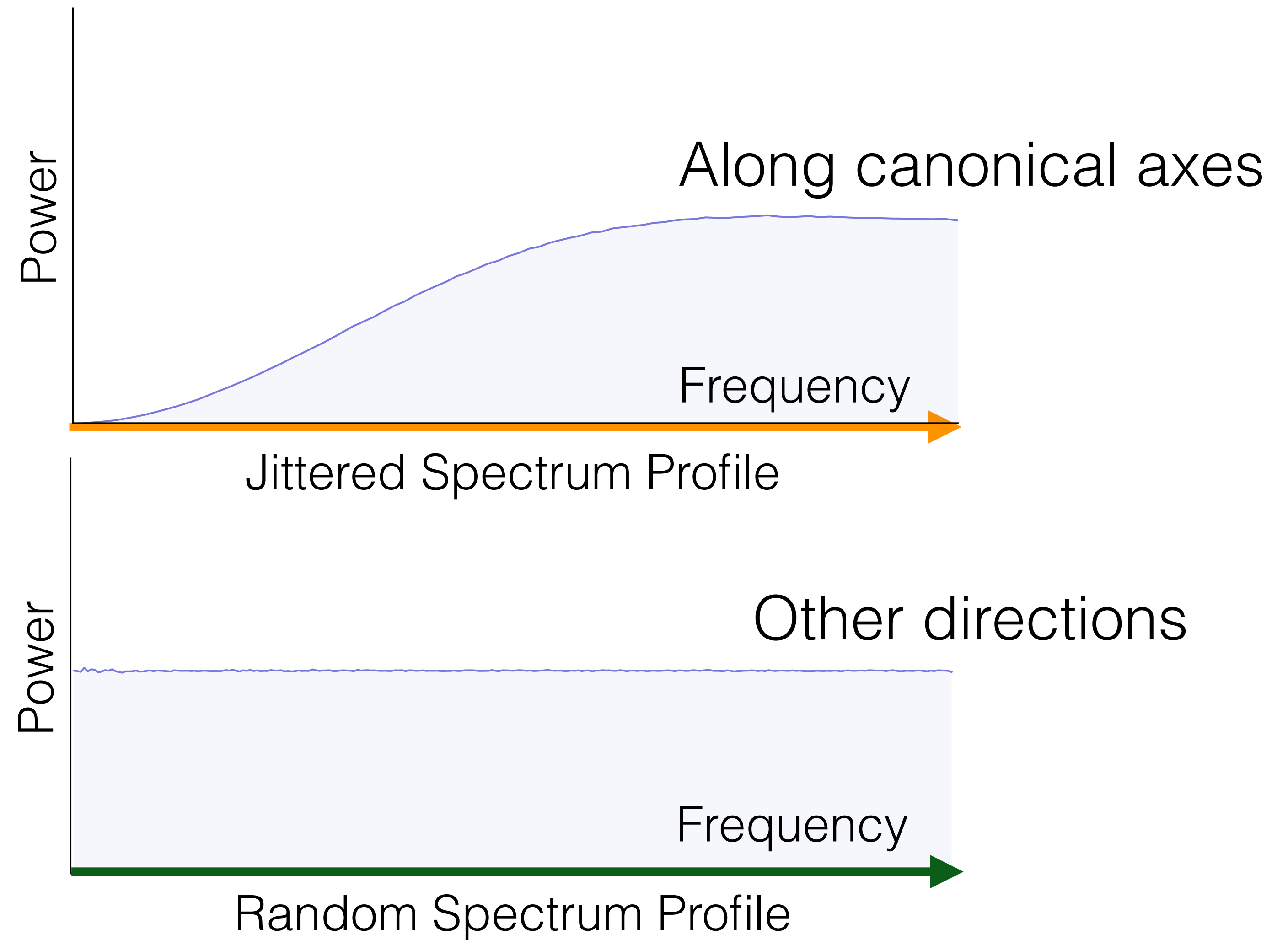


Convergence Analysis for Anisotropic Sampling Spectra

Power Spectrum

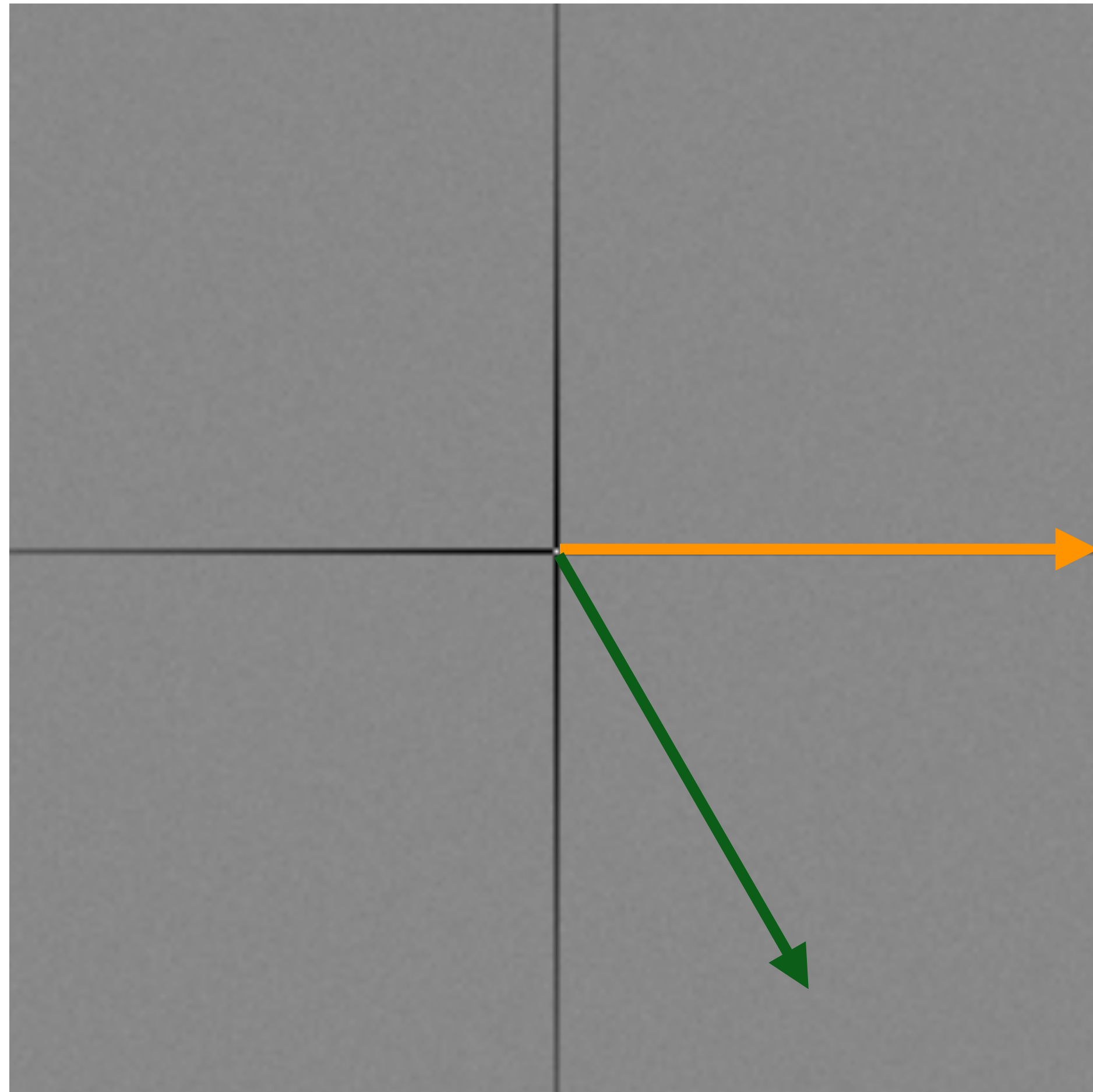


Radial Power Spectrum

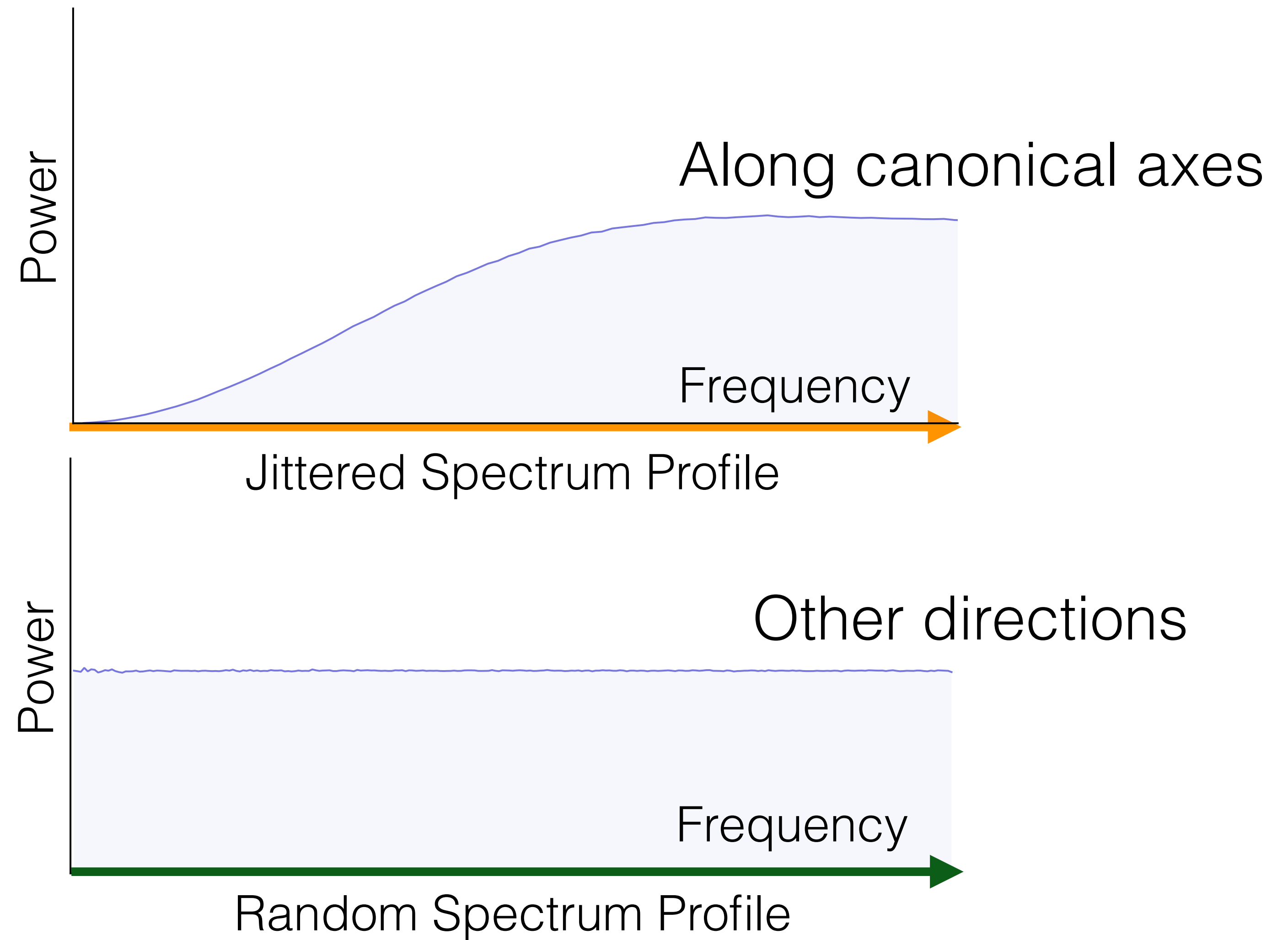


Convergence Analysis for Anisotropic Sampling Spectra

Power Spectrum

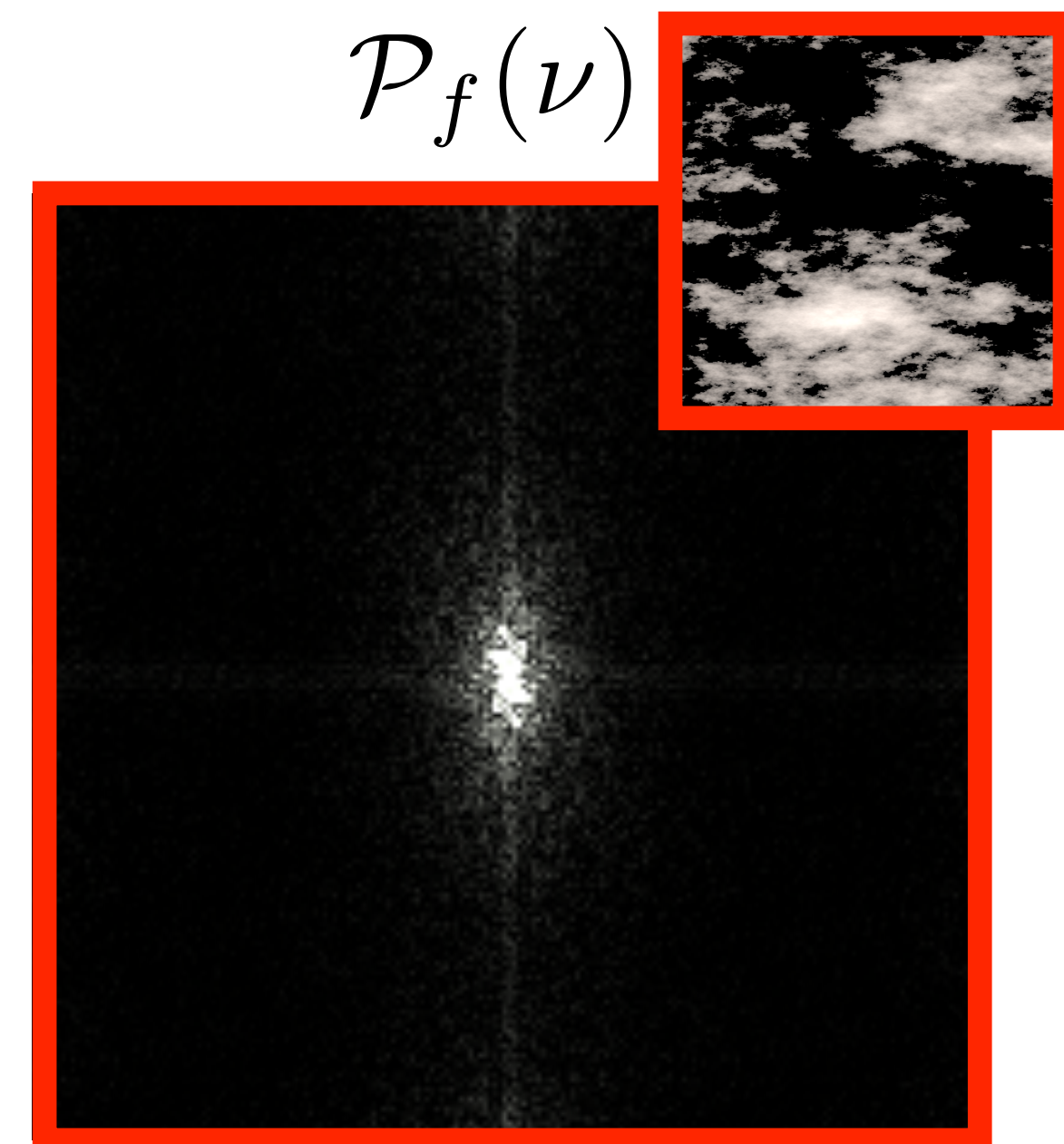
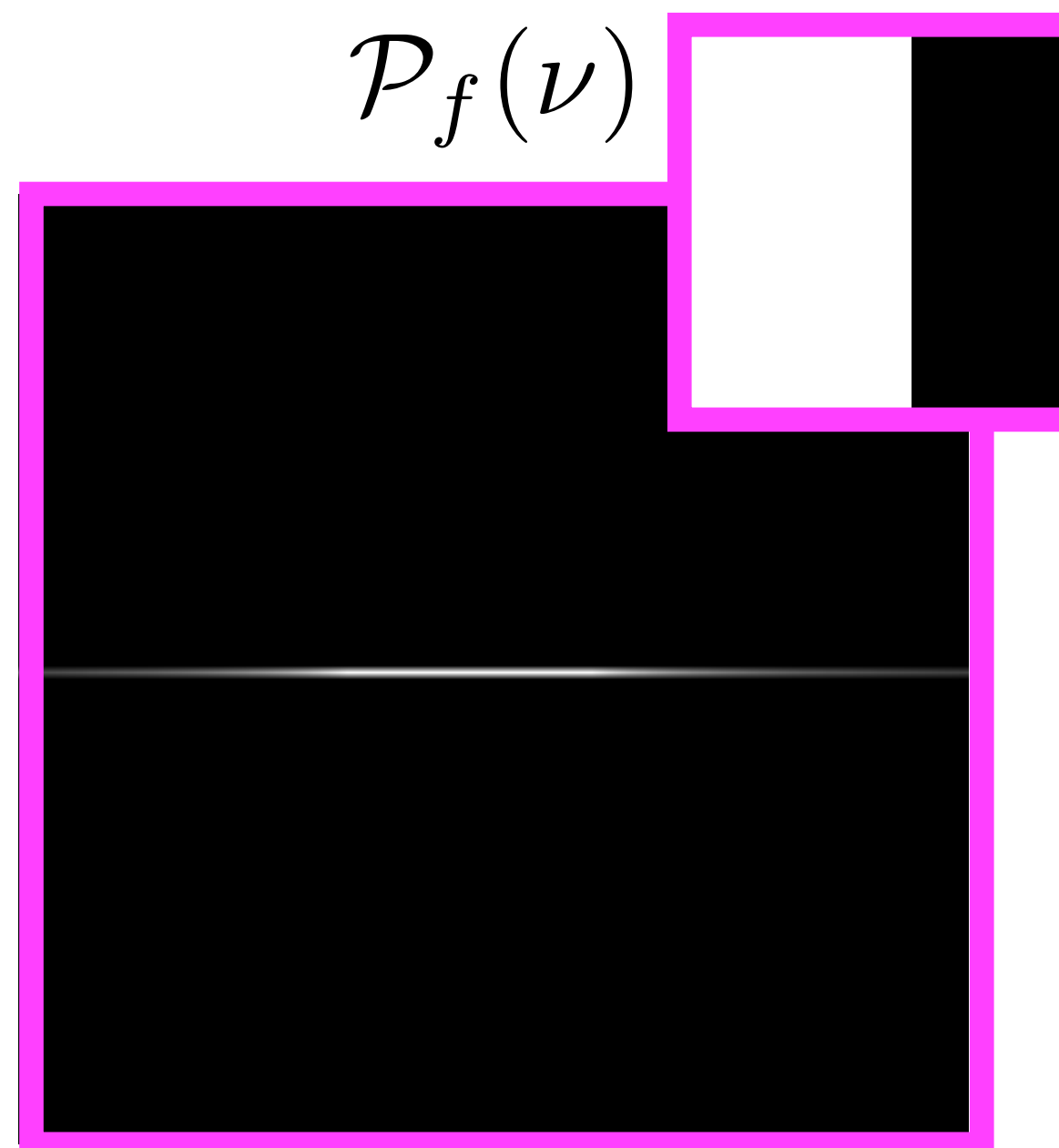
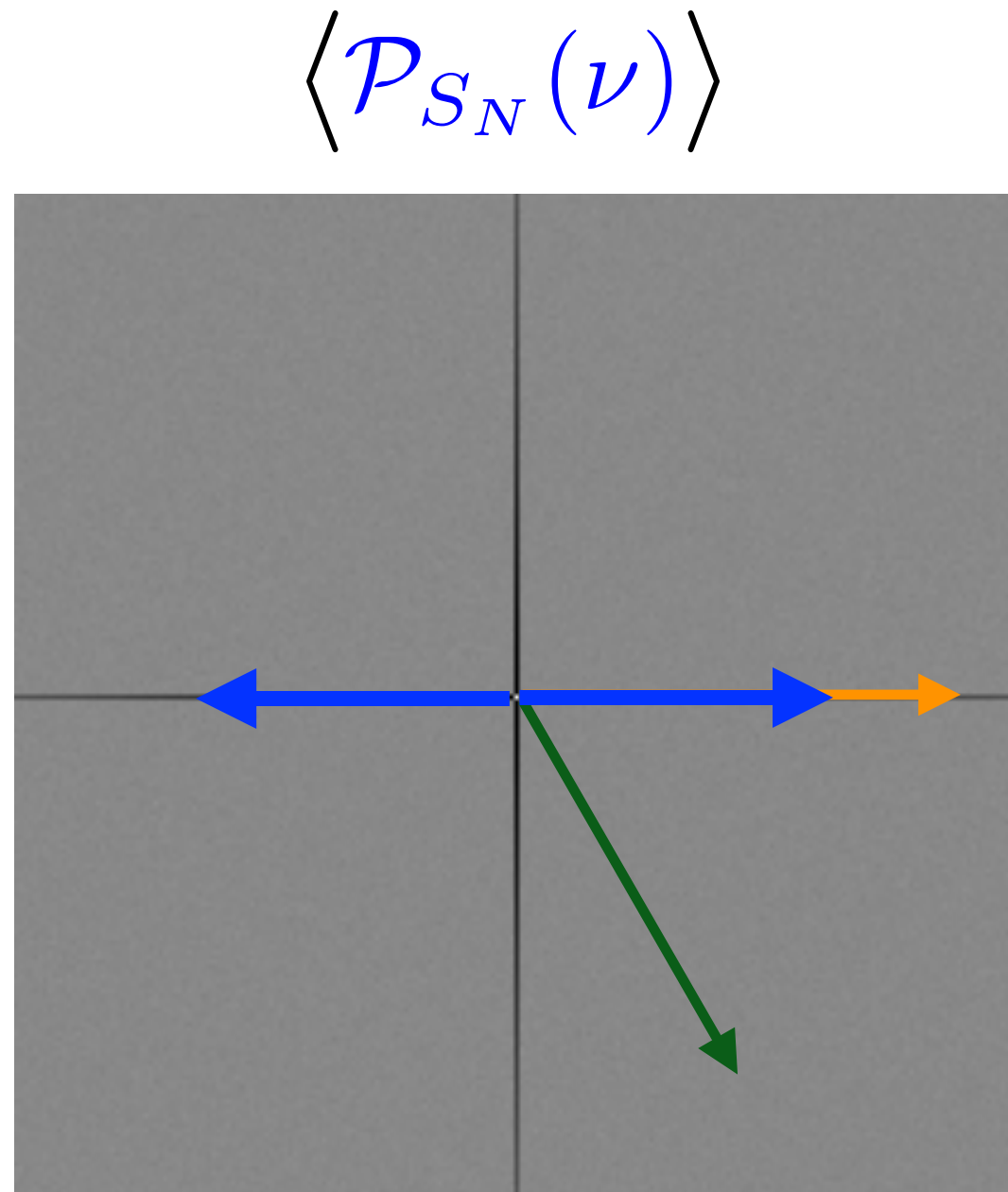


Radial Power Spectrum

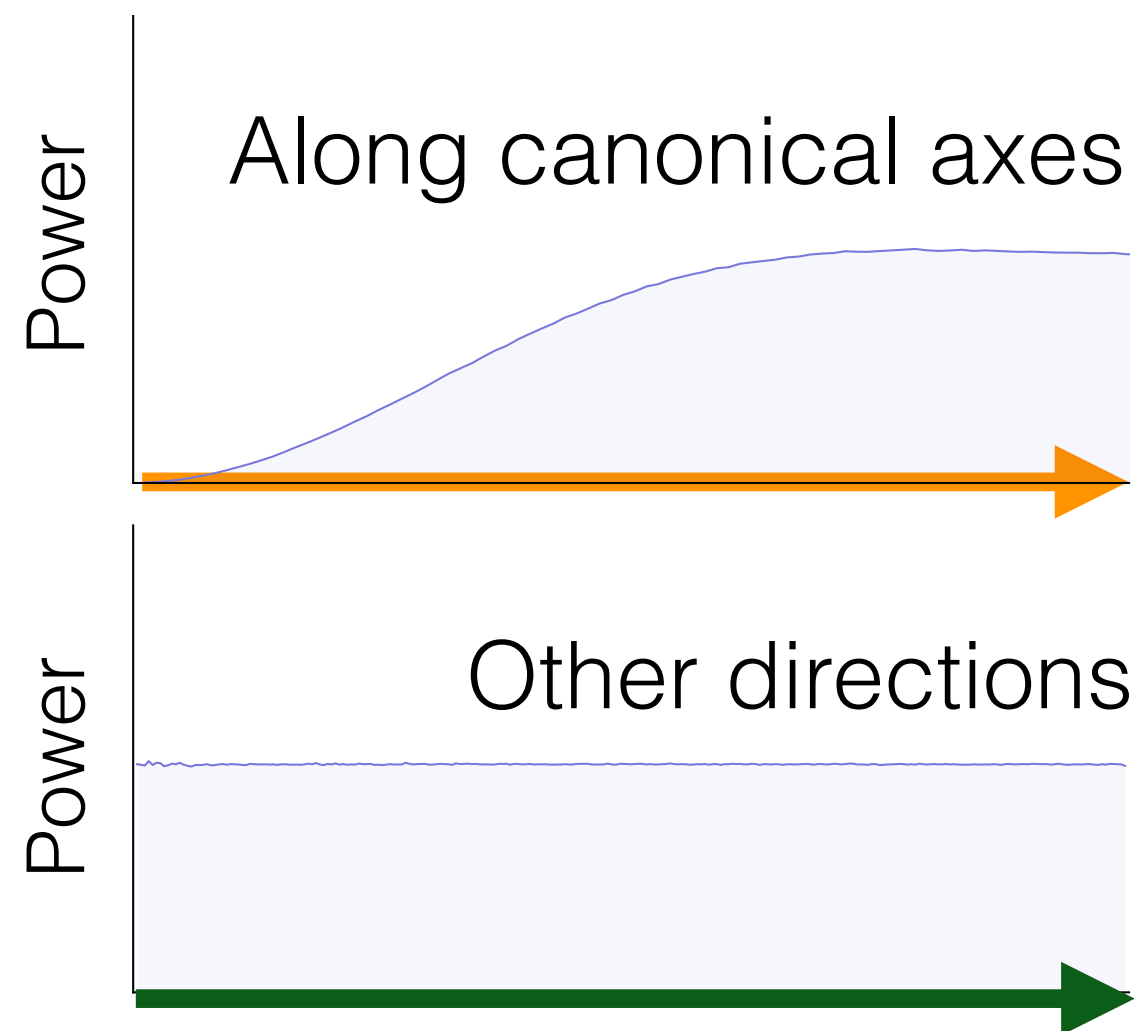


Convergence Analysis for Anisotropic Sampling Spectra

Spectrum



Radial Spectrum



$$O(N^{-2})$$

No Impact

$$\left. \begin{array}{l} O(N^{-2}) \\ O(N^{-1}) \end{array} \right\} O(N^{-1})$$

Variance due to N-rooks Sampler

$$\text{Var}(I_N) = \int_{\Omega} f(\vec{x}) \langle \mathcal{P}_{S_N}(\nu) \rangle \times \mathcal{P}_f(\nu) d\nu = \int_{\Omega} \dots d\nu$$

$f(\vec{x})$ is represented by a square with a white left half and a black right half, outlined in magenta.

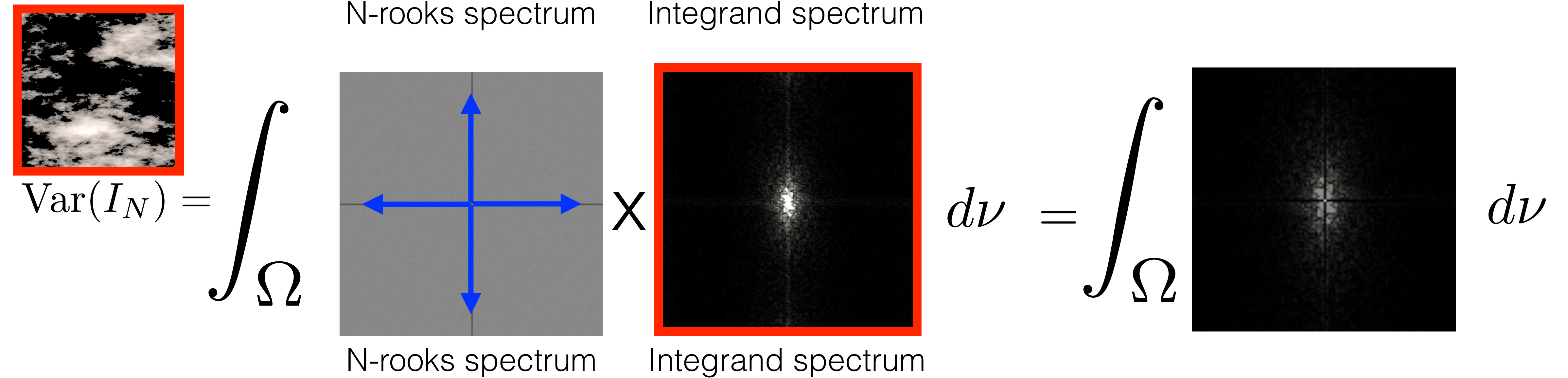
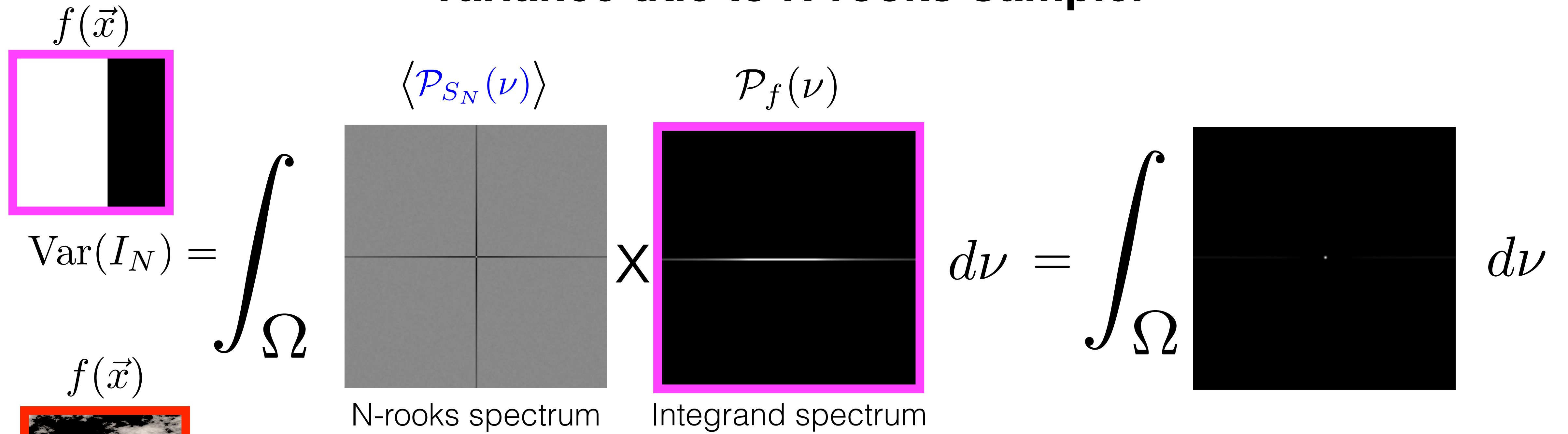
$\langle \mathcal{P}_{S_N}(\nu) \rangle$ is represented by a gray square divided into four quadrants by a vertical line, with a blue double-headed arrow across the horizontal center.

$\mathcal{P}_f(\nu)$ is represented by a black square with a white horizontal line across the middle, outlined in magenta.

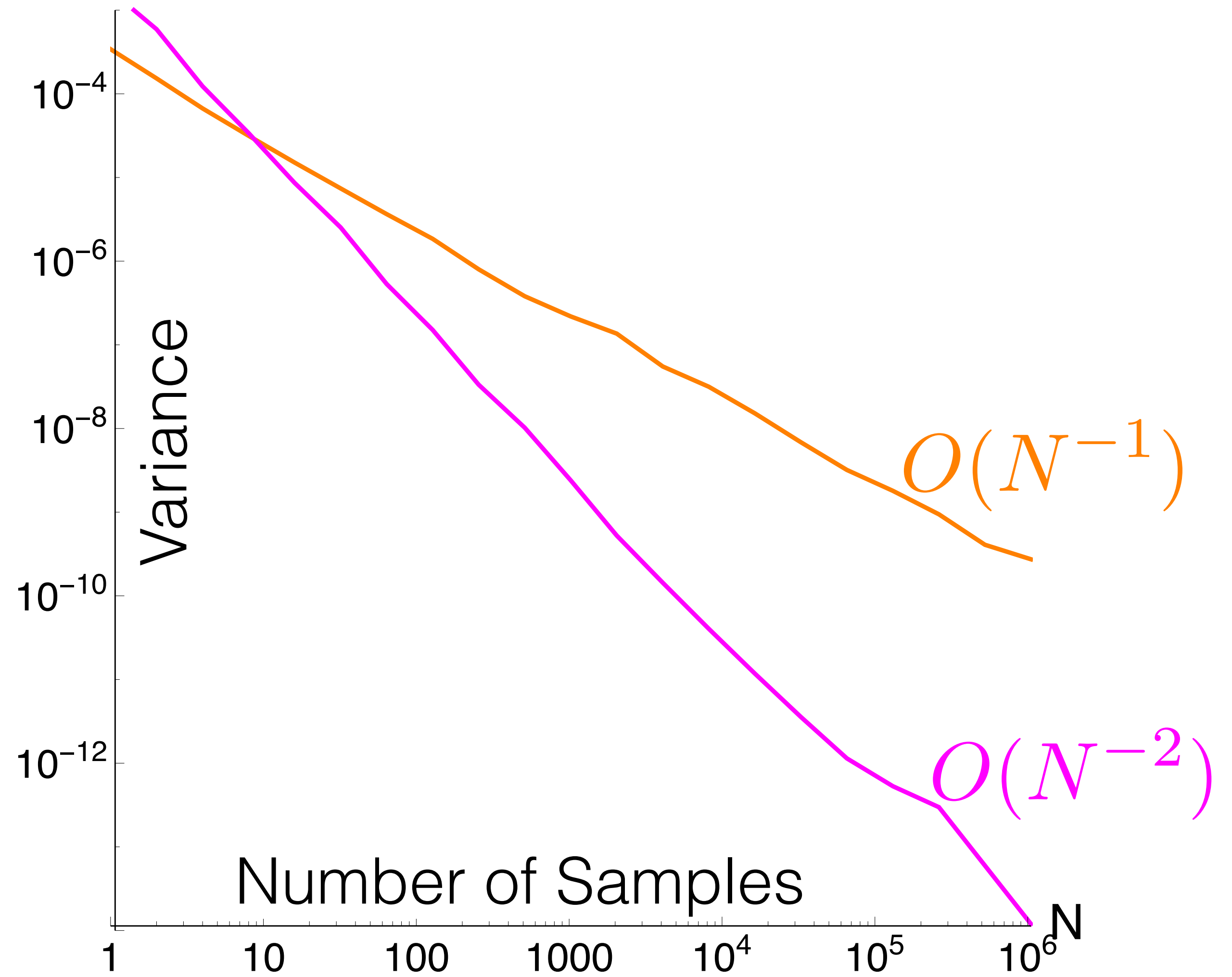
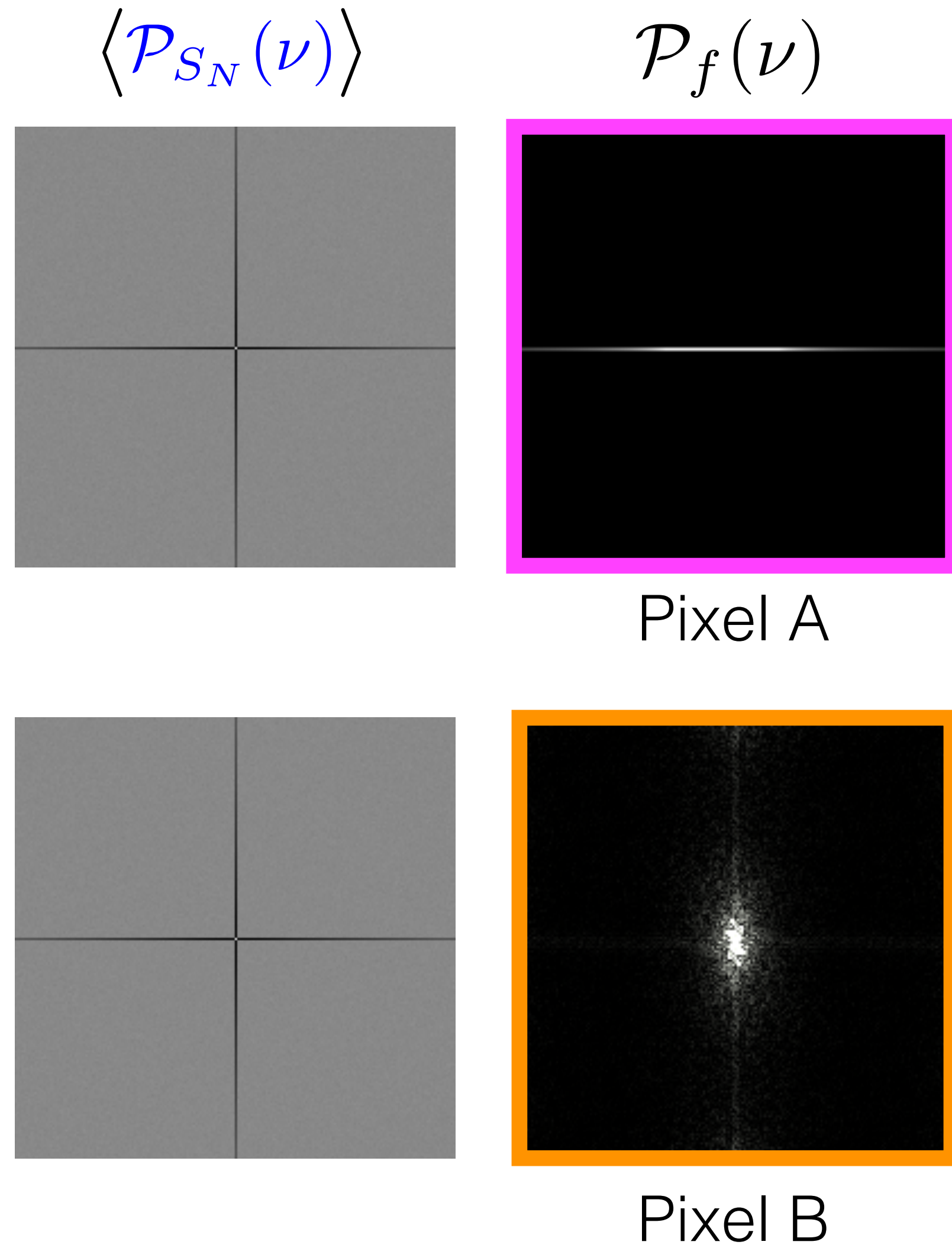
$d\nu$ is represented by a solid black square.

The diagram shows the variance $\text{Var}(I_N)$ as an integral over Ω of the product of $f(\vec{x})$, $\langle \mathcal{P}_{S_N}(\nu) \rangle$, and $\mathcal{P}_f(\nu)$. This is equated to an integral over Ω of the product of $d\nu$ and the $\mathcal{P}_f(\nu)$ spectrum.

Variance due to N-rooks Sampler

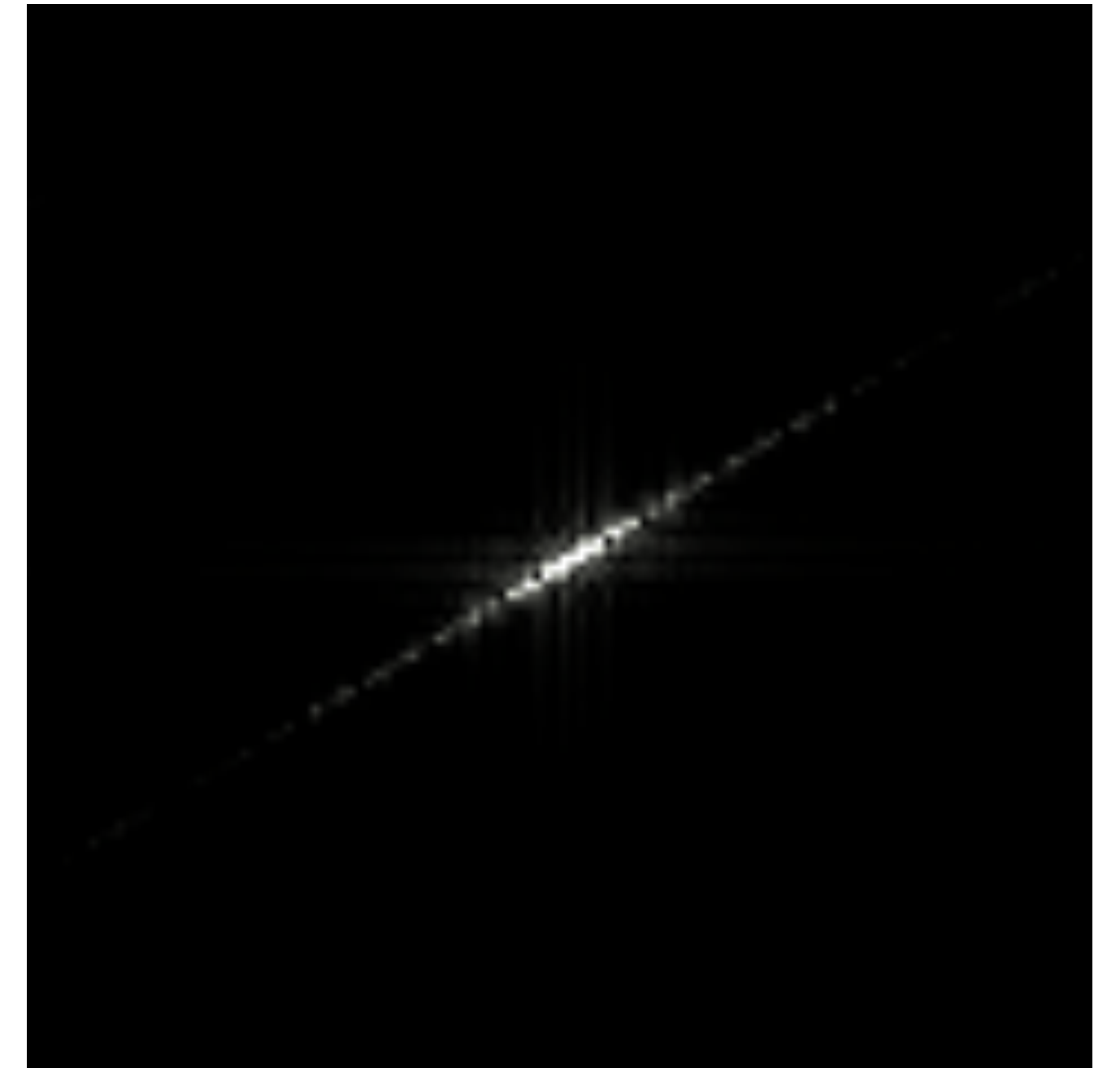


Variance Convergence of Latin Hypercube (N-rooks)



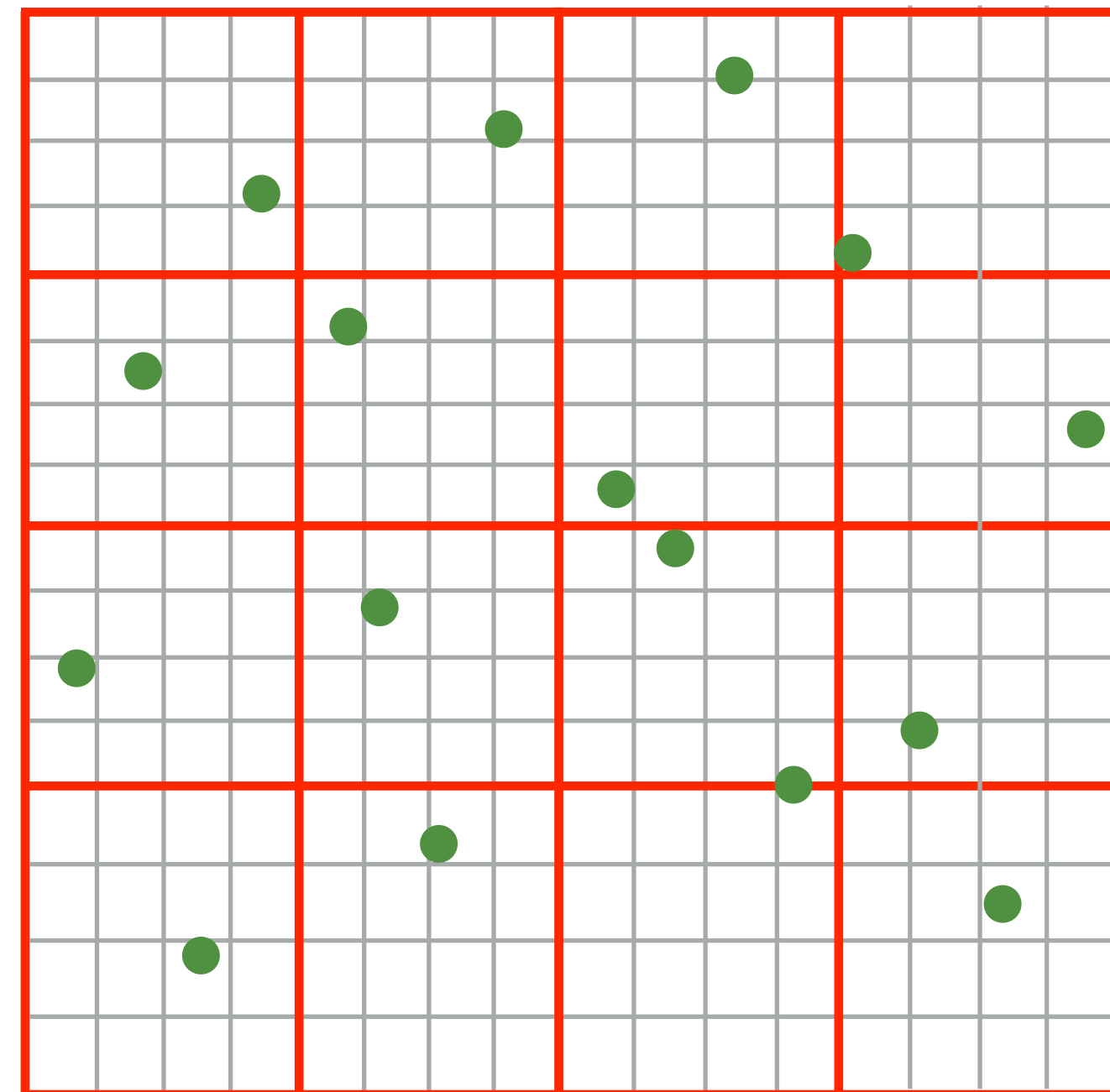
Non-Axis Aligned Integrand Spectra

$$\mathcal{P}_f(\nu)$$



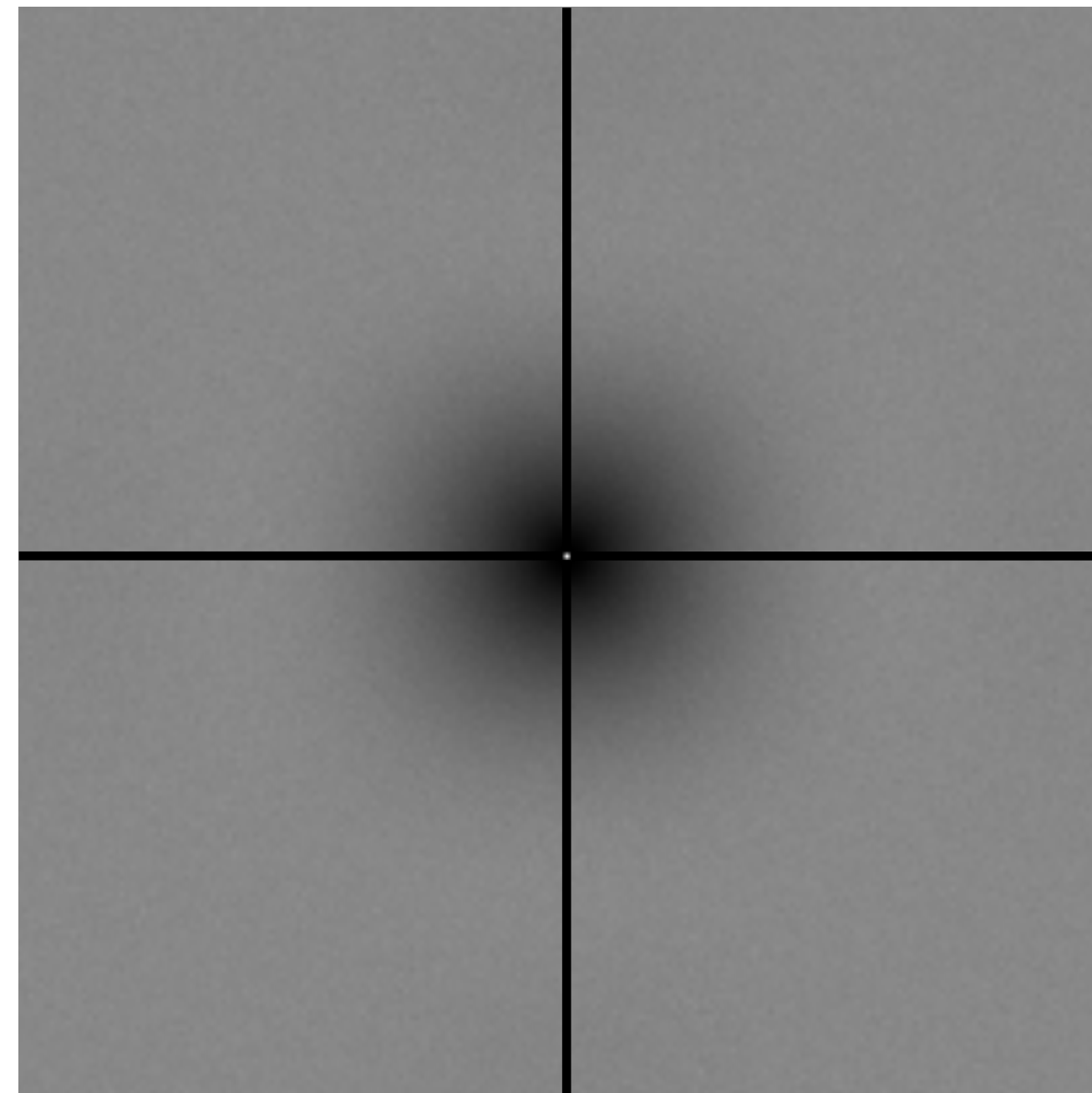
Integrand Spectrum

Non-Axis Aligned Integrand Spectra



Multi-jittered Samples

$$\langle \mathcal{P}_{S_N}(\nu) \rangle$$



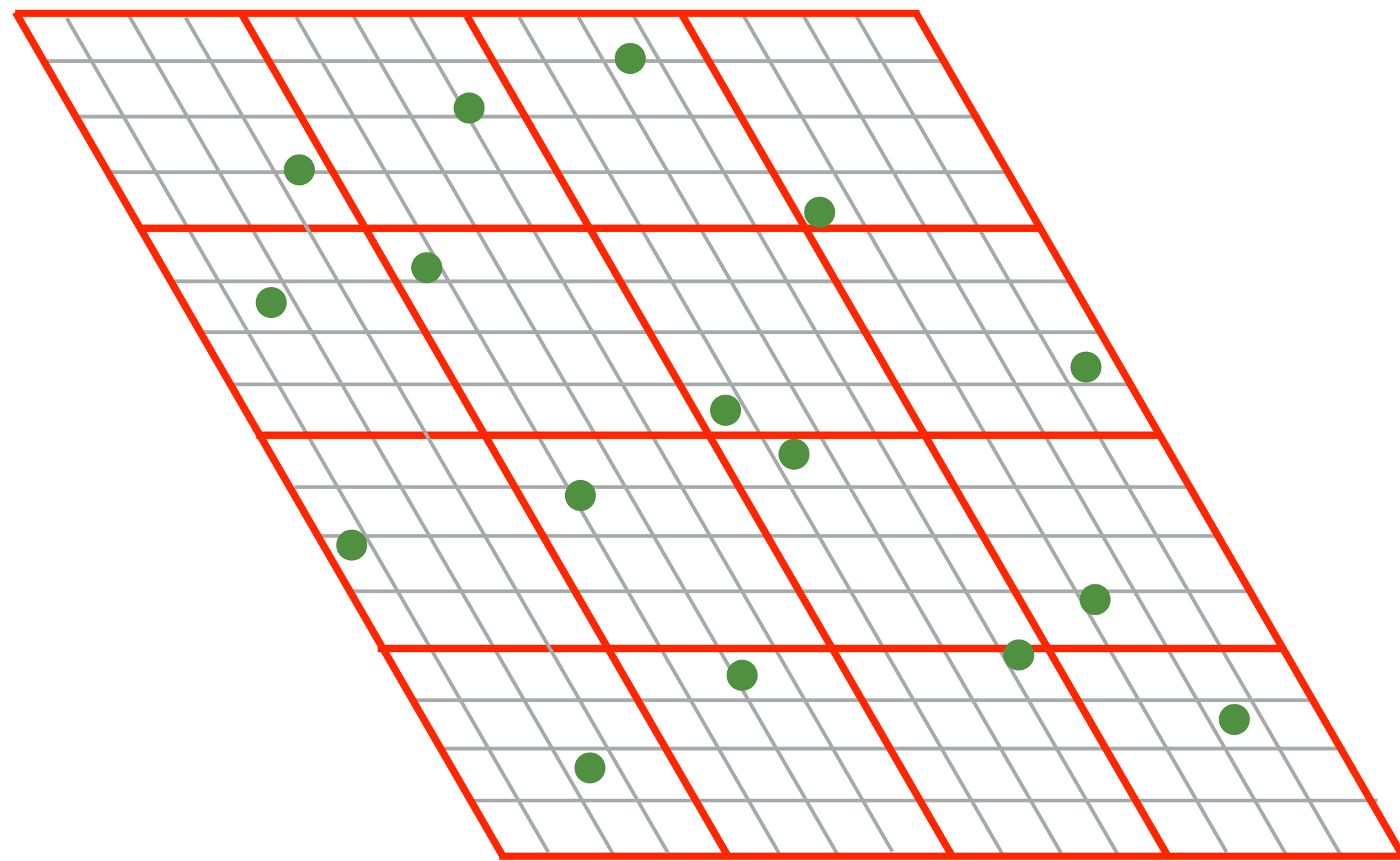
Sampling Spectrum

$$\mathcal{P}_f(\nu)$$



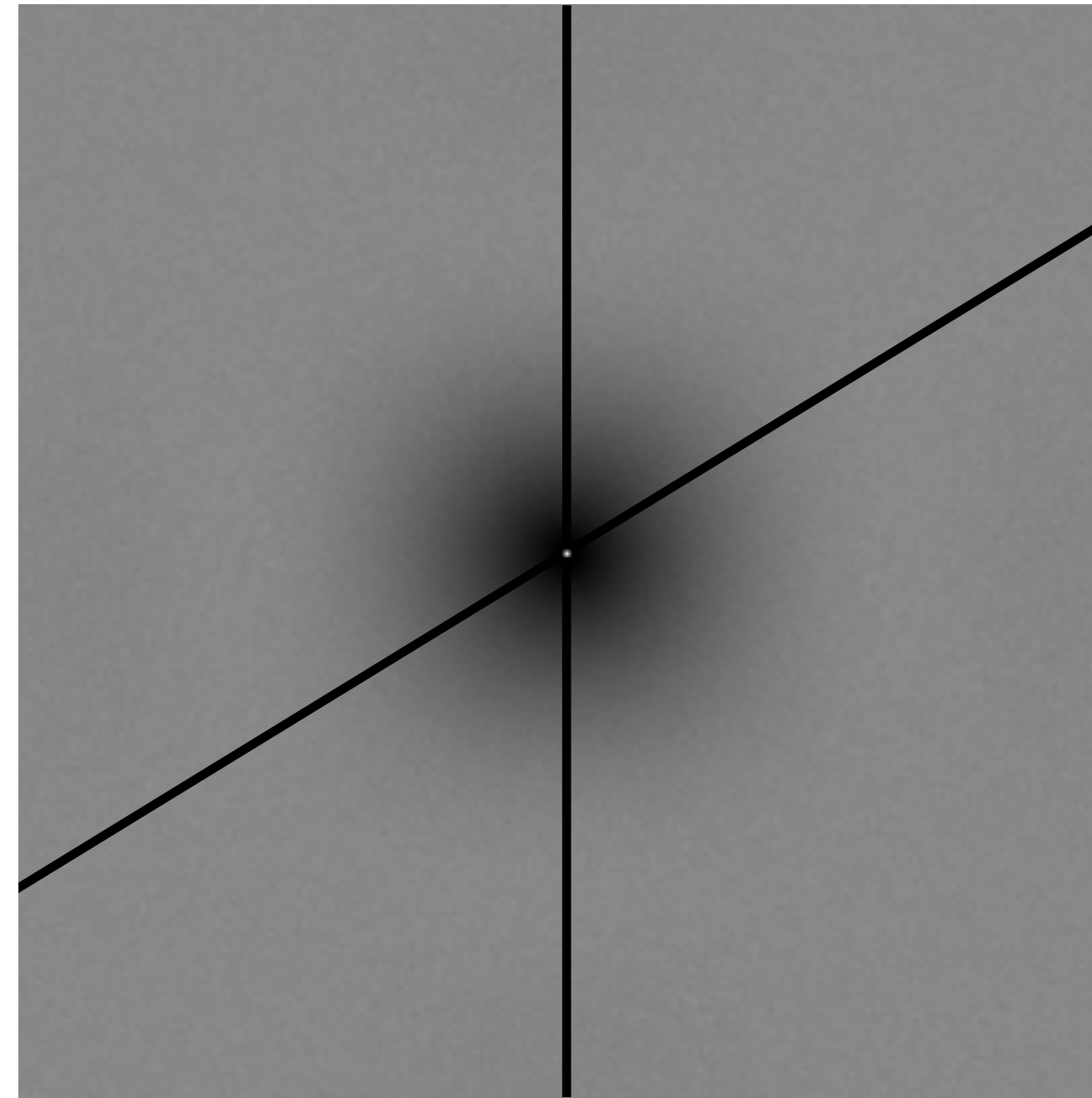
Integrand Spectrum

Shearing Multi-Jittered Samples



Sheared Samples

$$\langle \mathcal{P}_{S_N}(\nu) \rangle$$



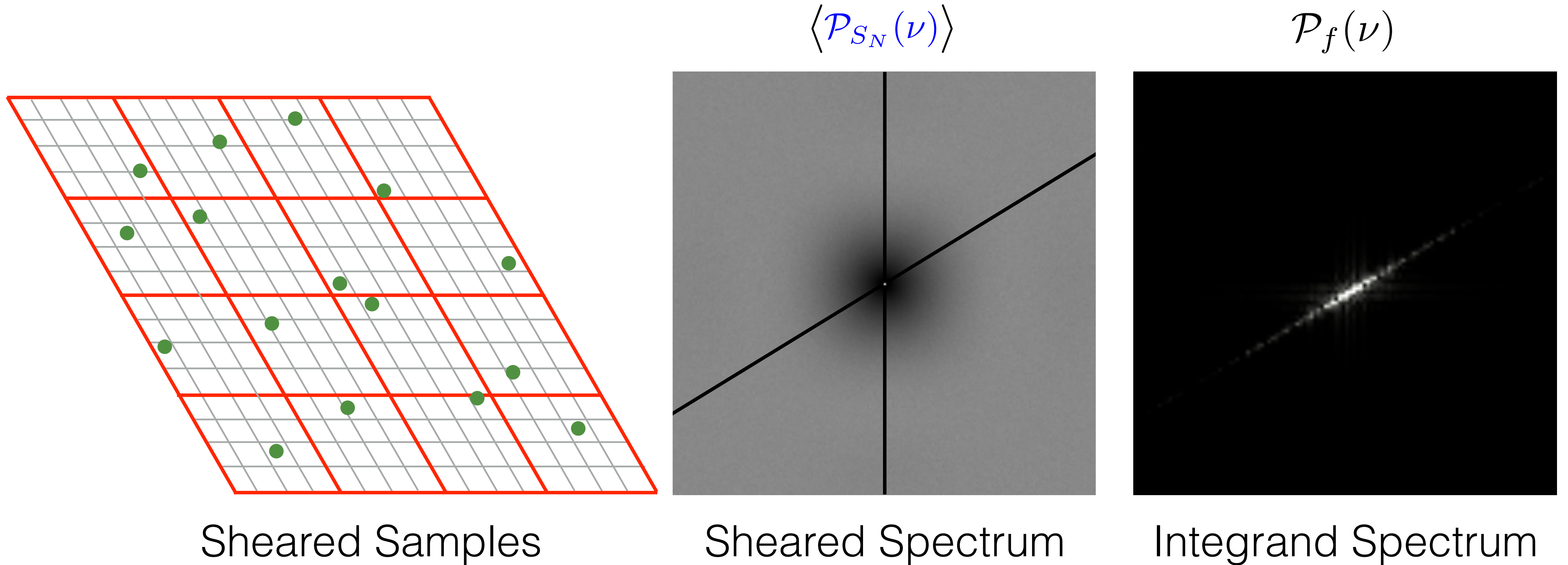
Sheared Spectrum

$$\mathcal{P}_f(\nu)$$



Integrand Spectrum

How can we determine the sample shearing parameters ?



Our Algorithm

- 1) Develop an oracle using the Frequency Analysis of Light Transport
- 2) Use this oracle to shear the samples
- 3) Perform Monte Carlo integration using the sheared samples

Frequency Analysis of Light Transport

Related Work

- Frequency Analysis of Light Transport Durand et al. [2005]
- Depth of Field Soler et al. [2009]
- Motion Blur Egan et al. [2009]
- Ambient Occlusion Egan et al. [2011] and more...

Related Work

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Related Work

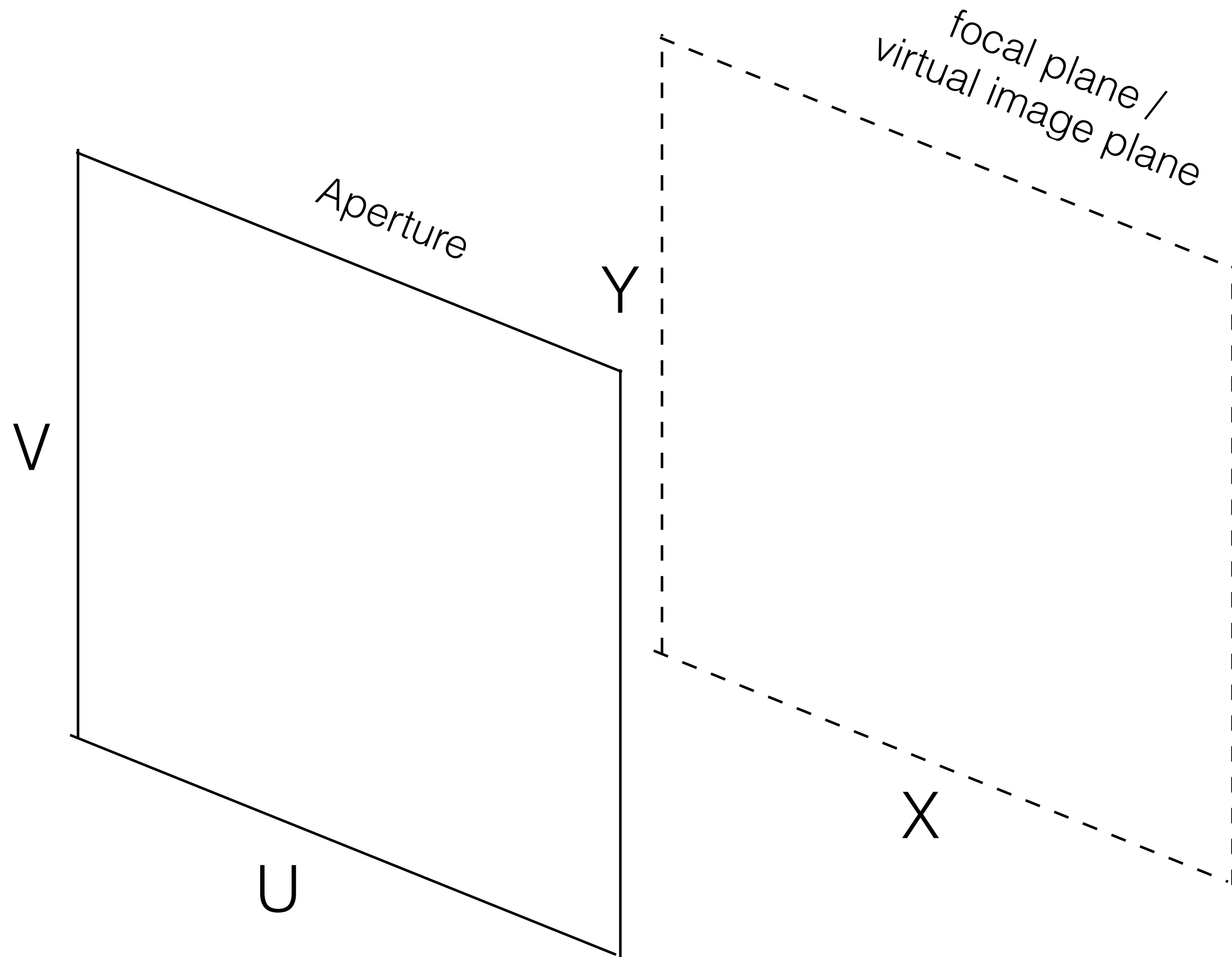
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- Depth of Field Soler et al. [2009]
- Motion Blur Egan et al. [2009]
- Ambient Occlusion Egan et al. [2011] and more...

Reconstruction

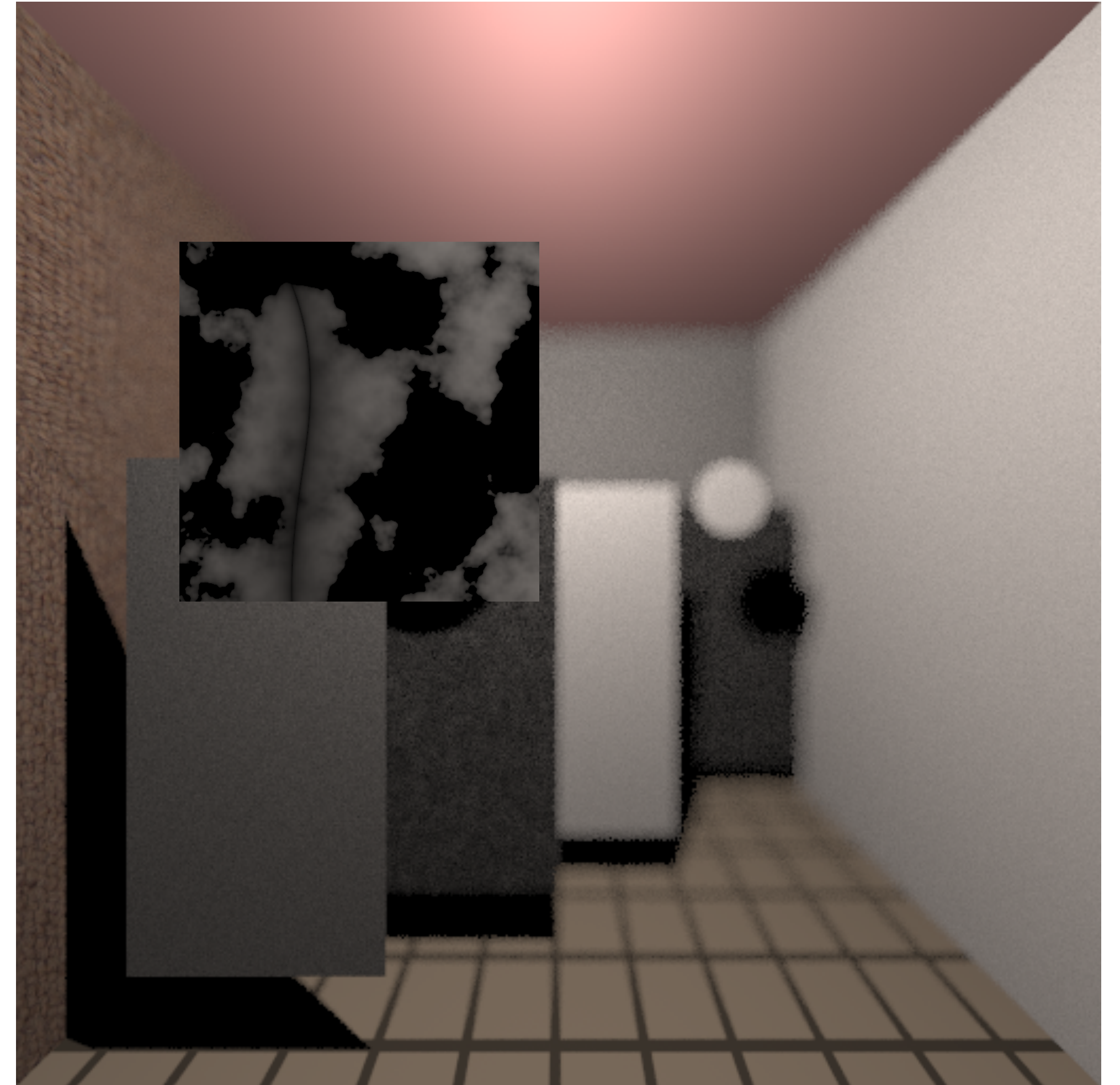
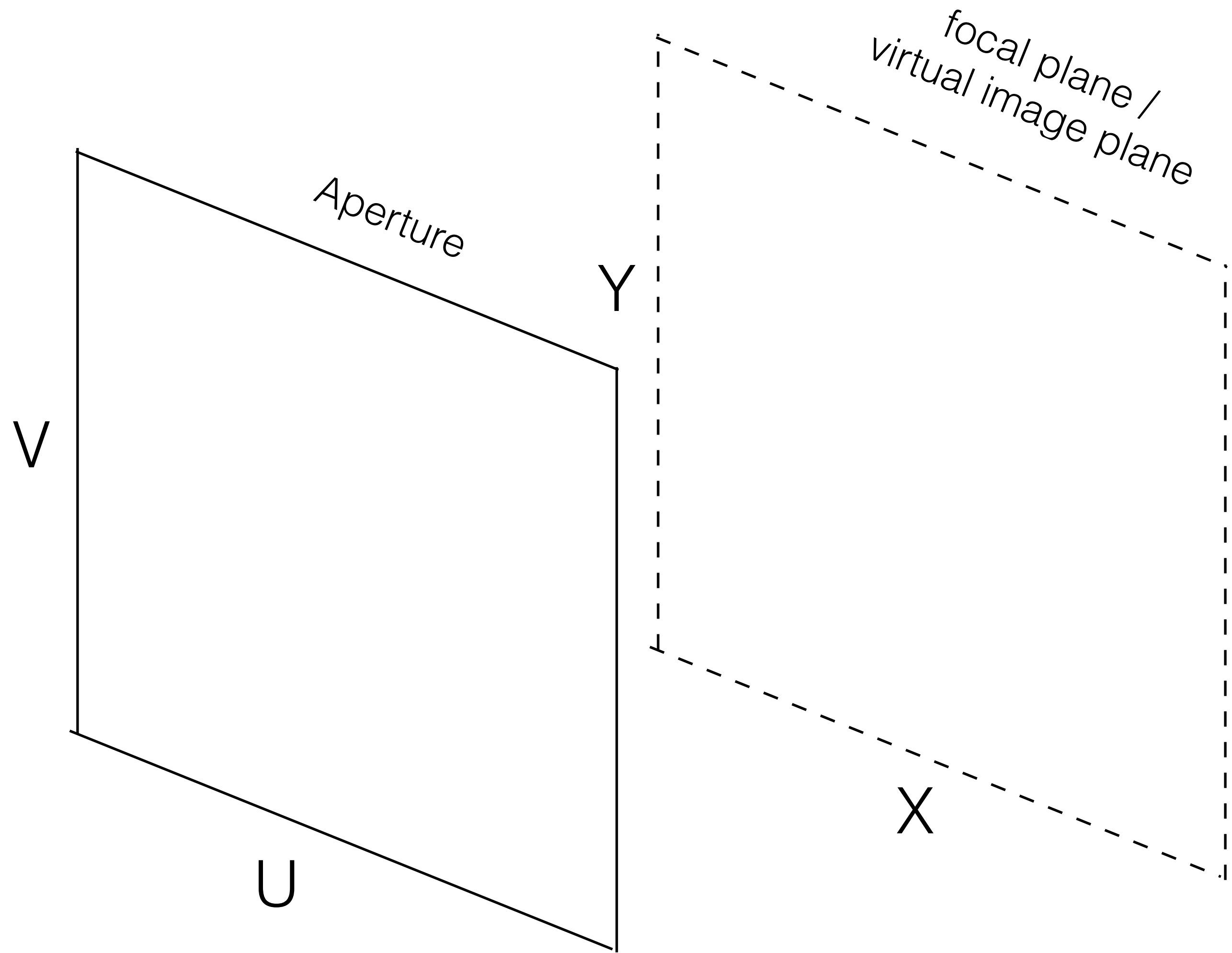
Our Work

Integration

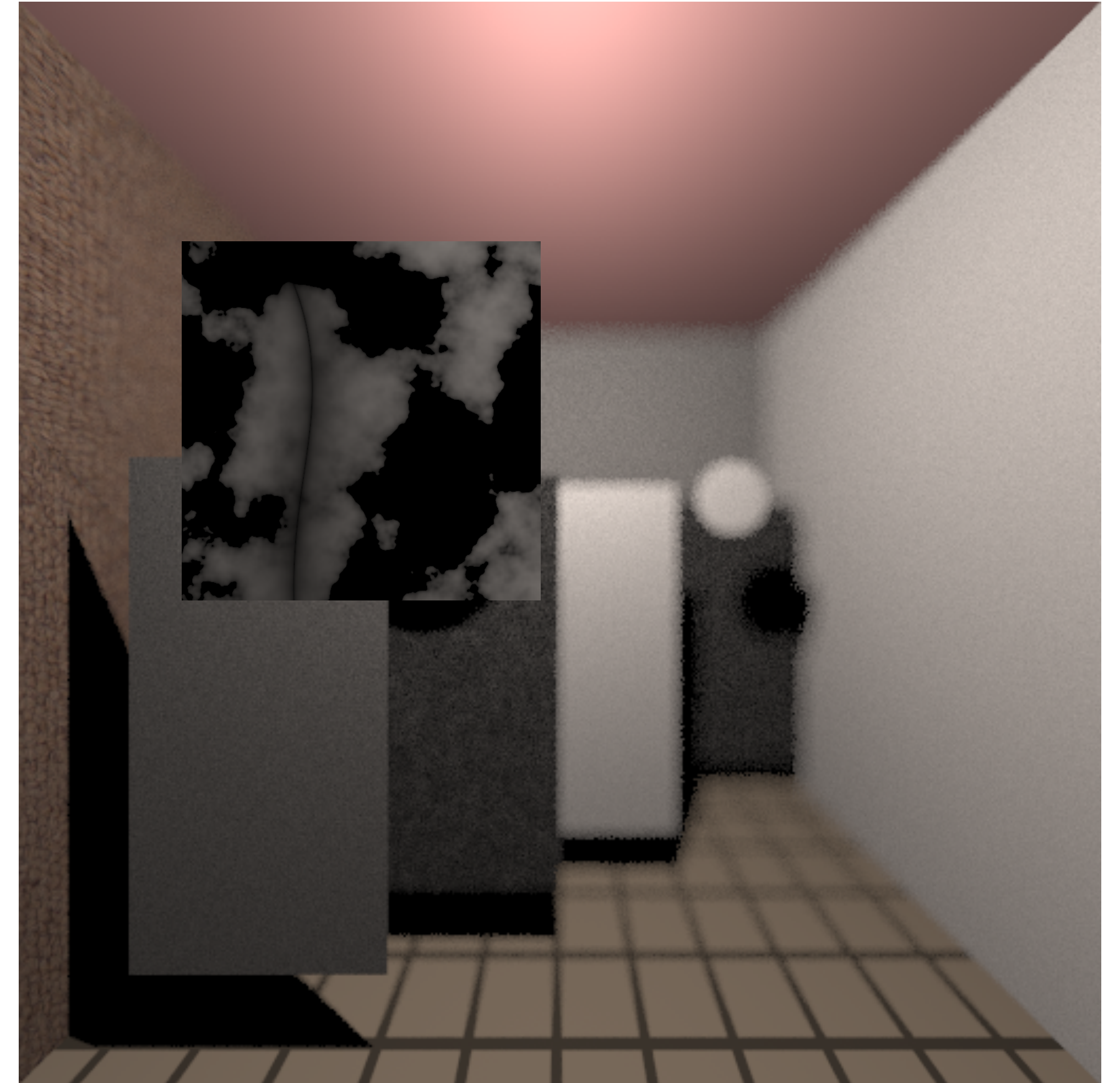
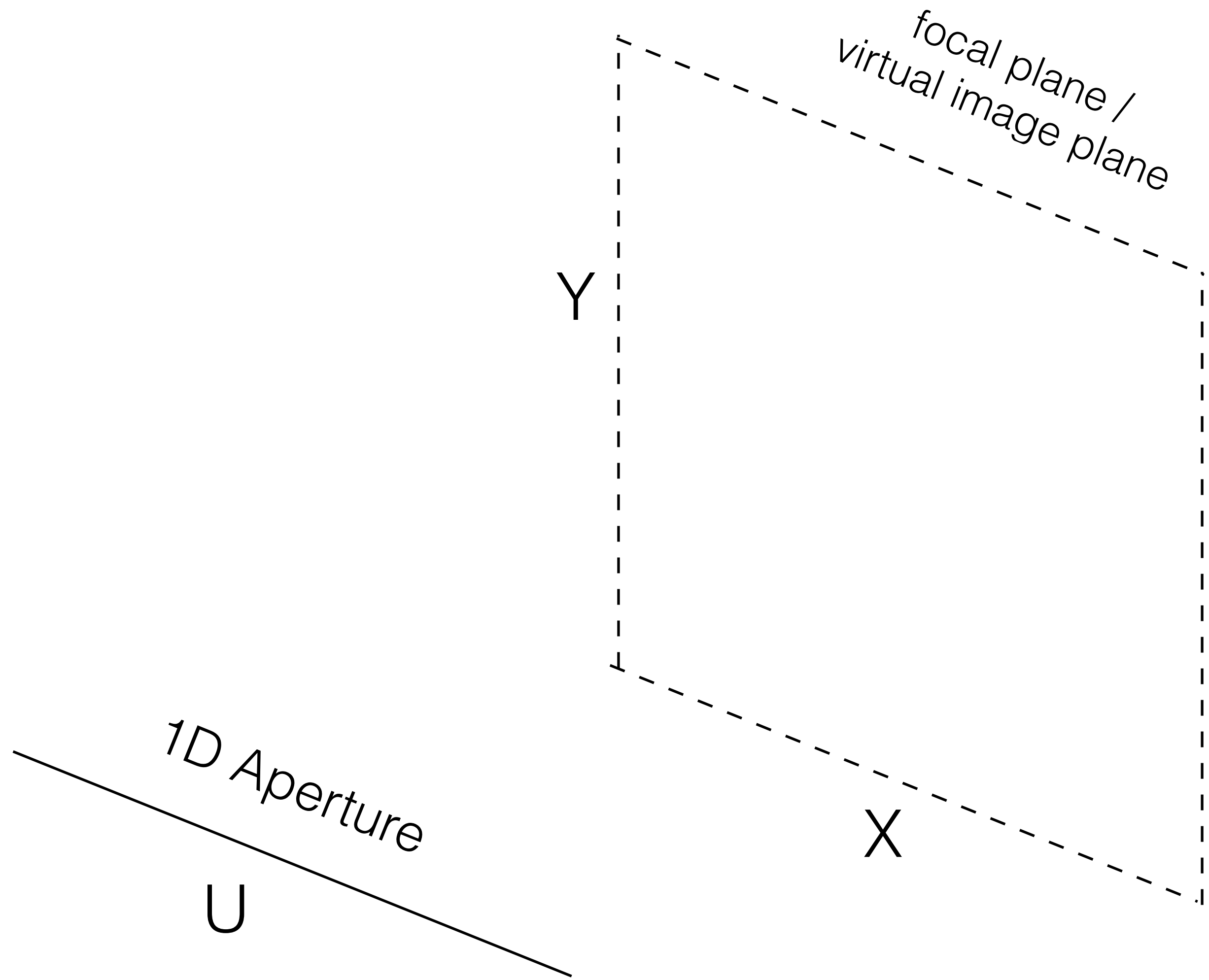
Depth of Field Analysis



Depth of Field Analysis

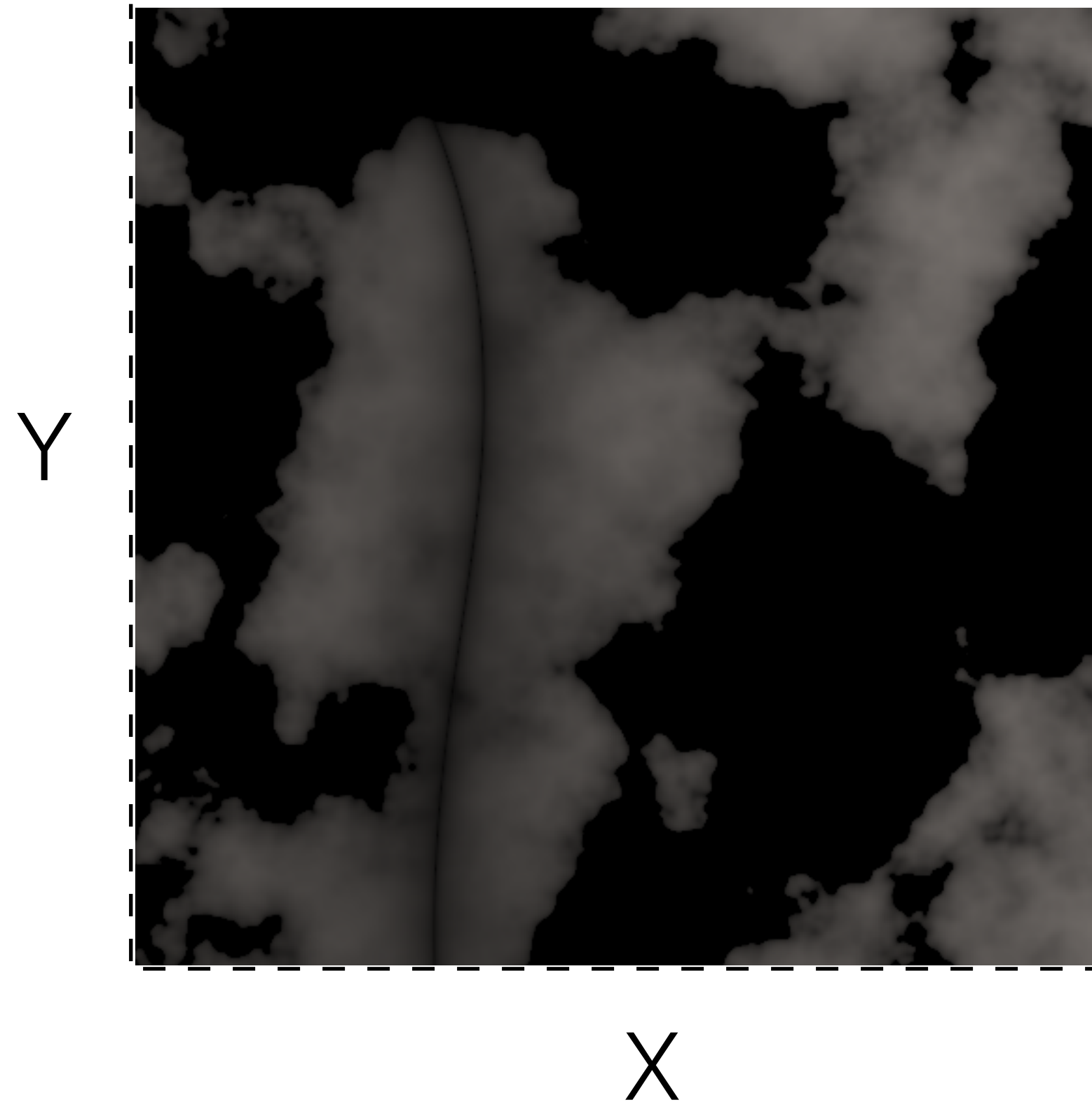


Depth of Field Analysis

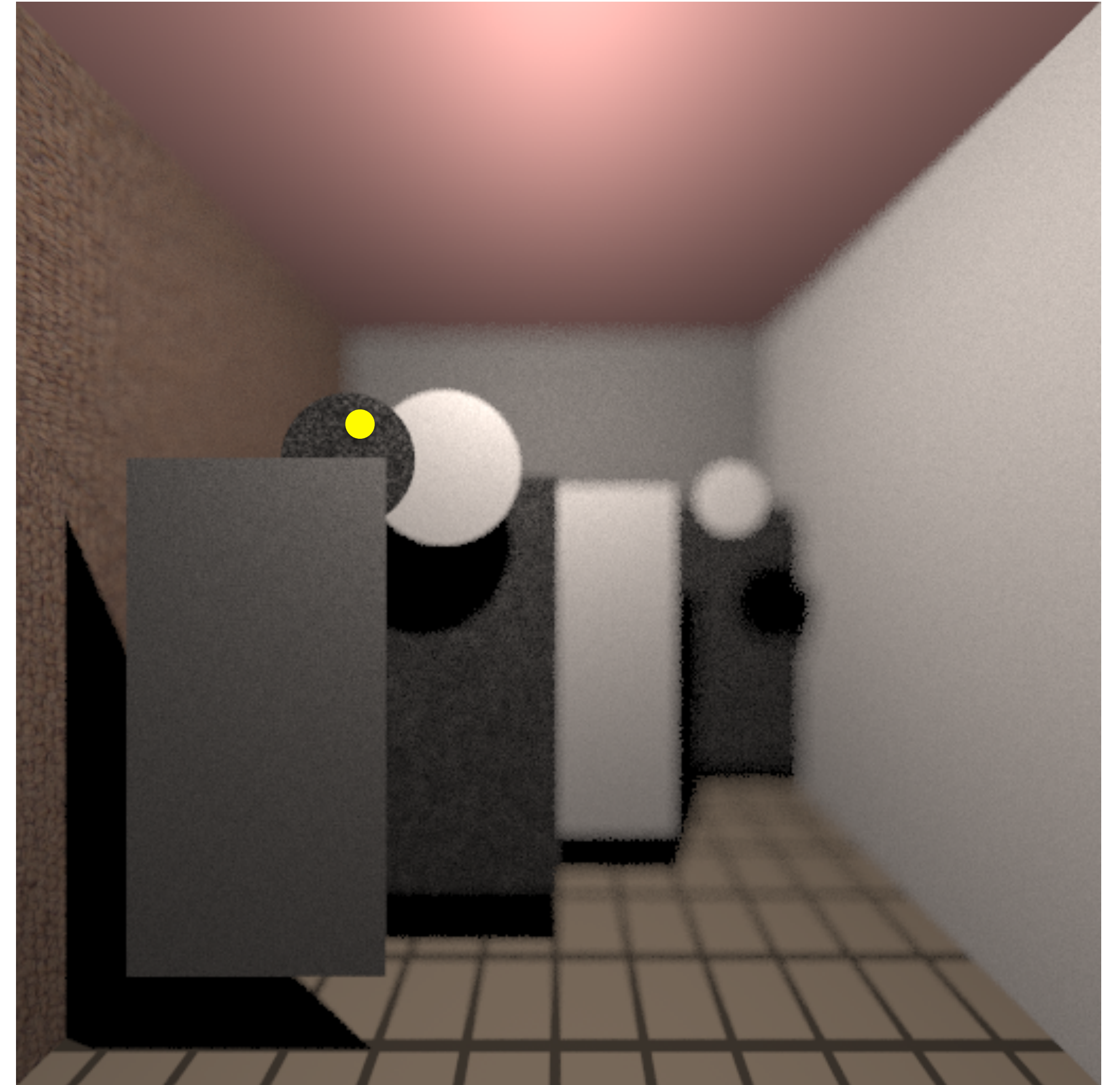
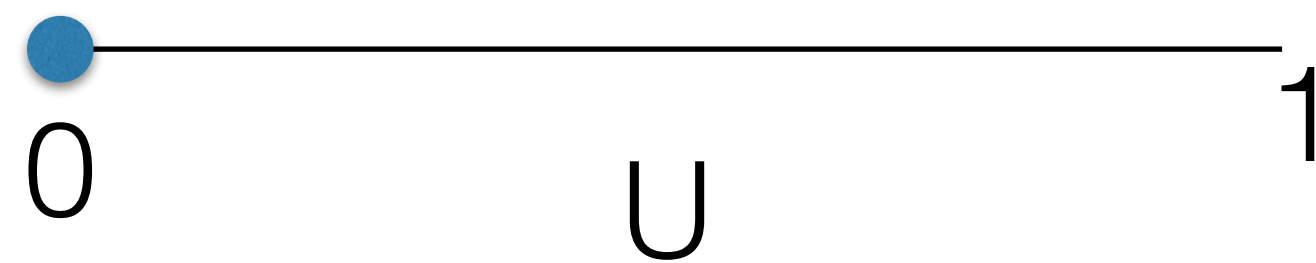


Depth of Field Analysis

focal plane /
virtual image plane

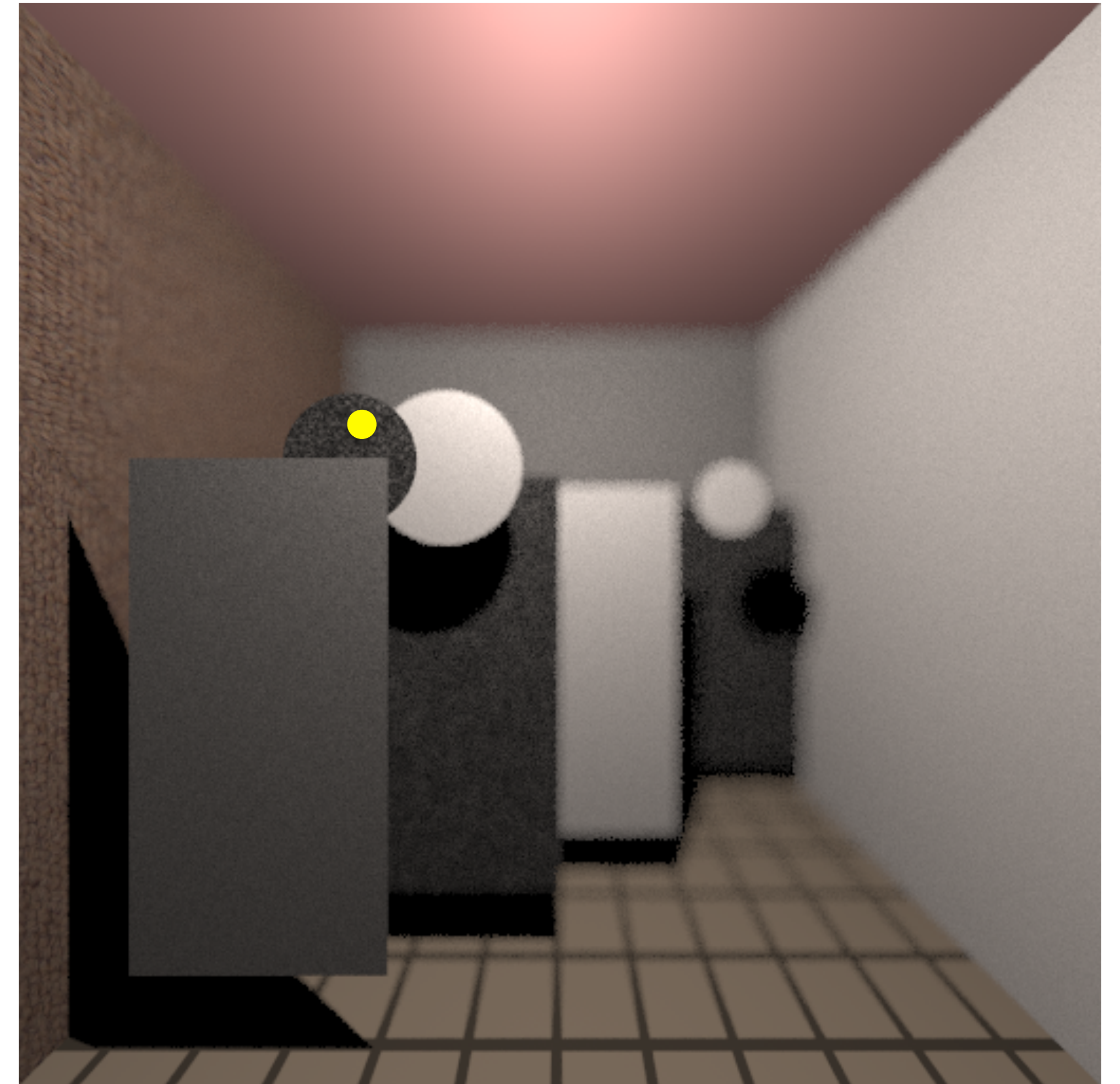
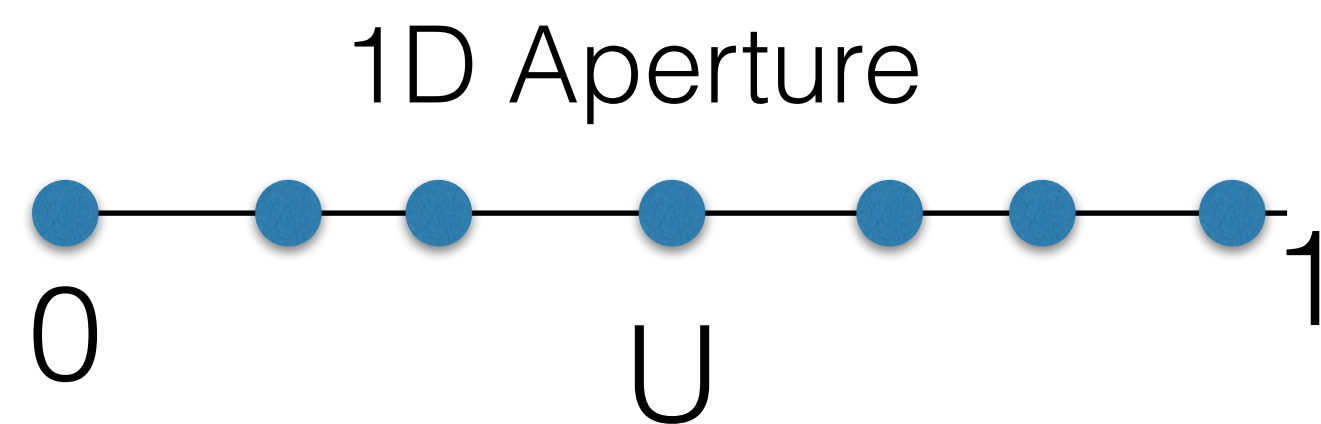
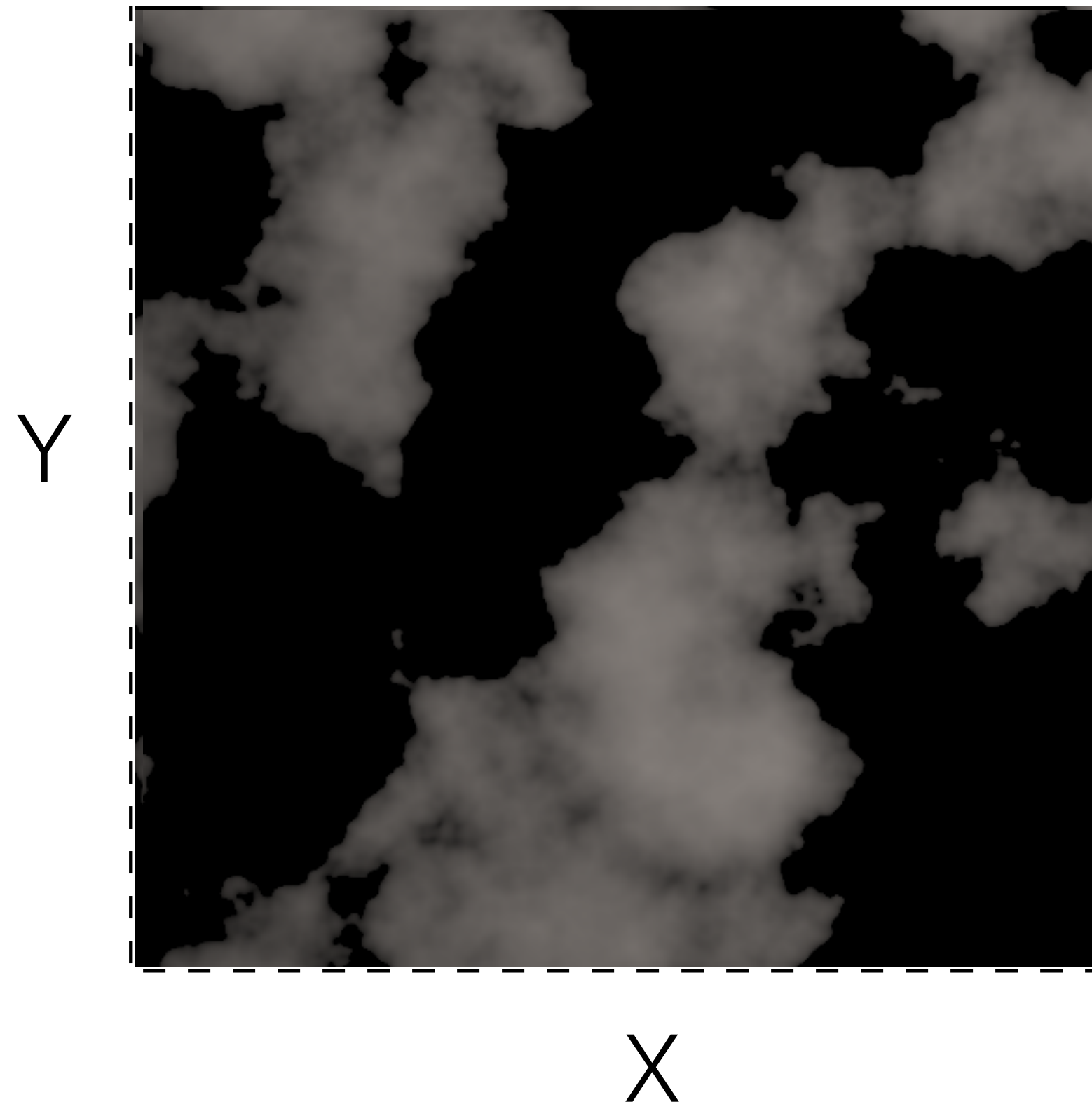


1D Aperture

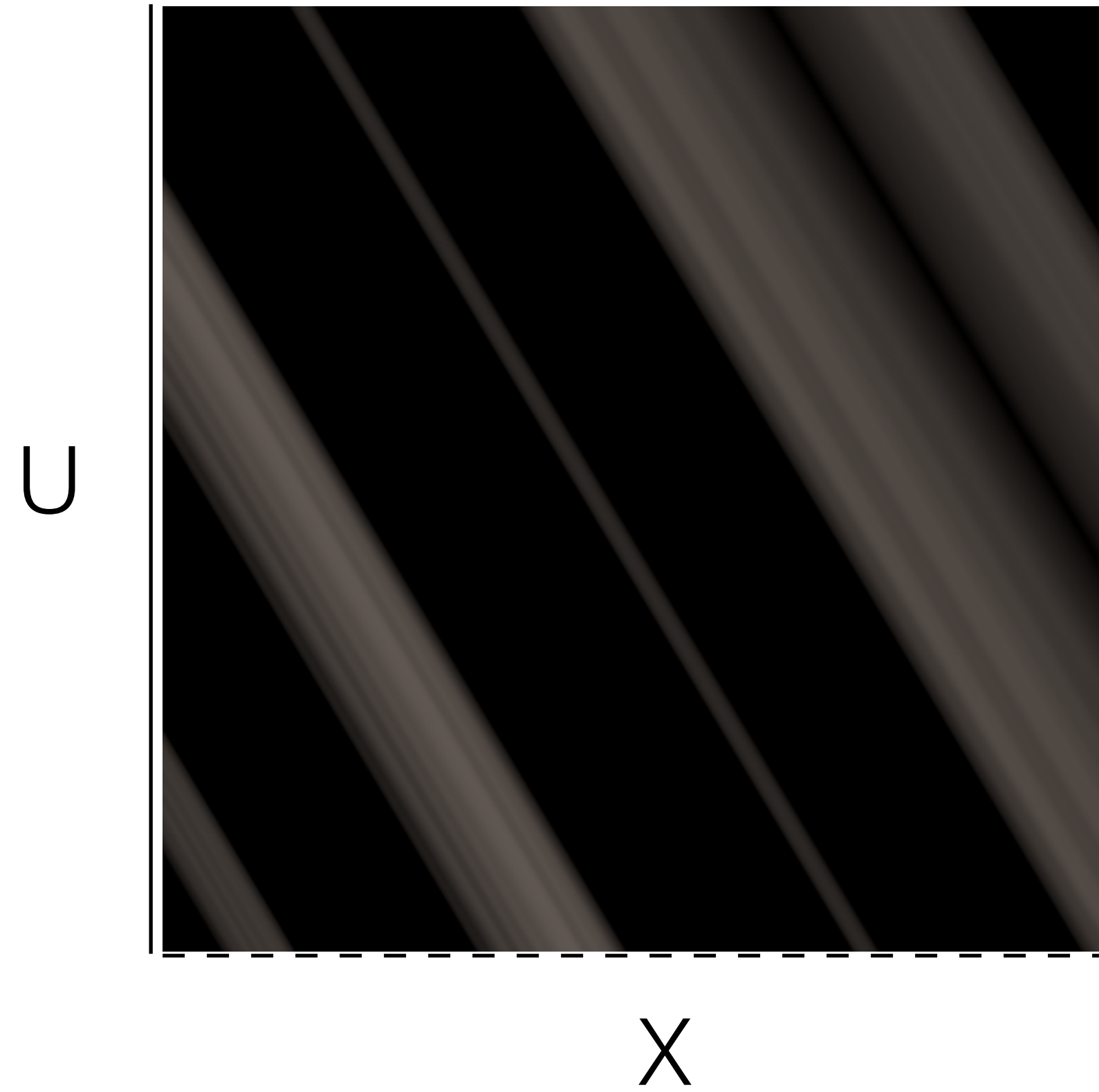


Depth of Field Analysis

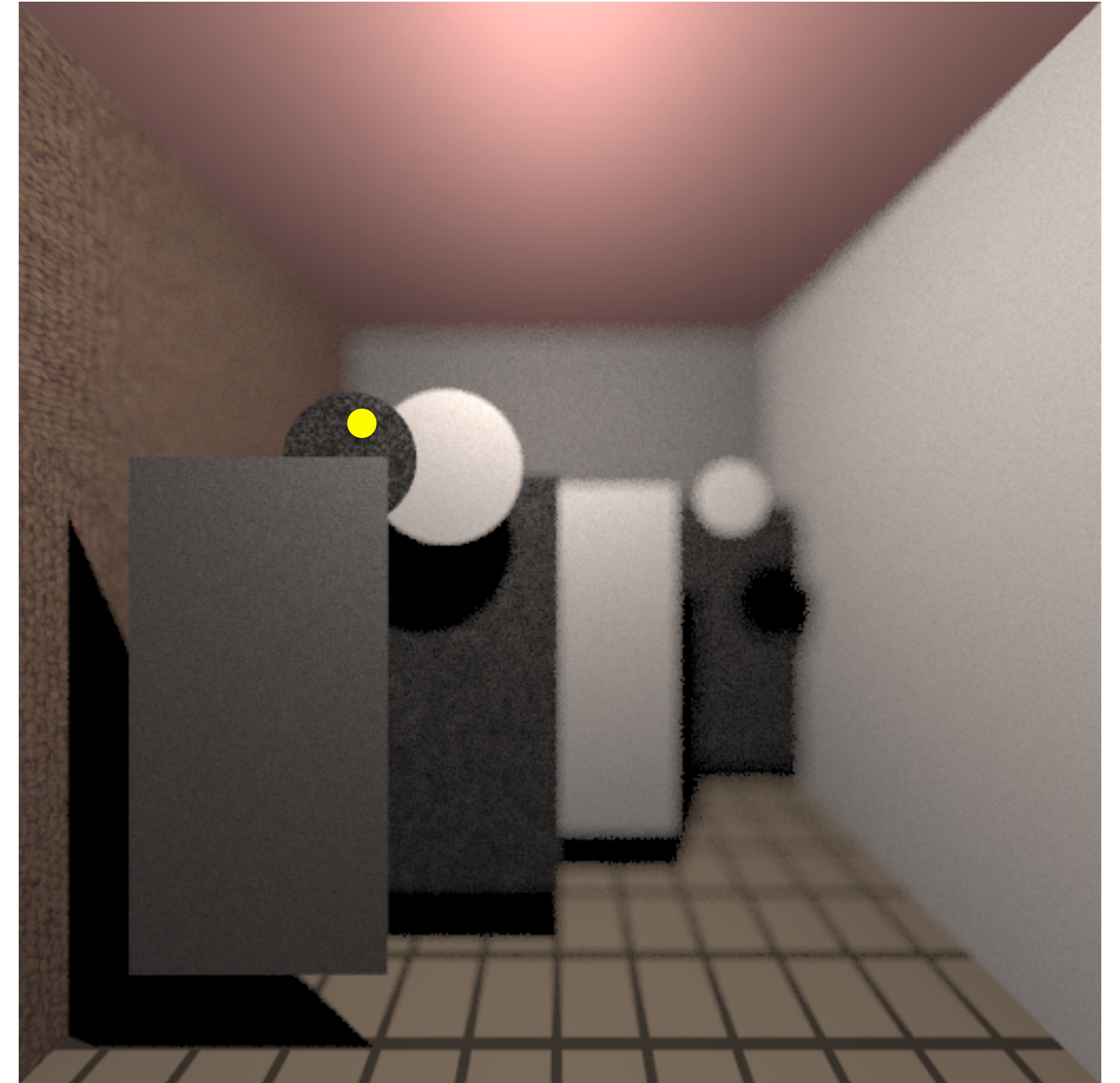
focal plane /
virtual image plane



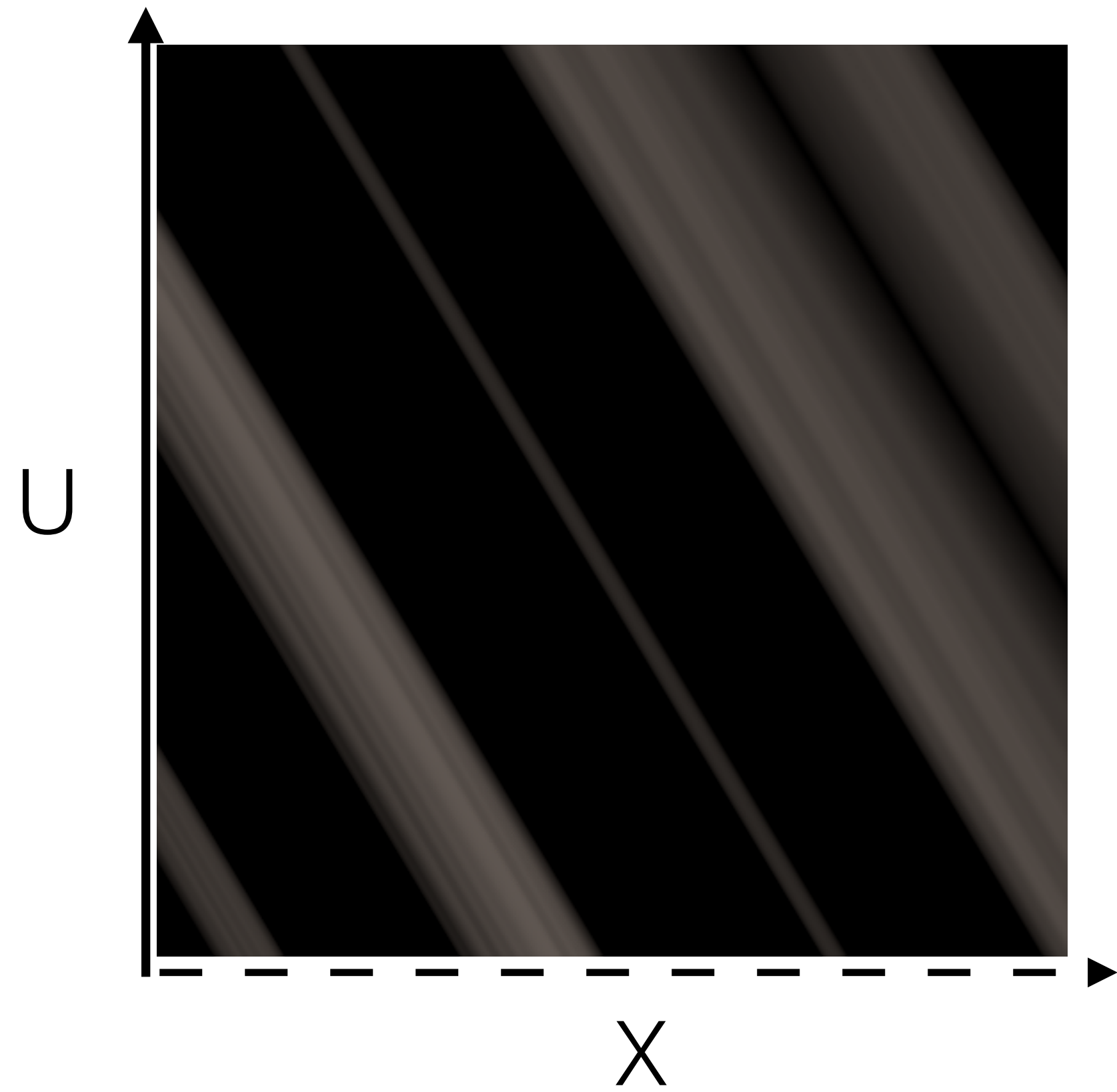
Depth of Field Analysis



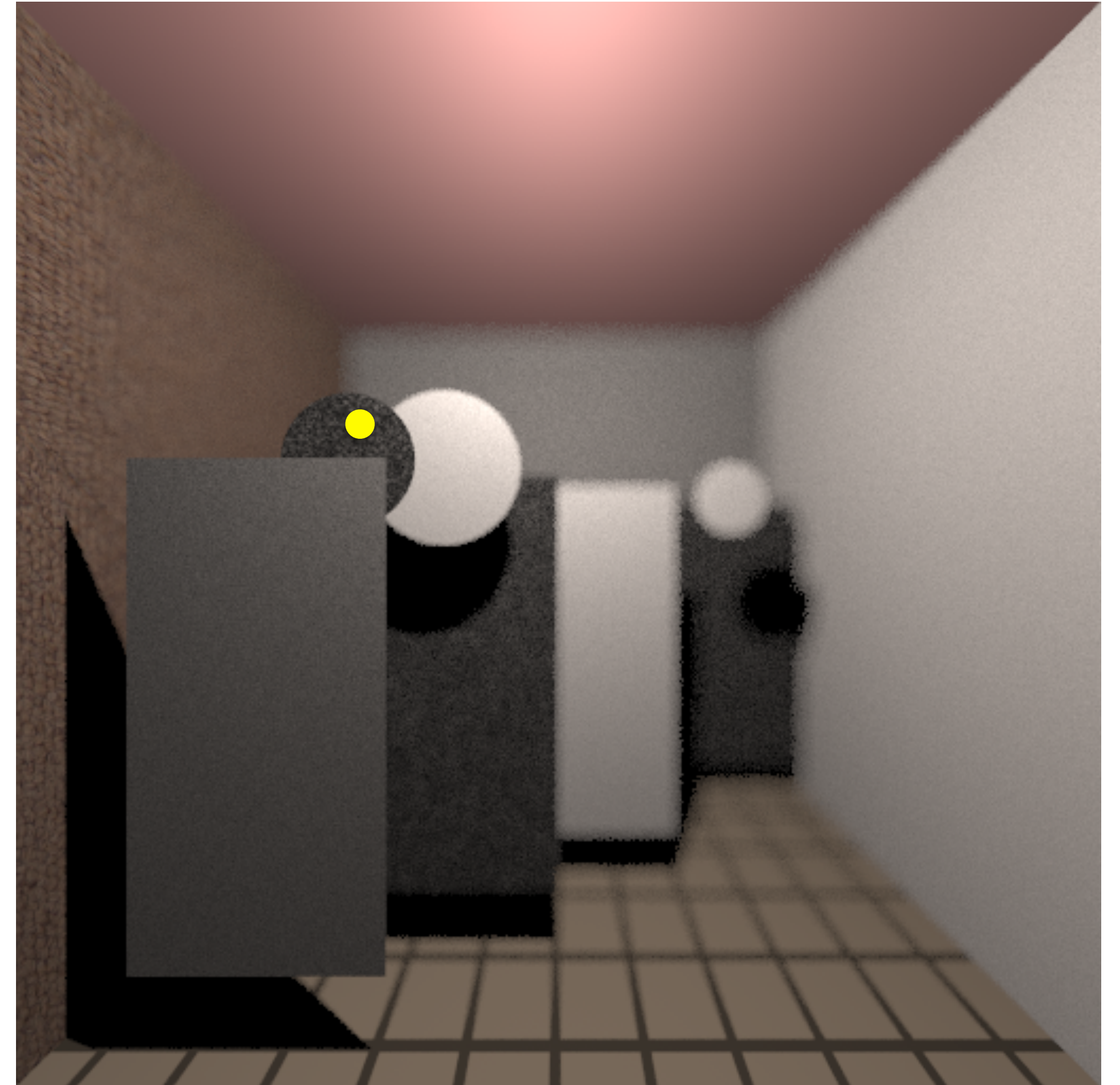
XU Slices



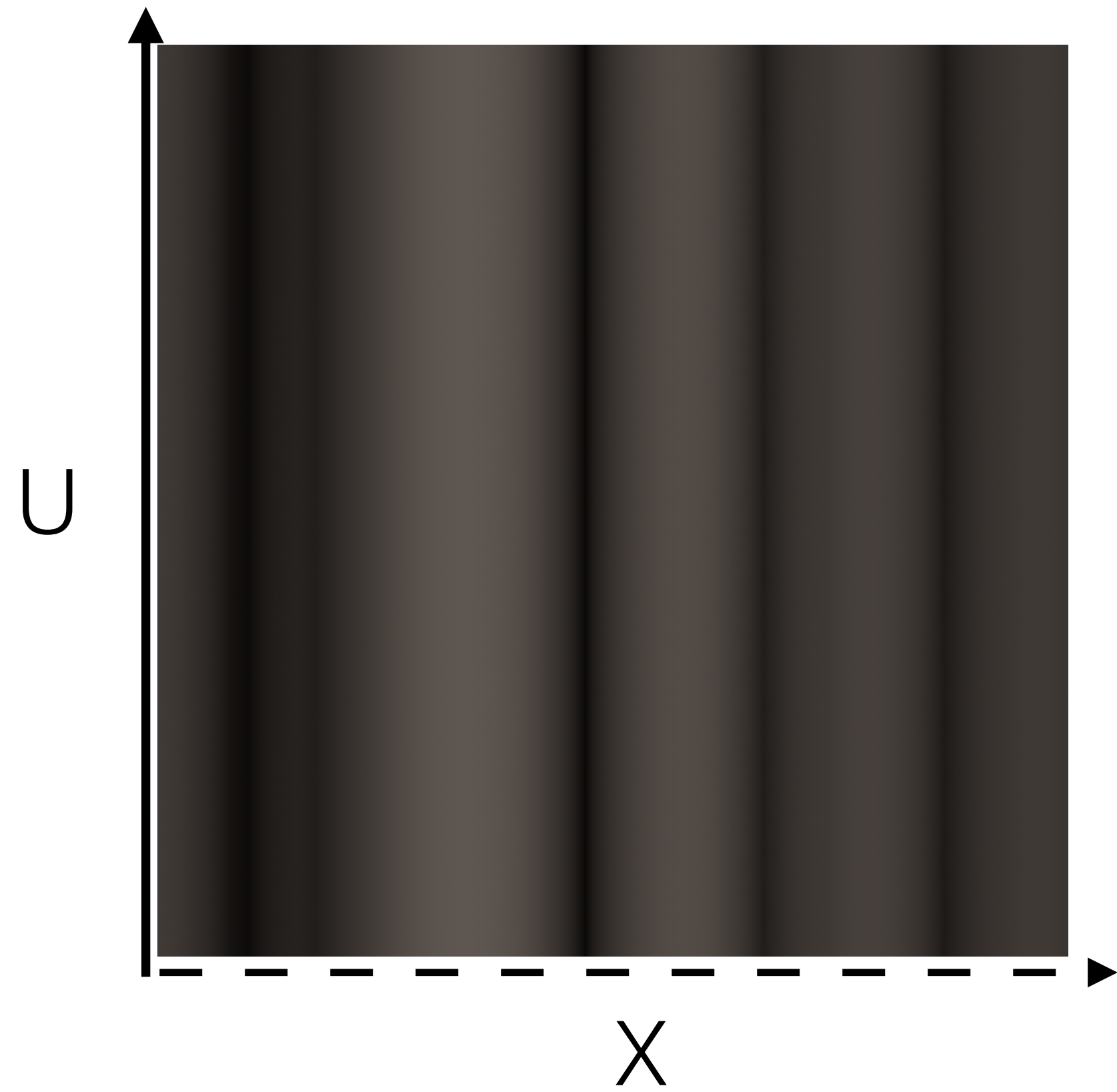
Depth of Field Analysis



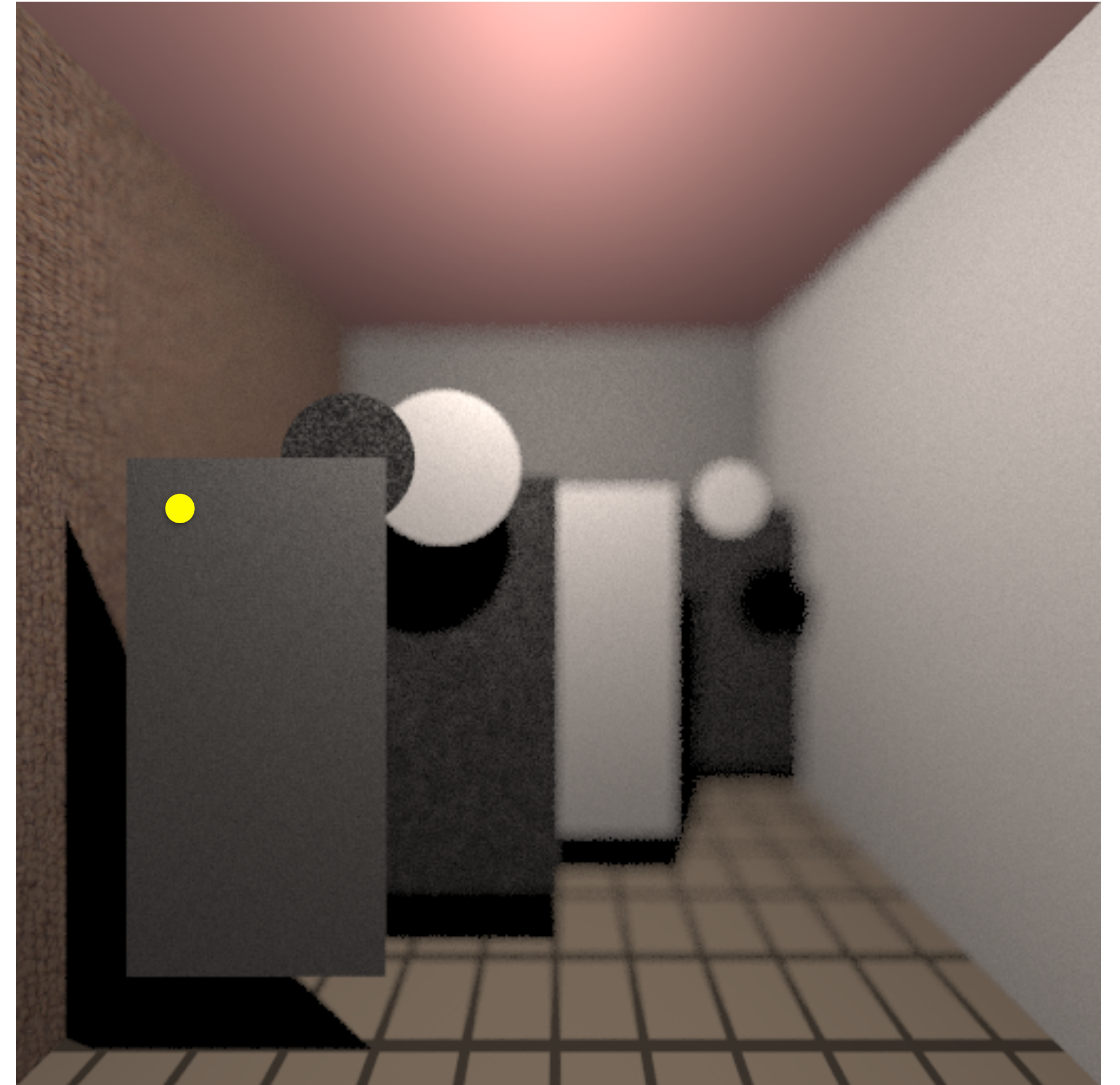
XU Slices



Depth of Field Analysis



XU Slices



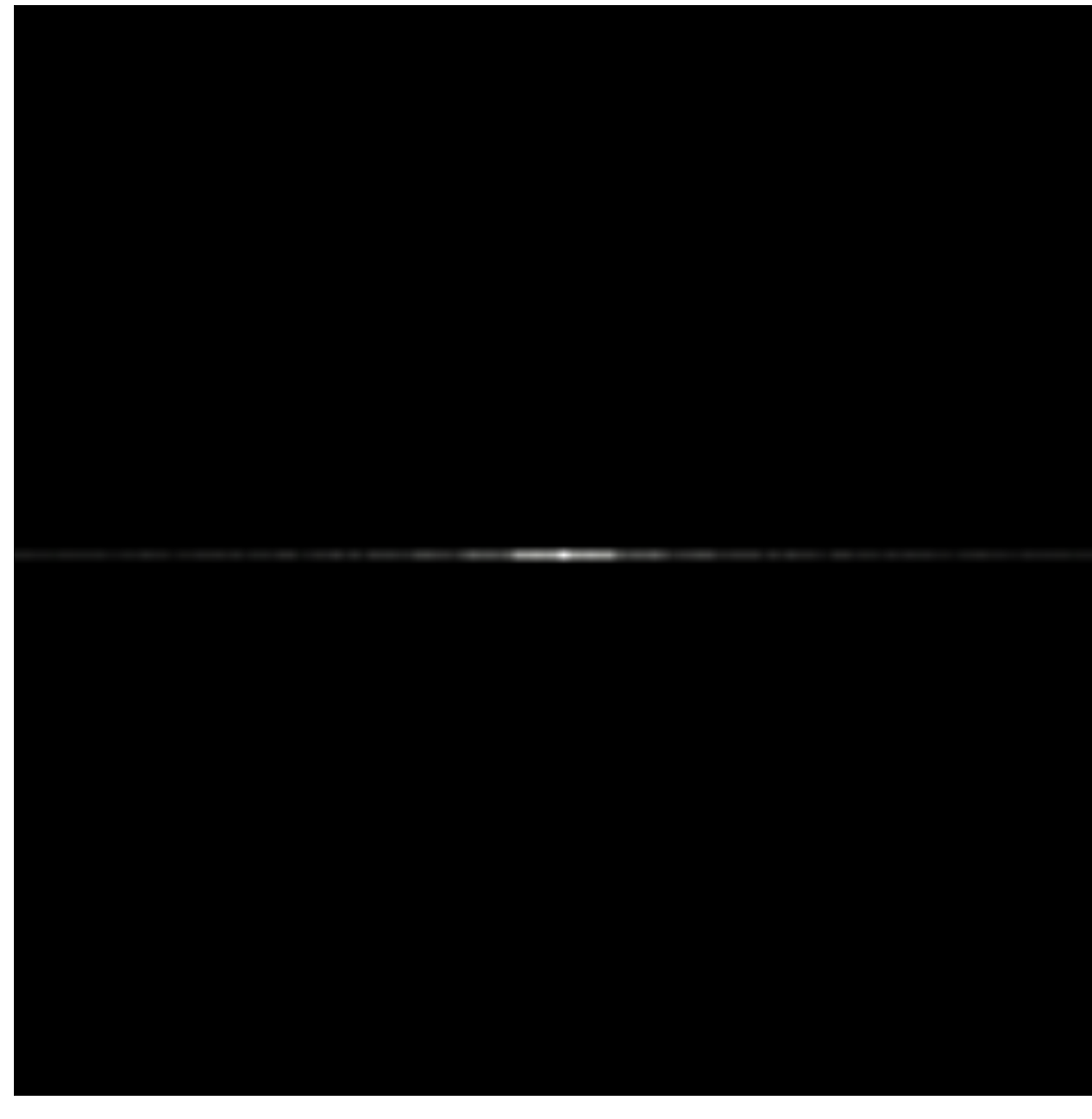
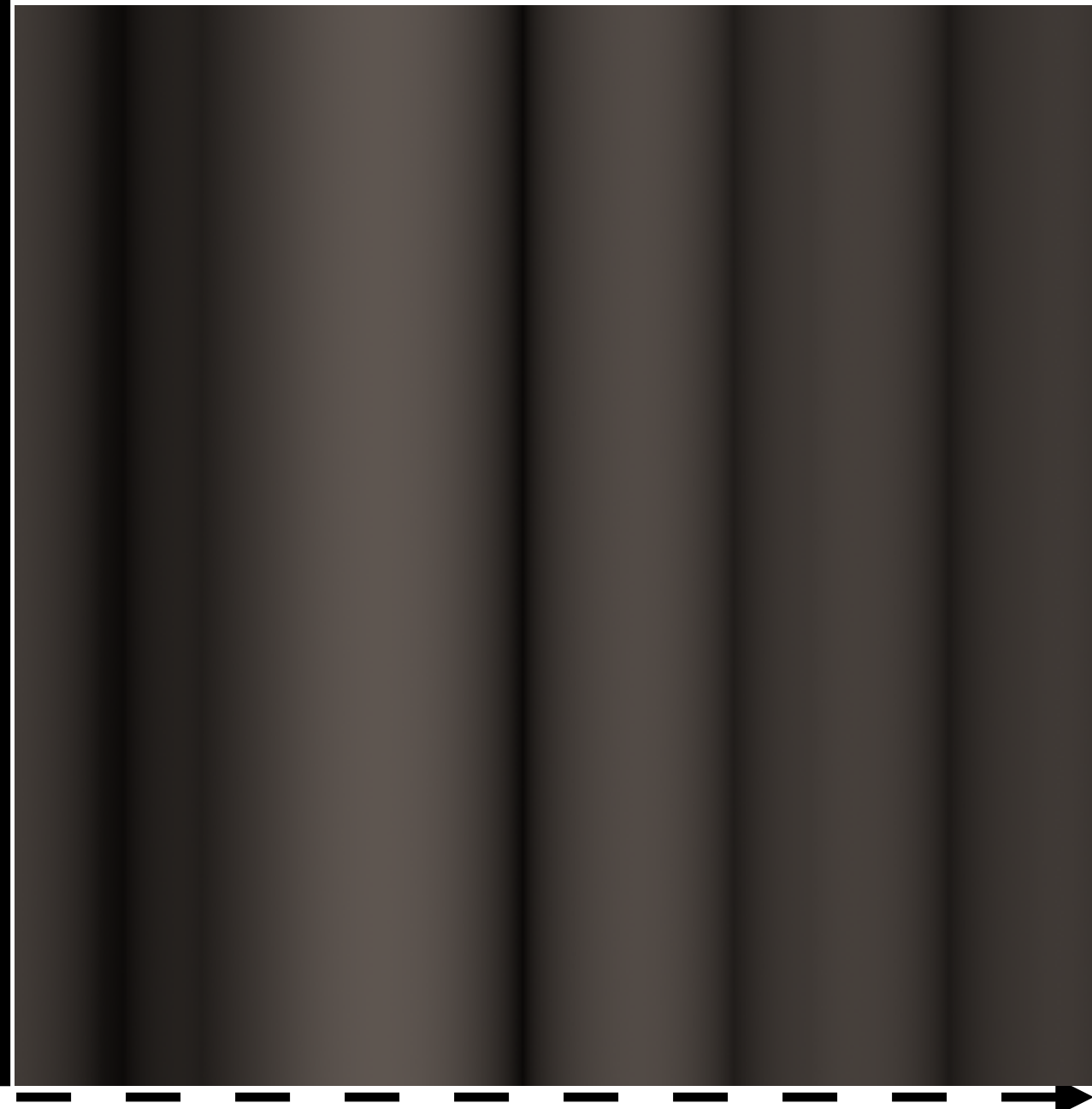
Depth of Field Analysis

Ray space

Spatial

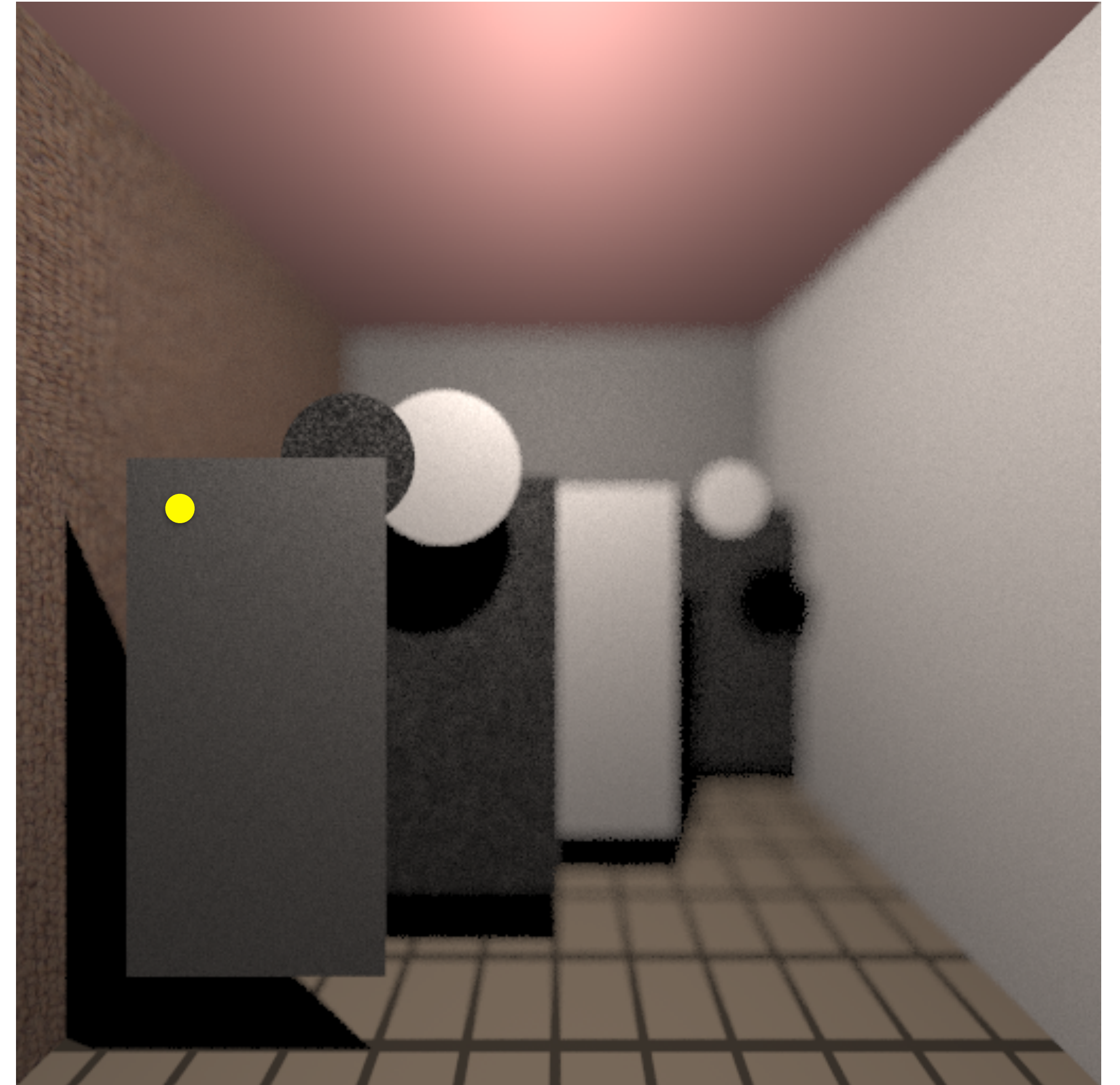
Fourier

U



X

XU Slices



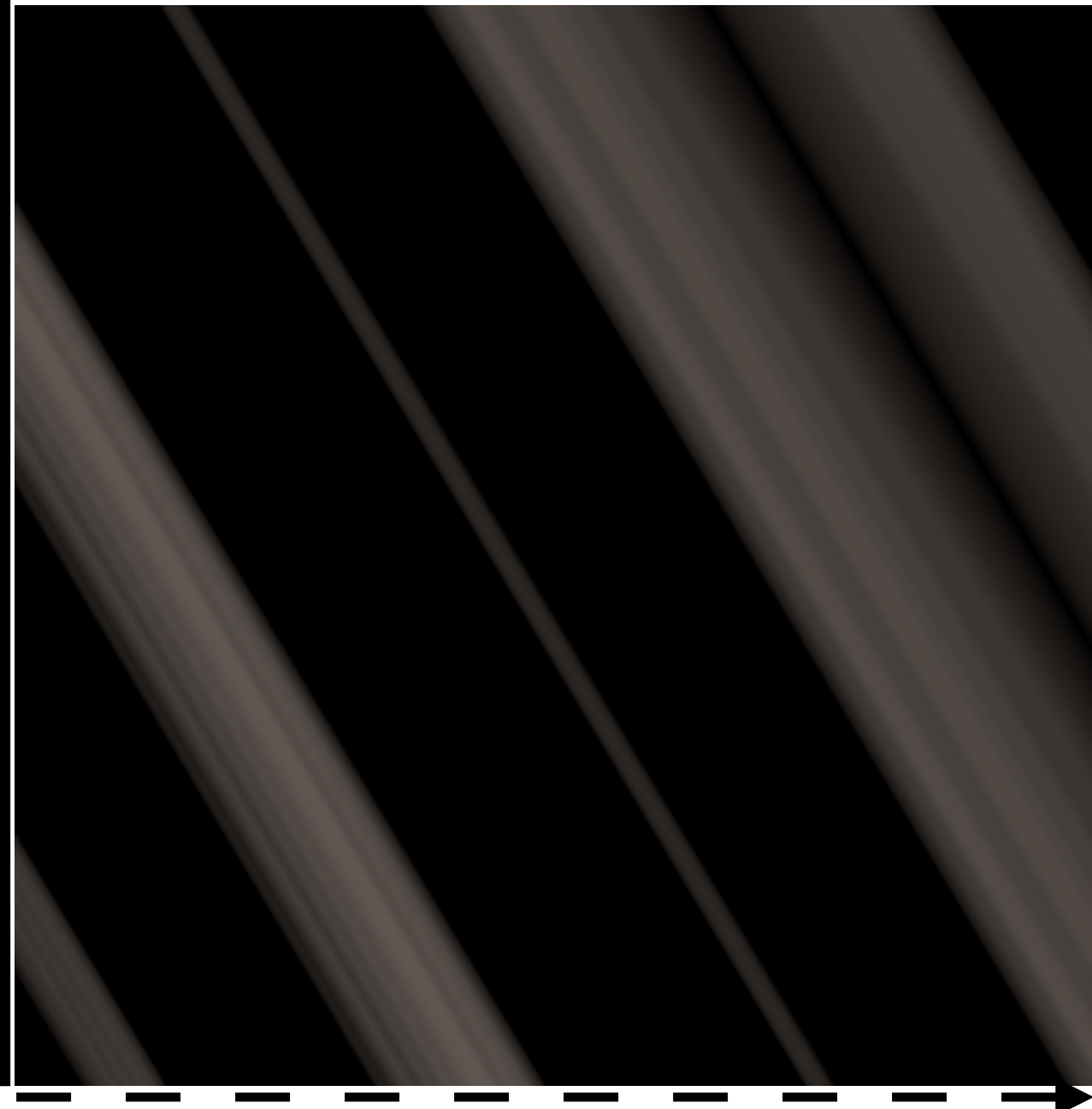
Depth of Field Analysis

Ray space

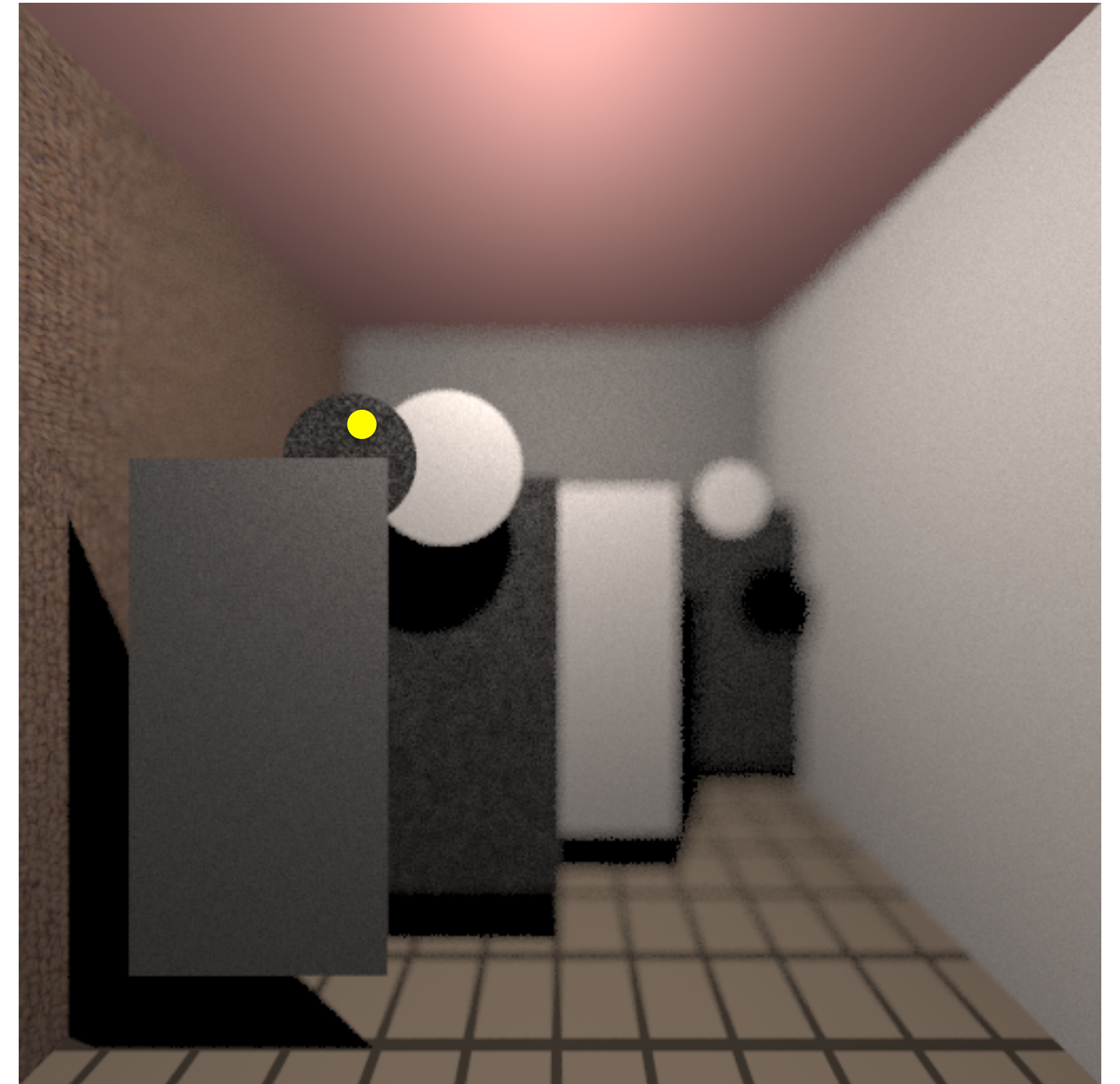
Spatial

Fourier

U



X
XU Slices



Durand et al. [2005]

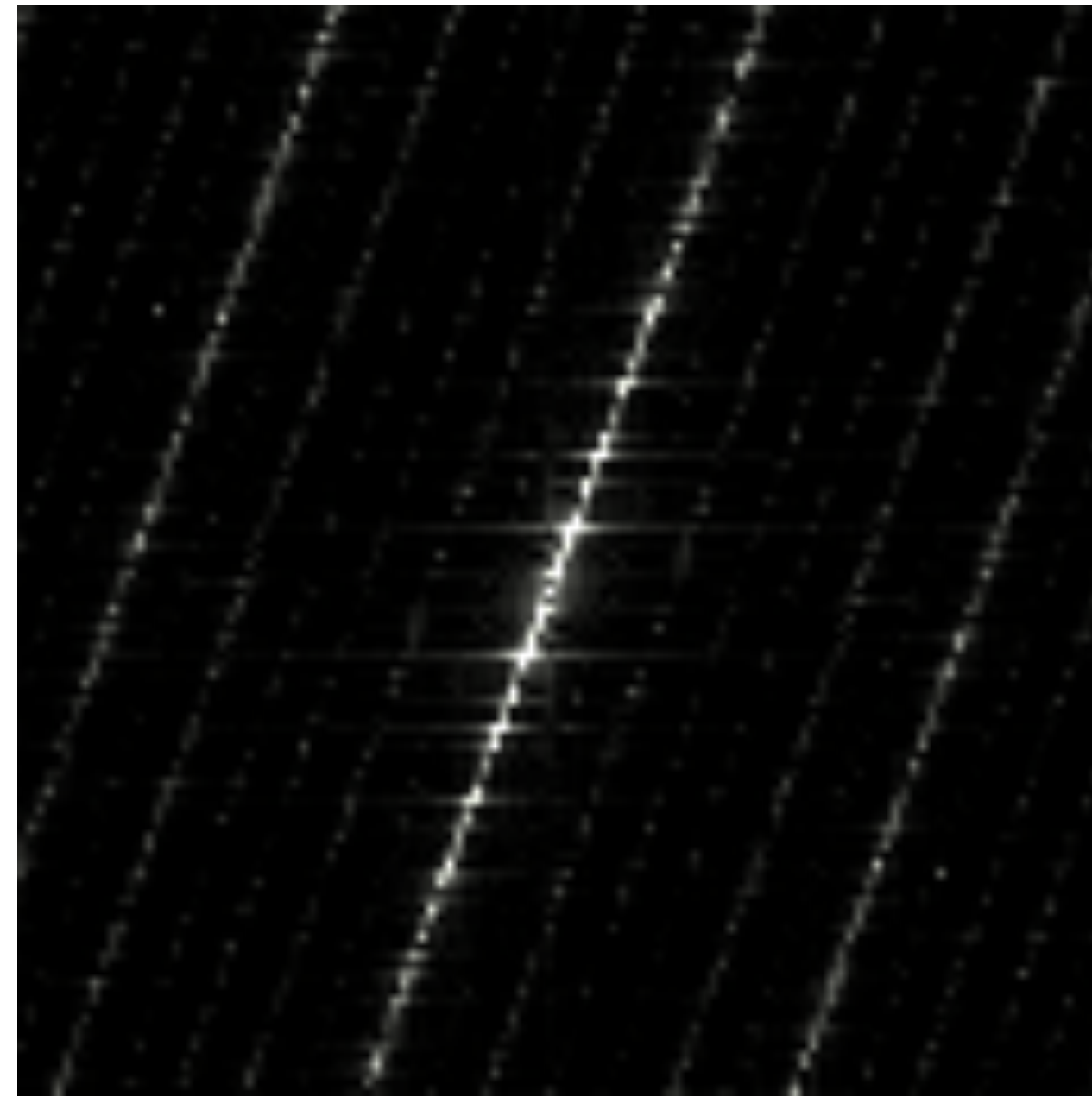
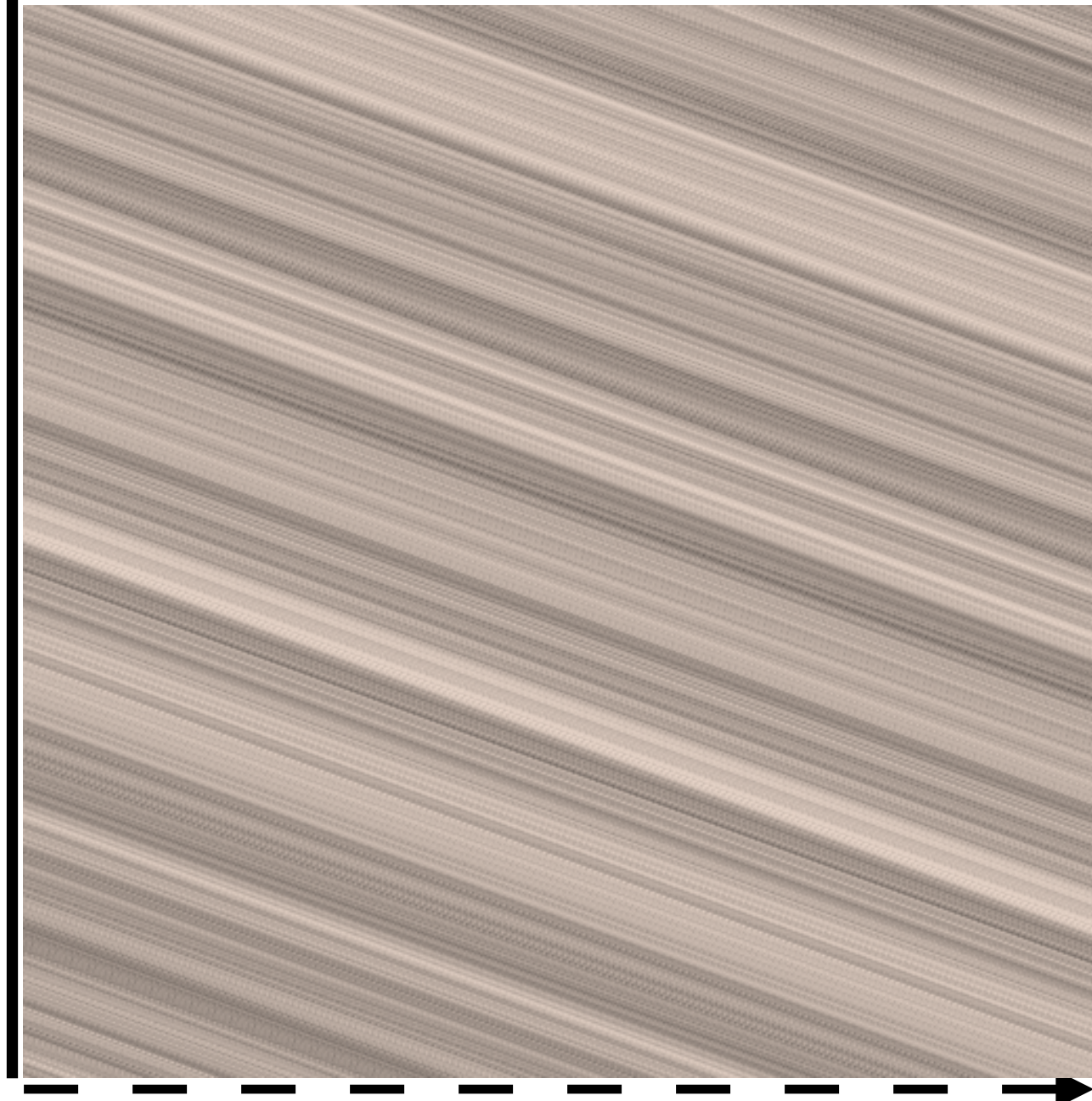
Depth of Field Analysis

Ray space

Spatial

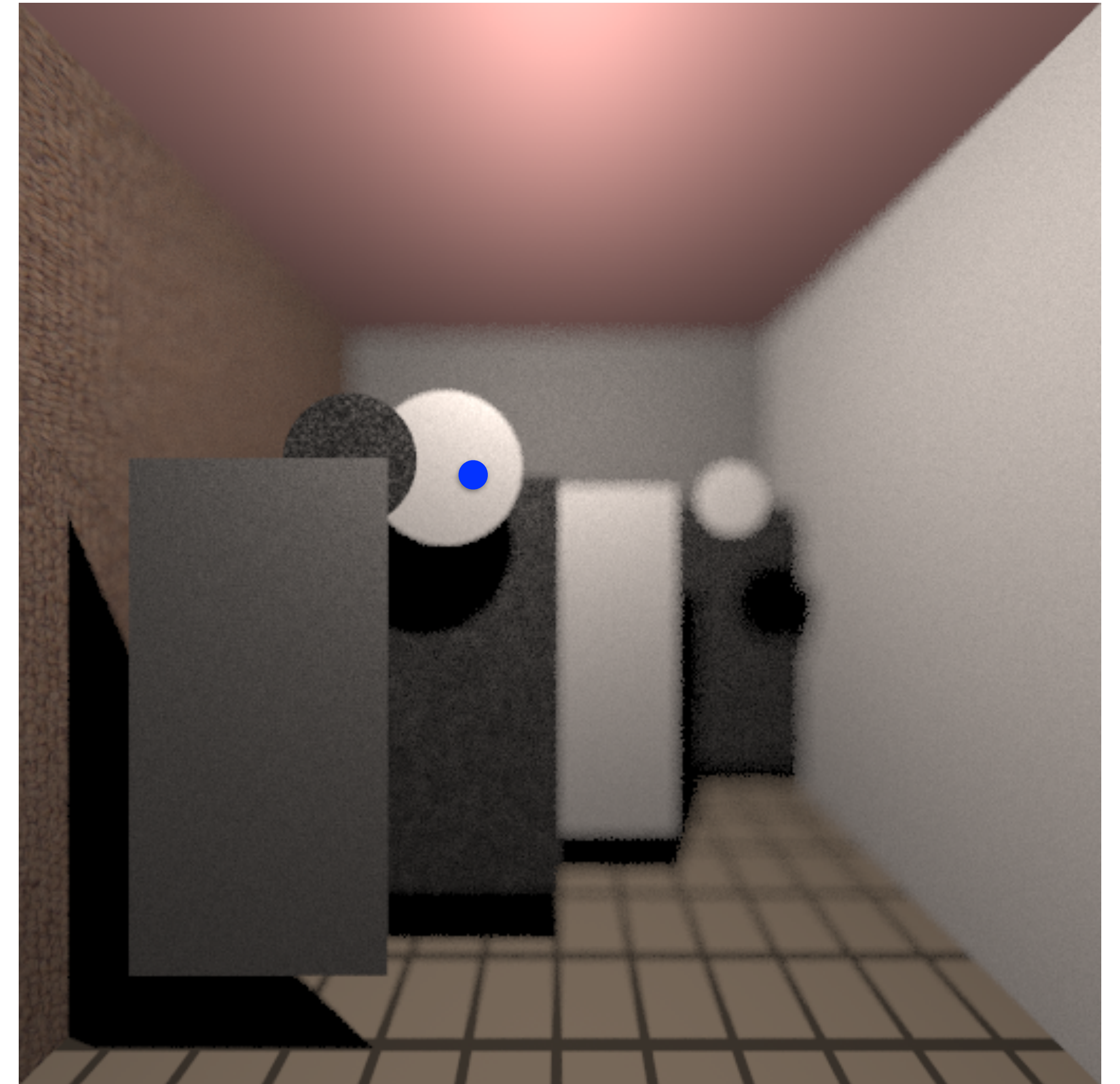
Fourier

U



X

XU Slices



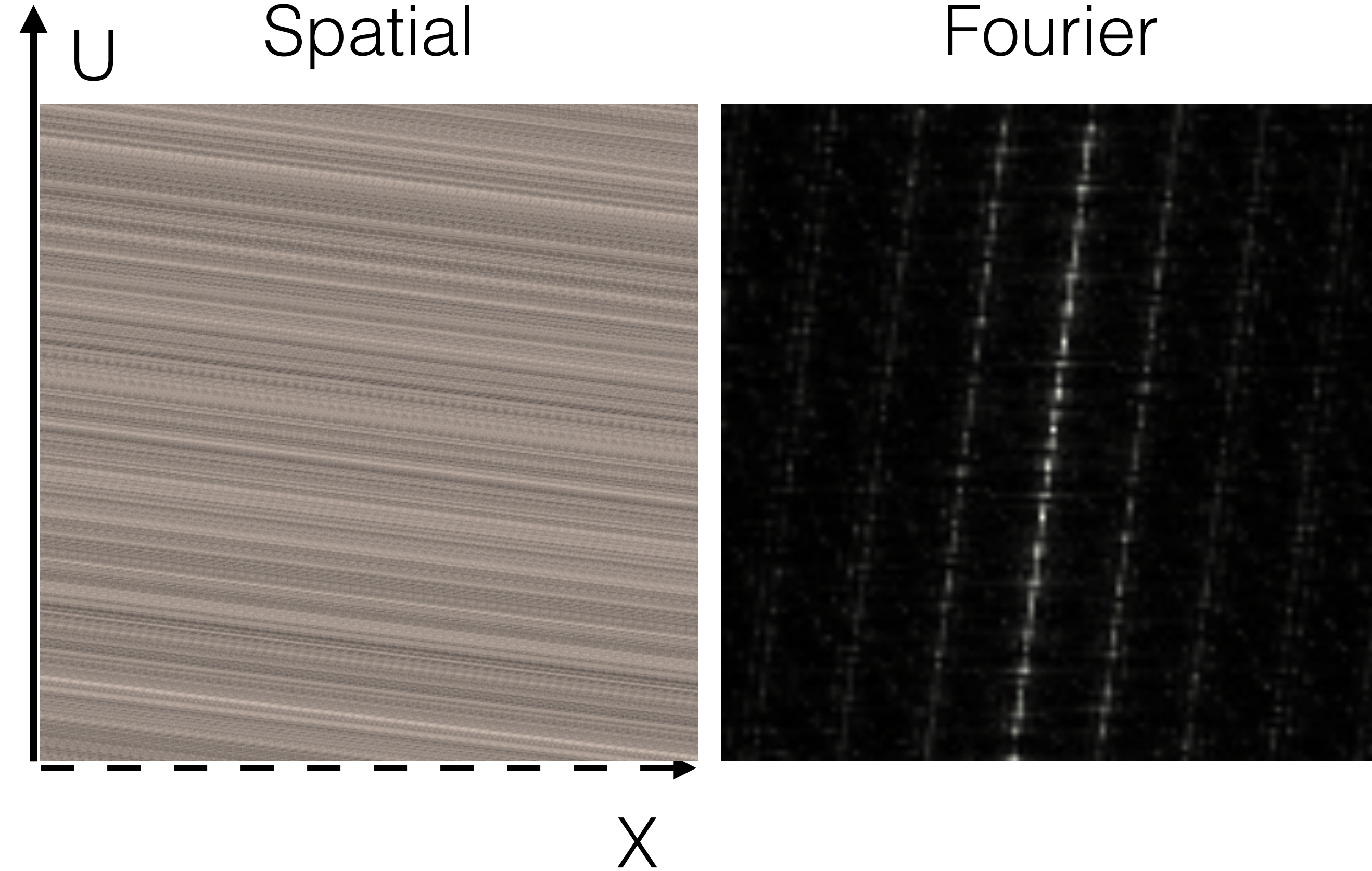
Durand et al. [2005]

Depth of Field Analysis

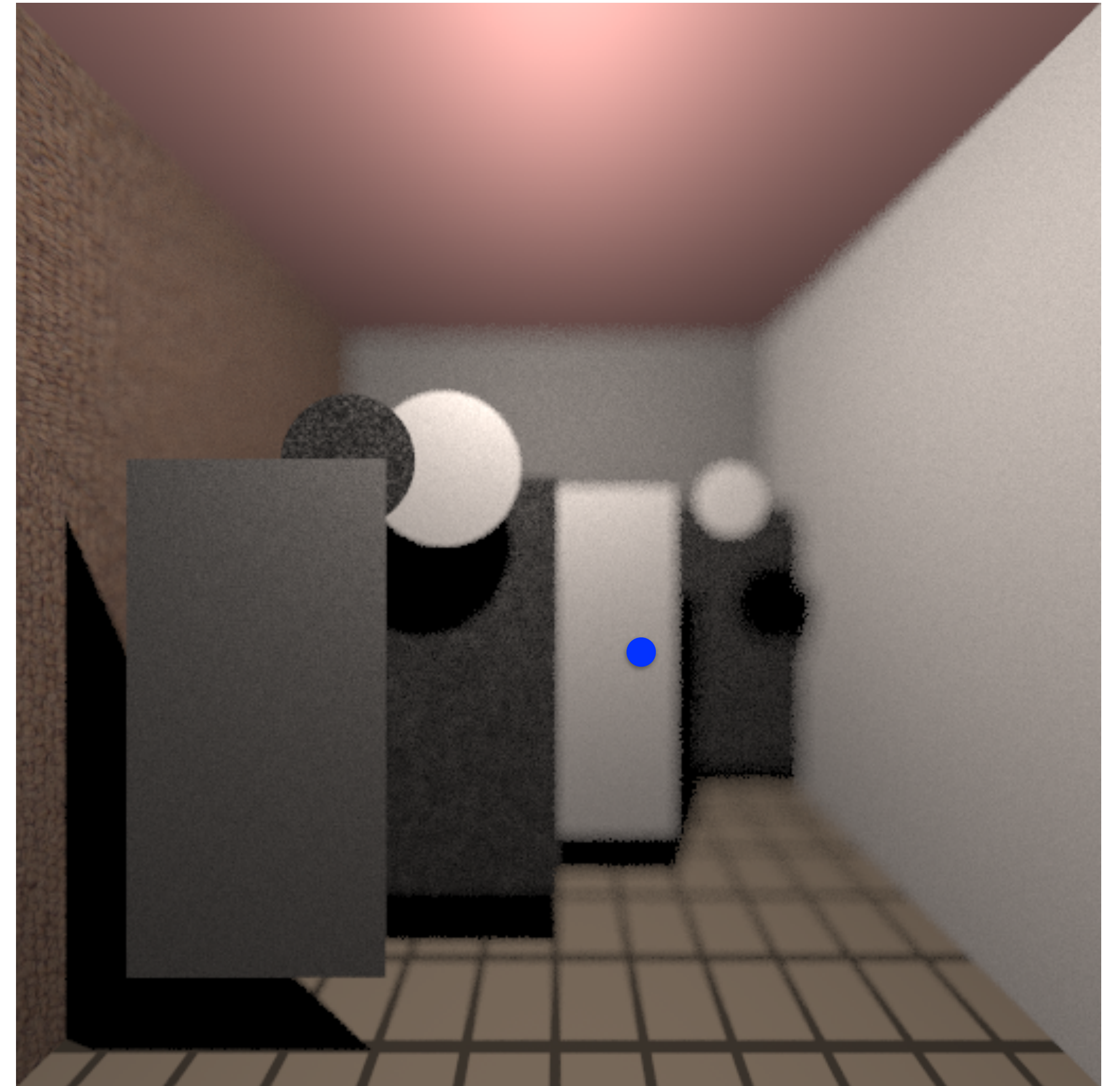
Ray space

Spatial

Fourier



XU Slices



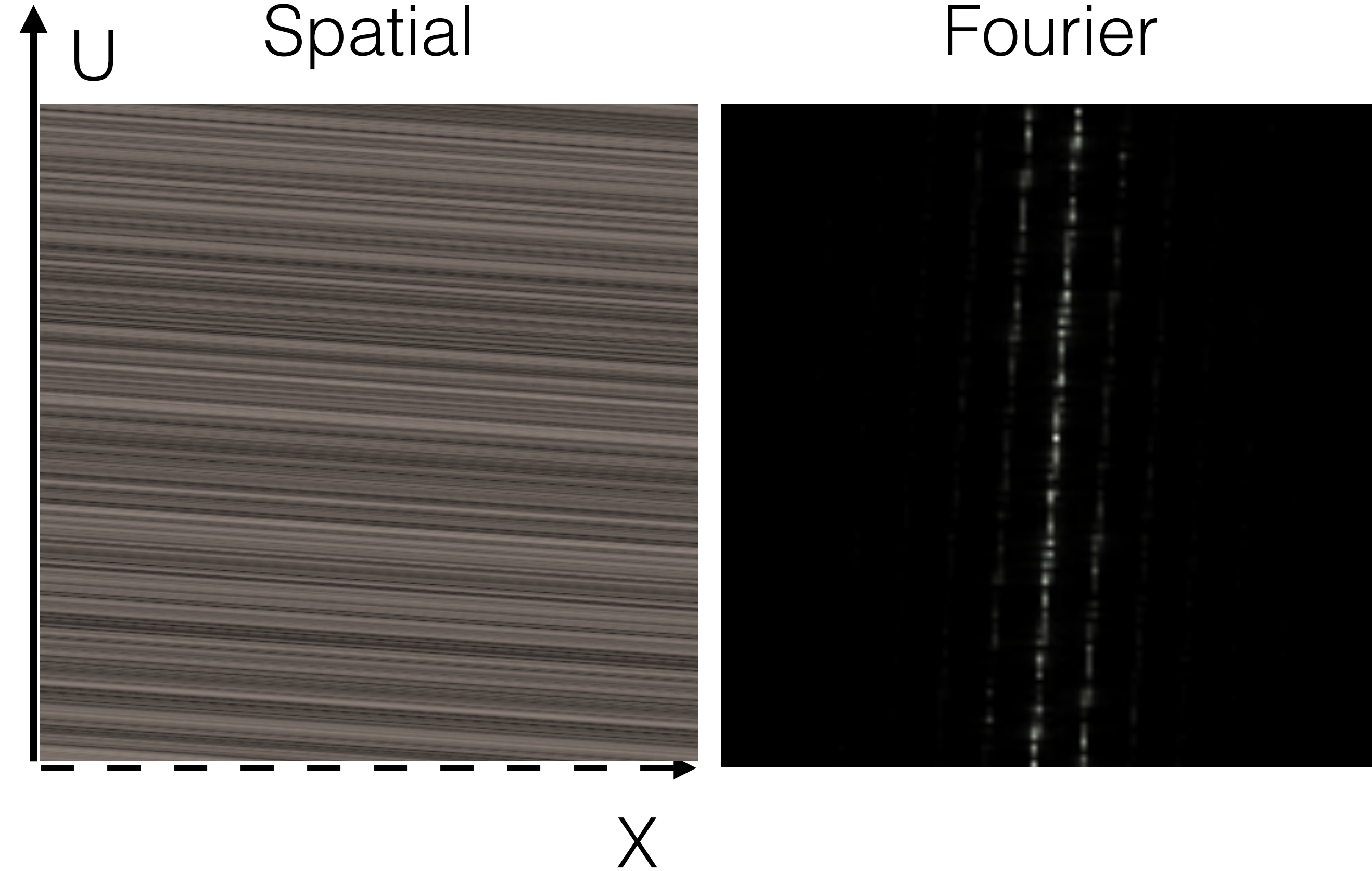
Durand et al. [2005]

Depth of Field Analysis

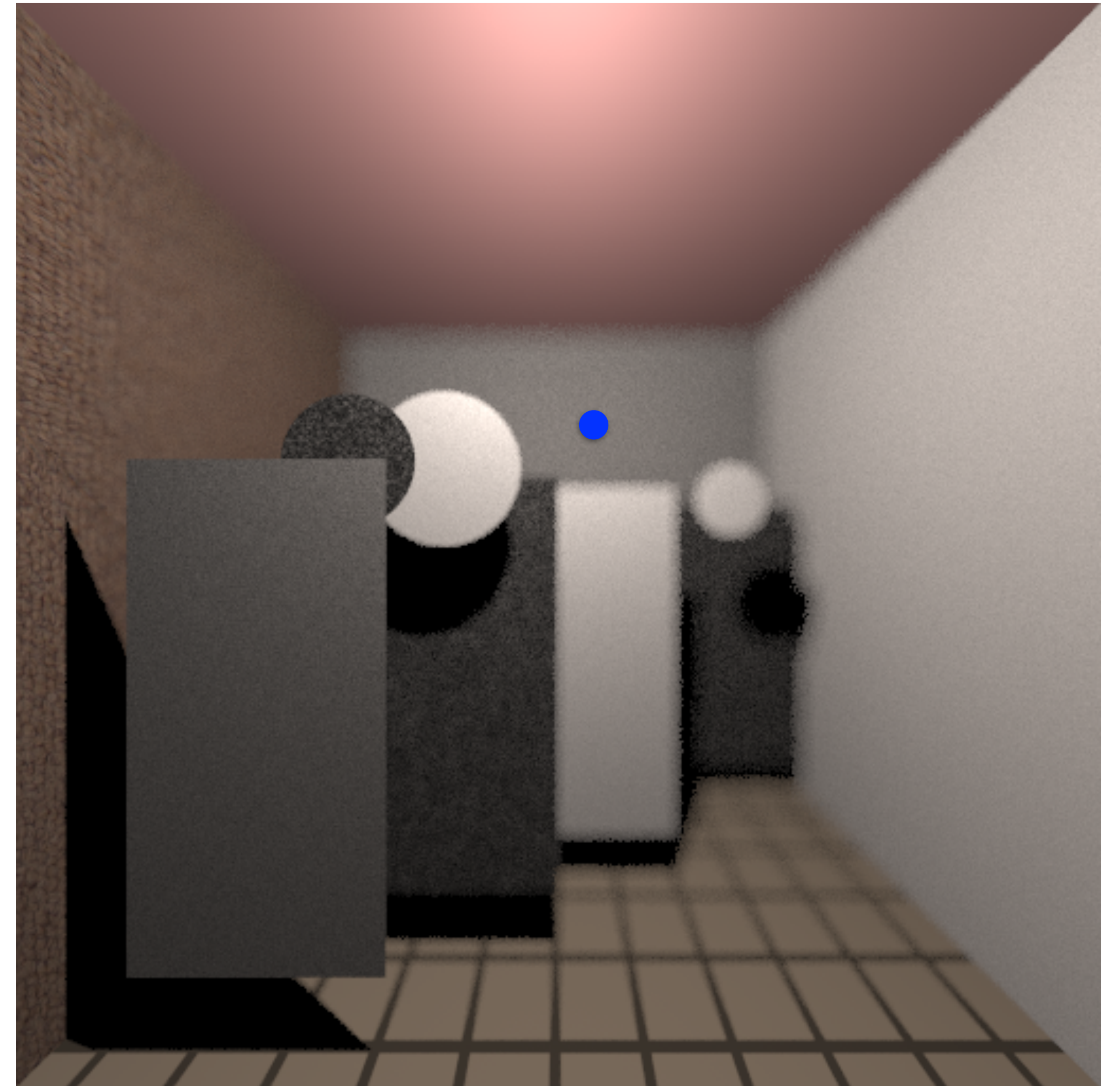
Ray space

Spatial

Fourier



XU Slices



Durand et al. [2005]

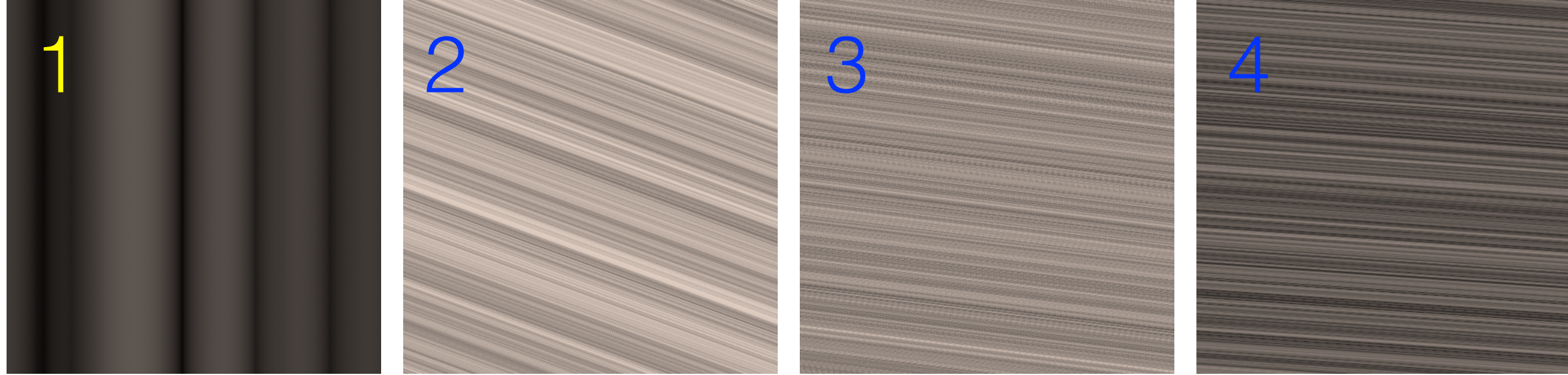
Light Field gets Sheared

$$x = x + u \frac{F - d}{d}, \quad F: \text{ focal distance}$$

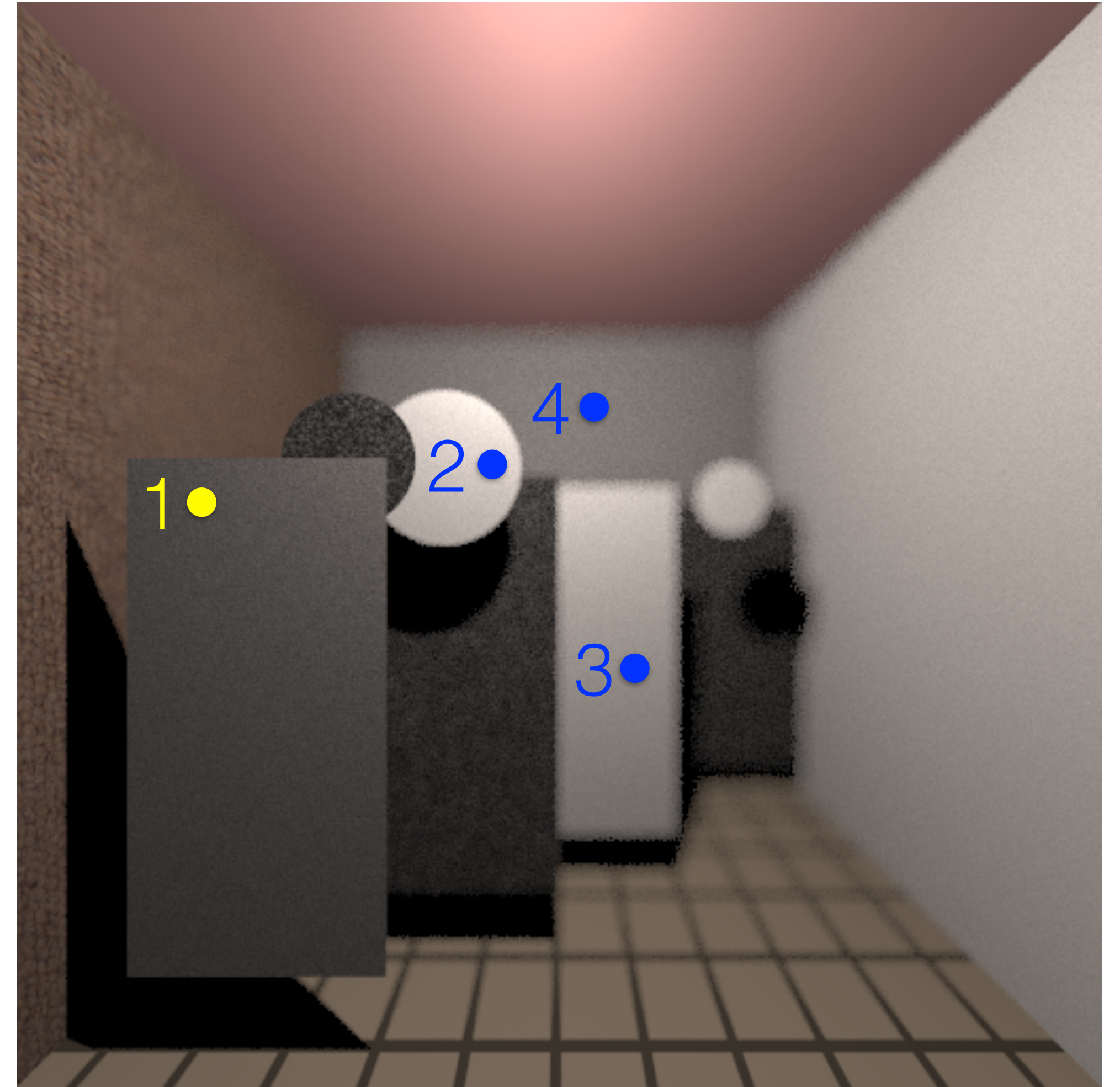
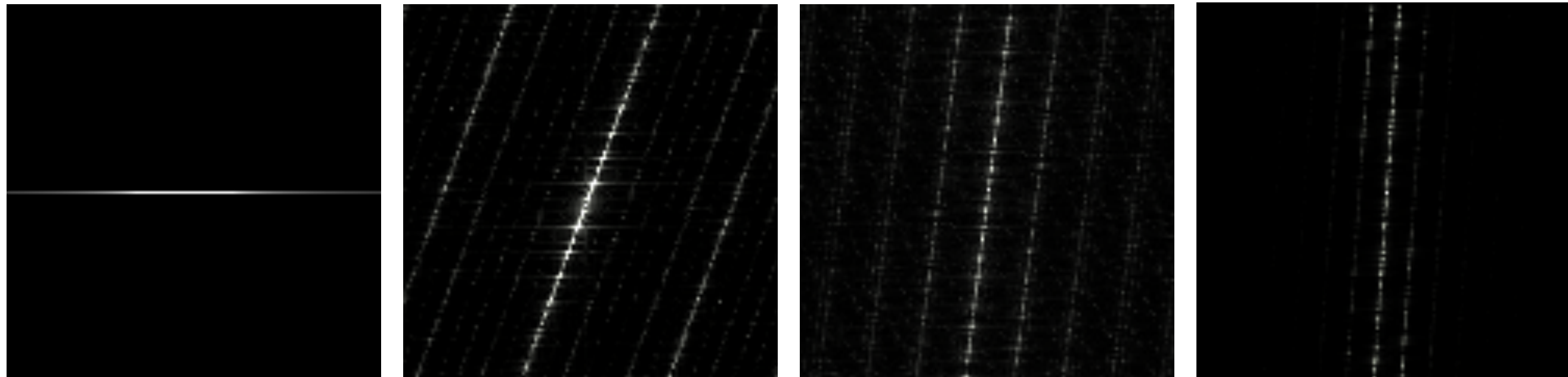
Shear increases with depth of the object



XU Slices



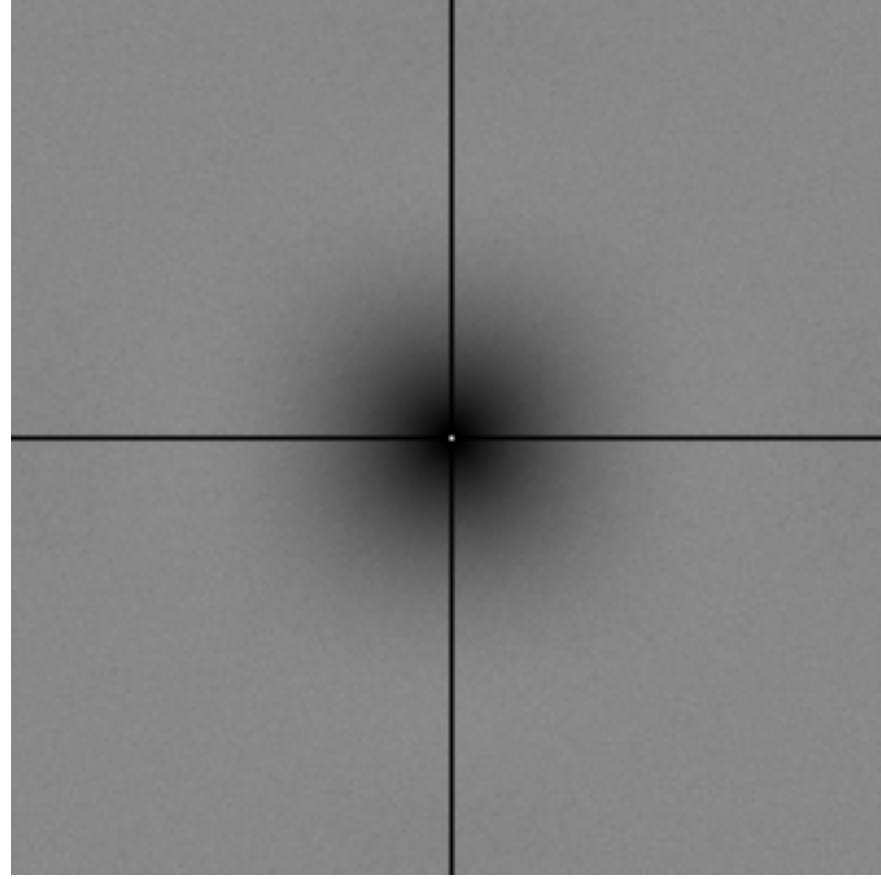
Spectra



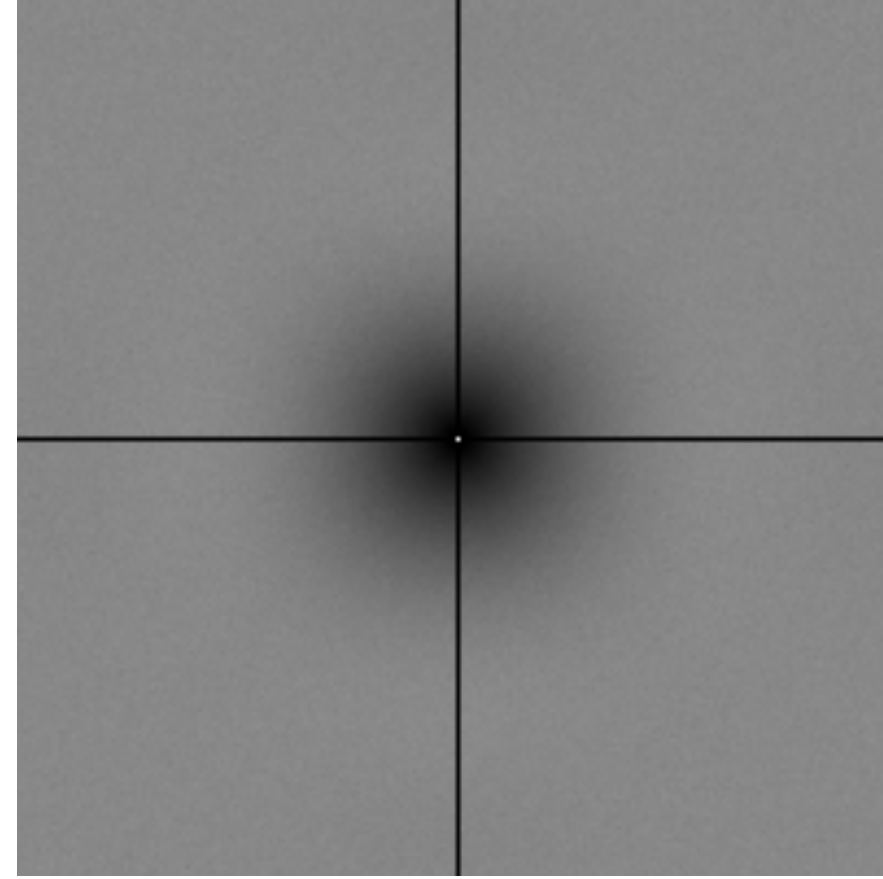
Spectra along Different Projections

Uncorrelated
Multi-jittered

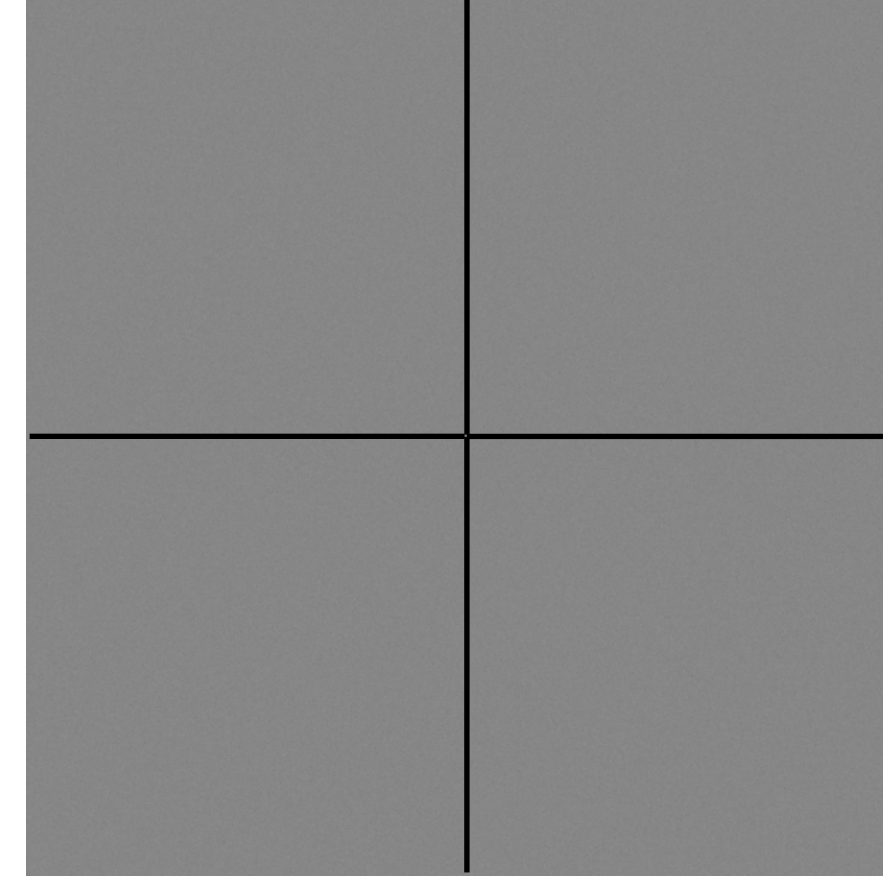
XY



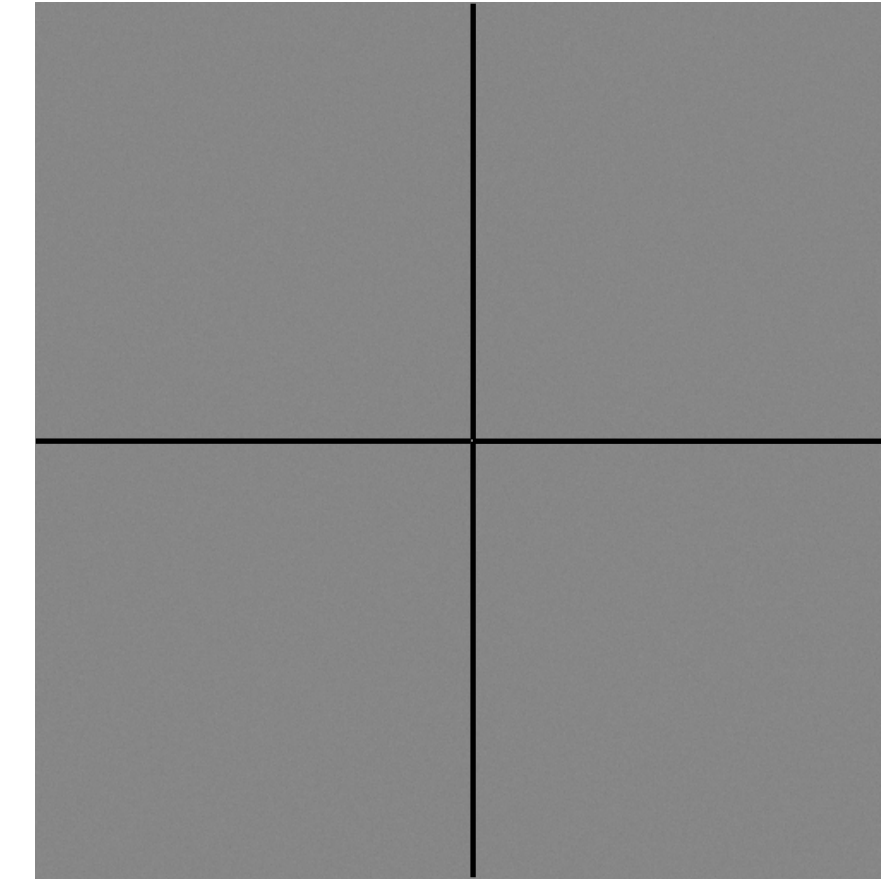
UV



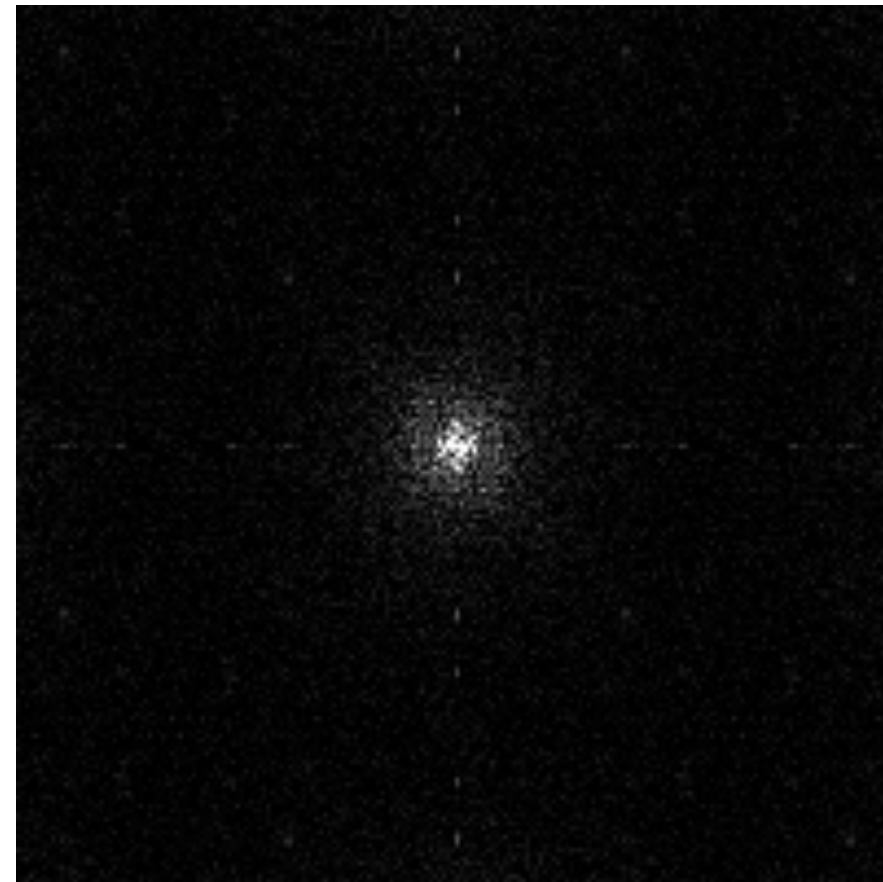
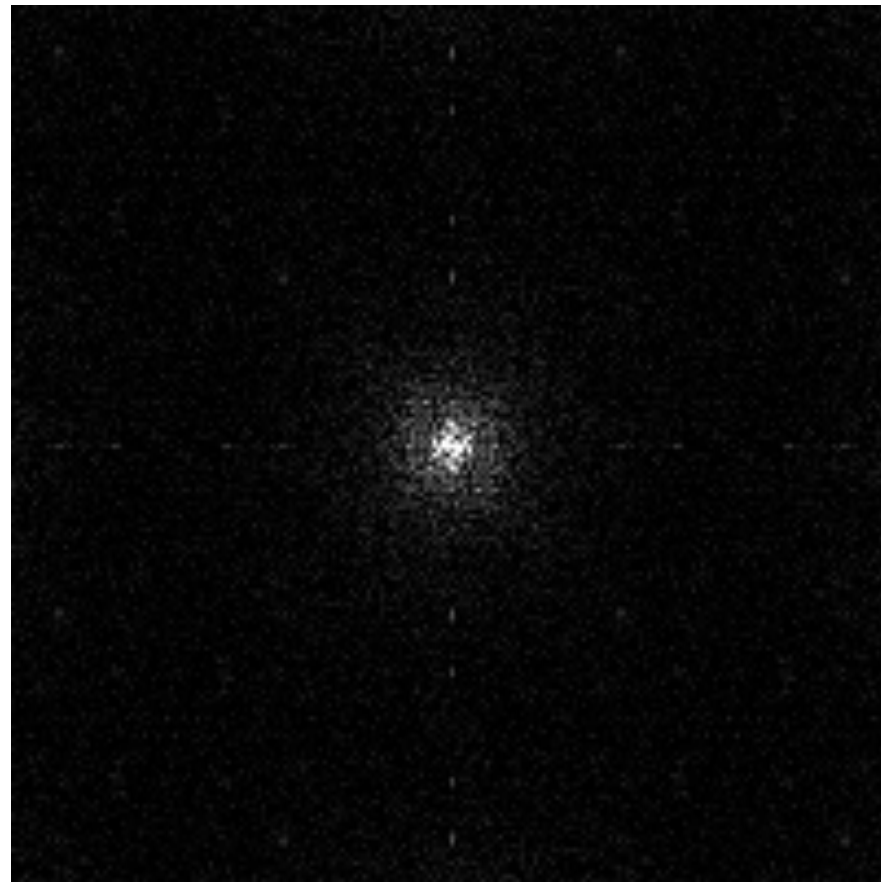
XU



YV

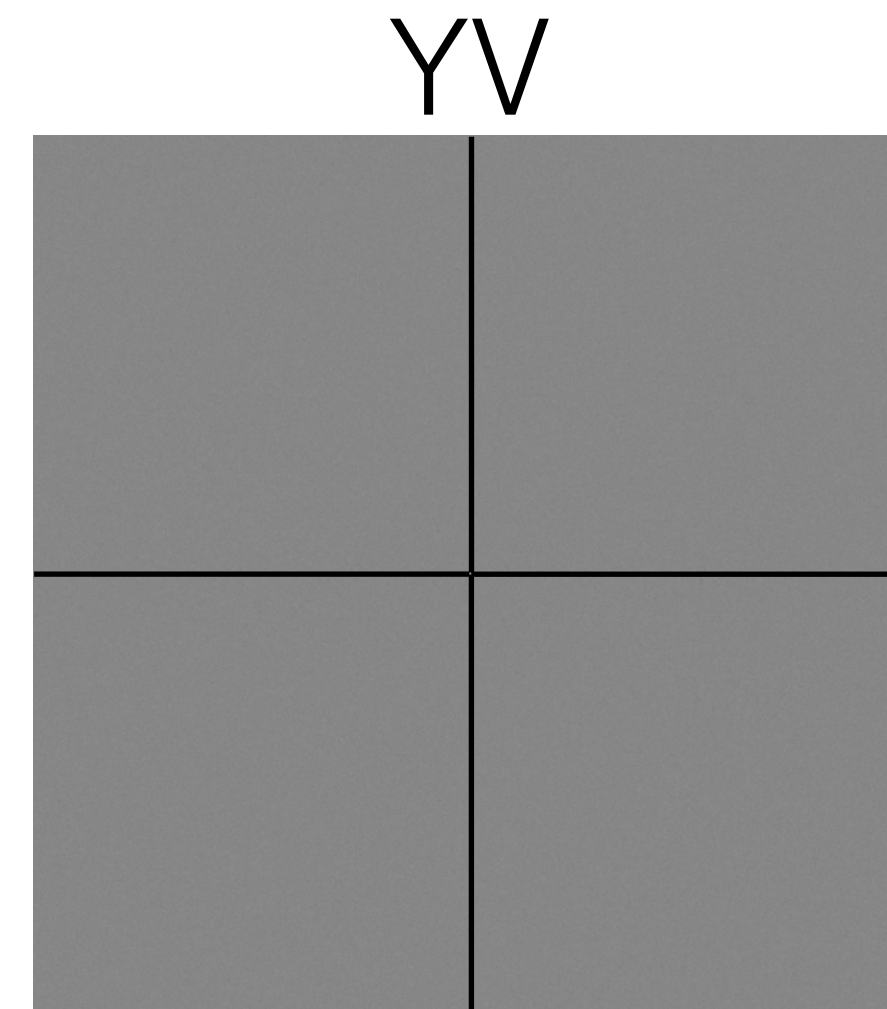
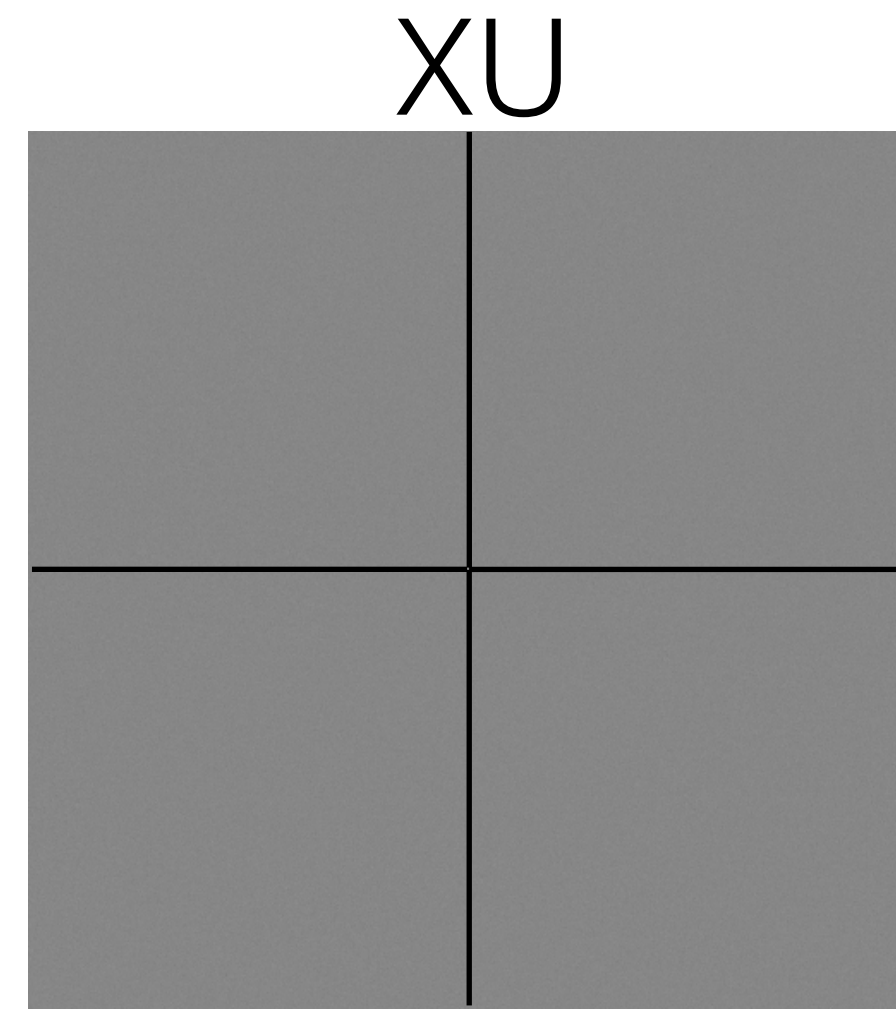


Integrand



Spectra along Different Projections

Uncorrelated
Multi-jittered

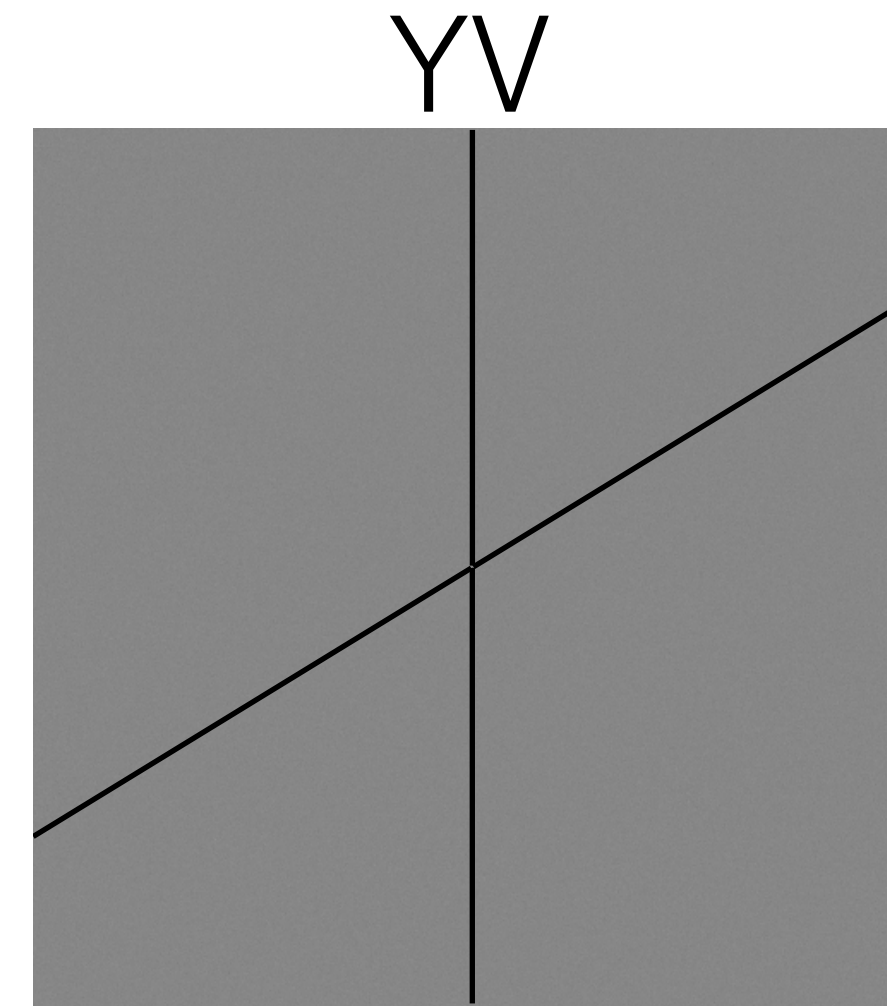
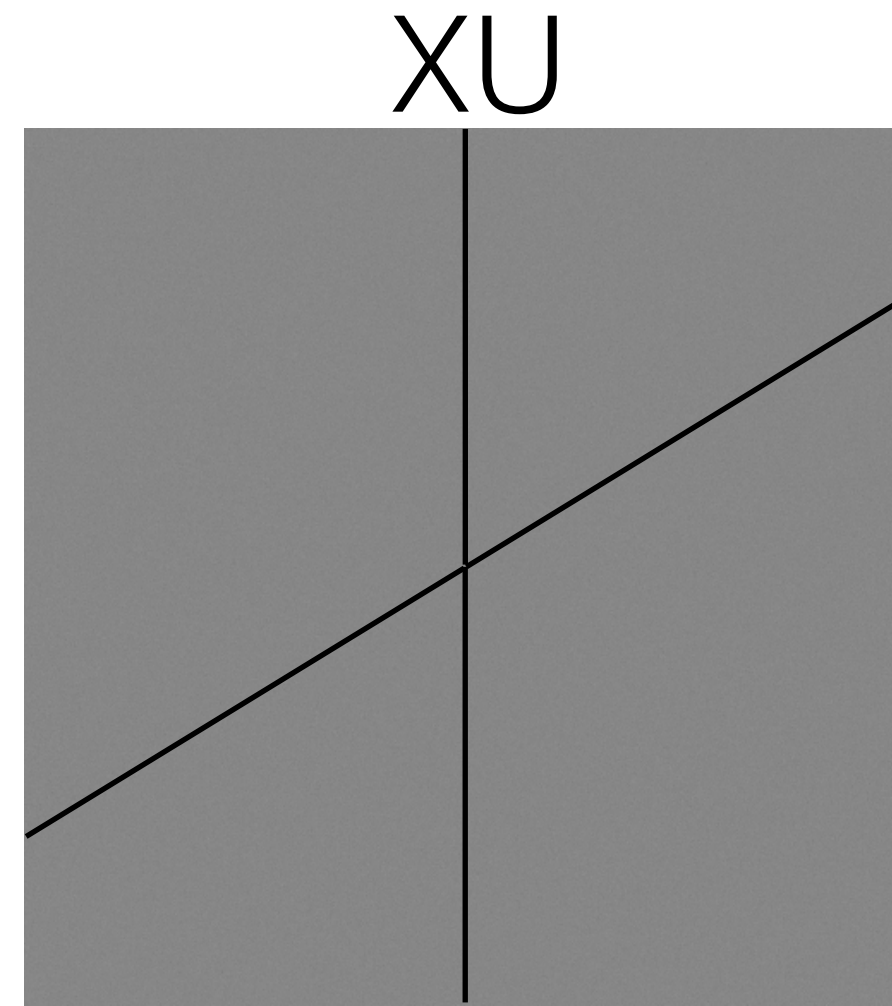


Integrand

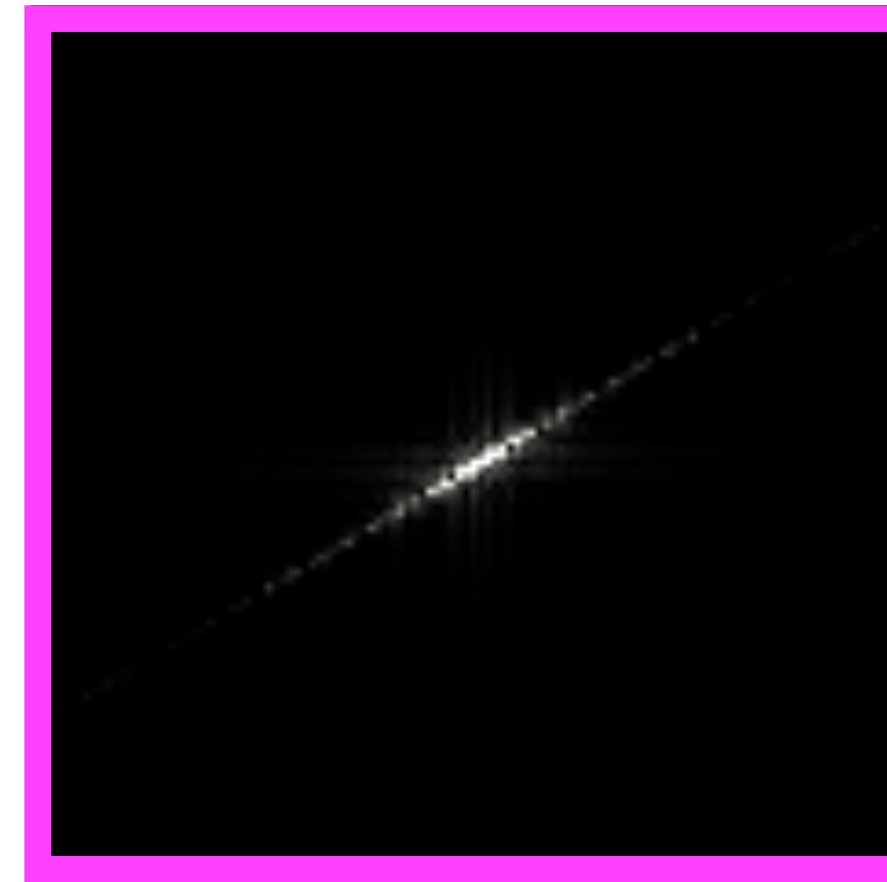


Spectra along Different Projections

Uncorrelated
Multi-jittered



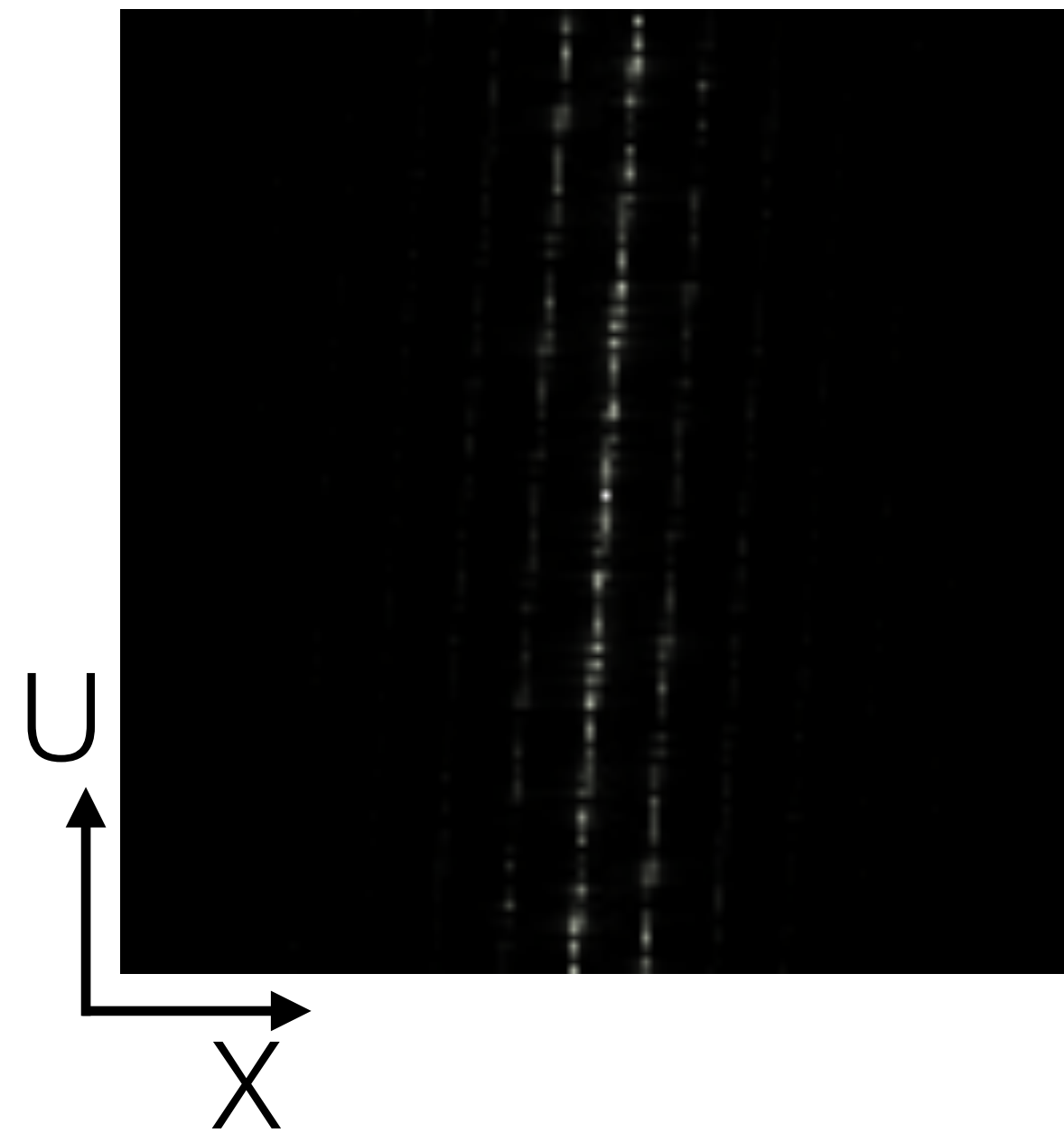
Integrand



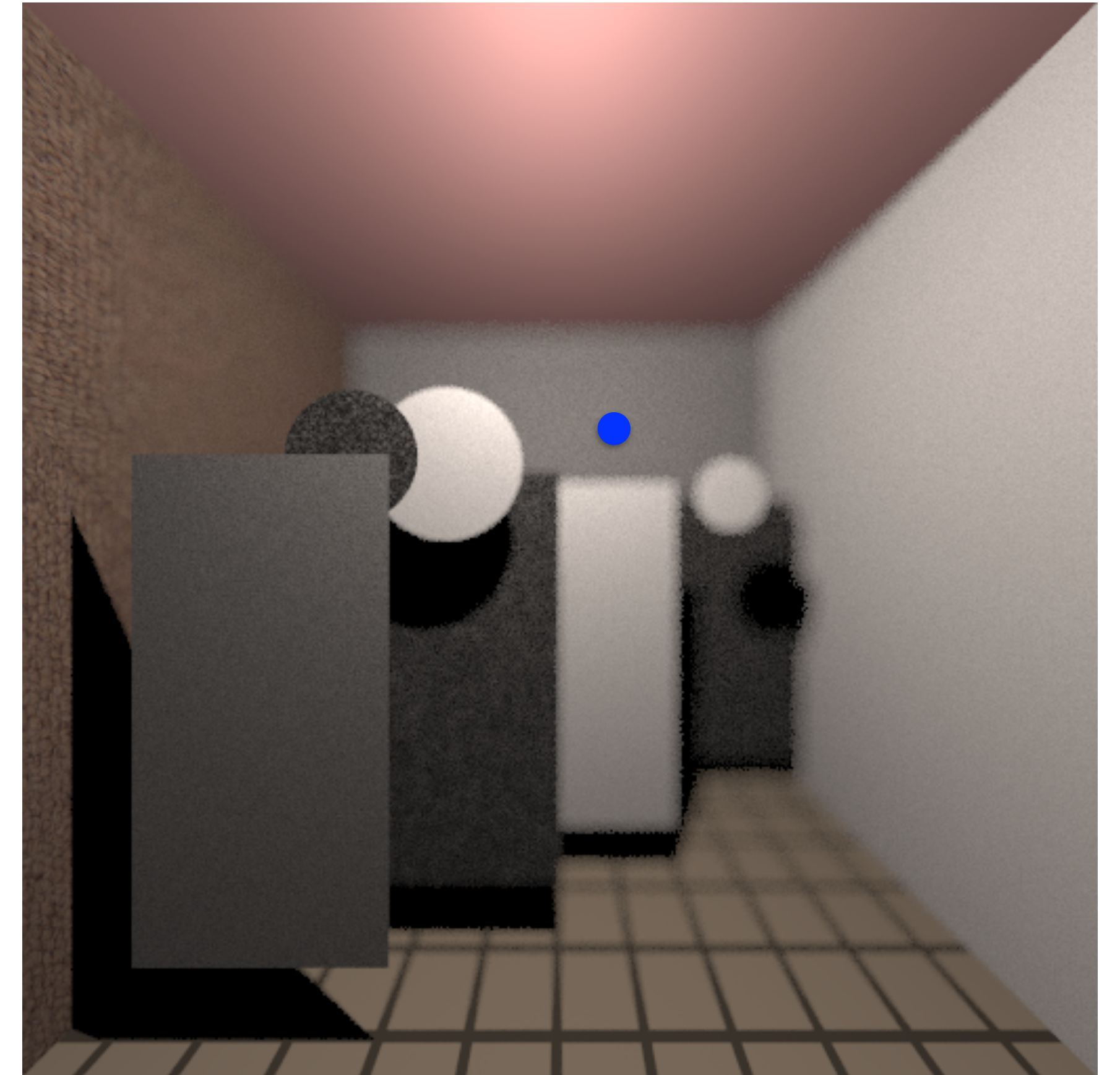
Variance & Convergence Analysis with Sheared Samples

Cornell Box Scene

XU Projection

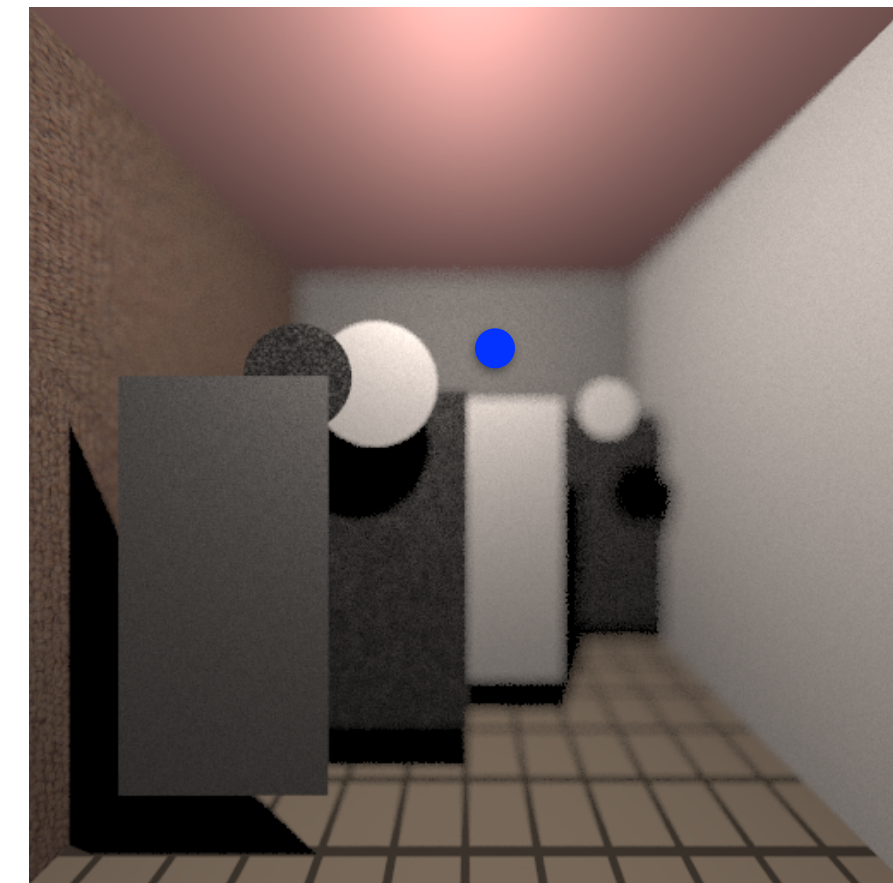


Integrand Spectrum

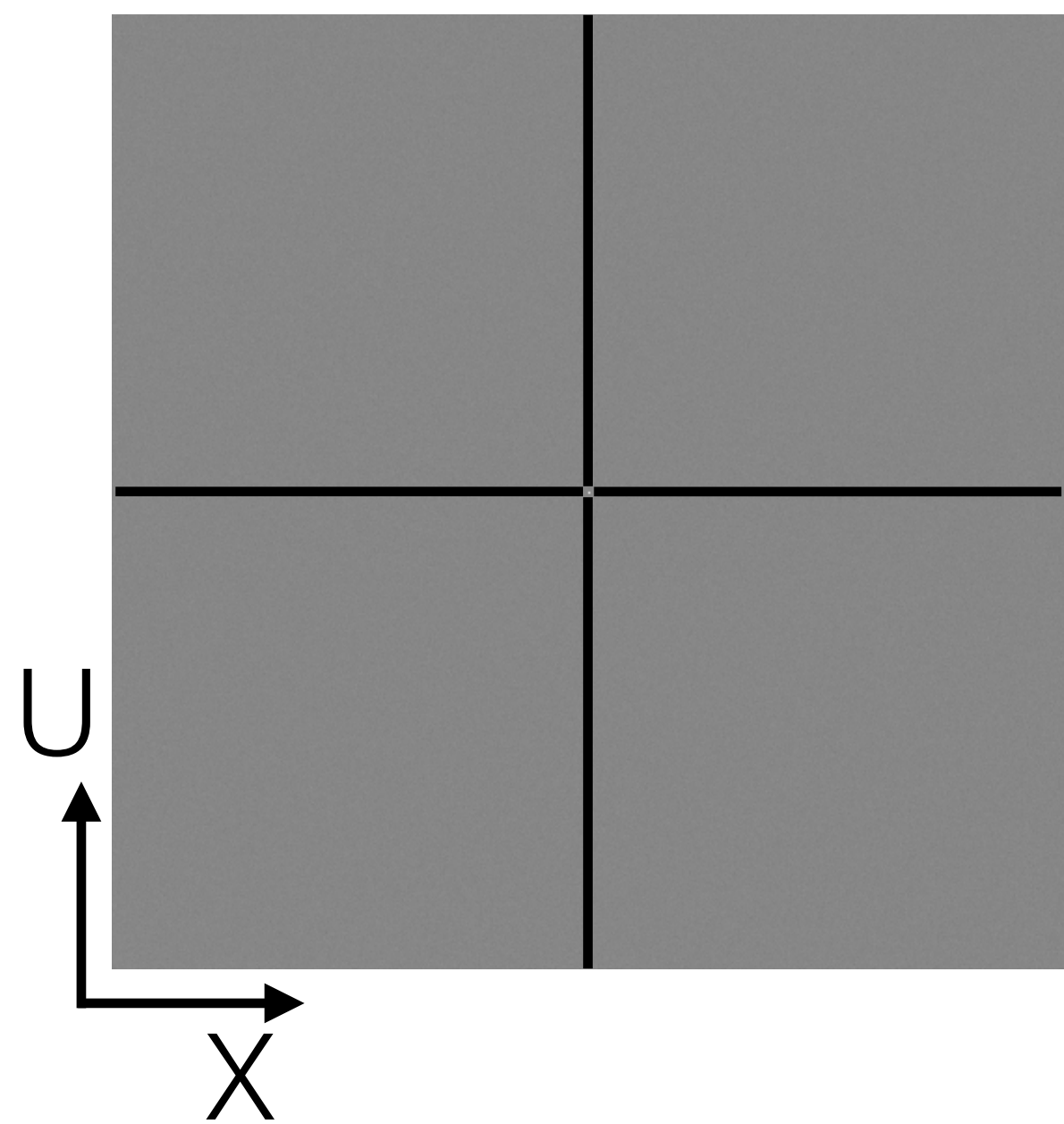


$$\int_x \int_y \int_u \int_v f(x, y, u, v) dv du dy dx$$

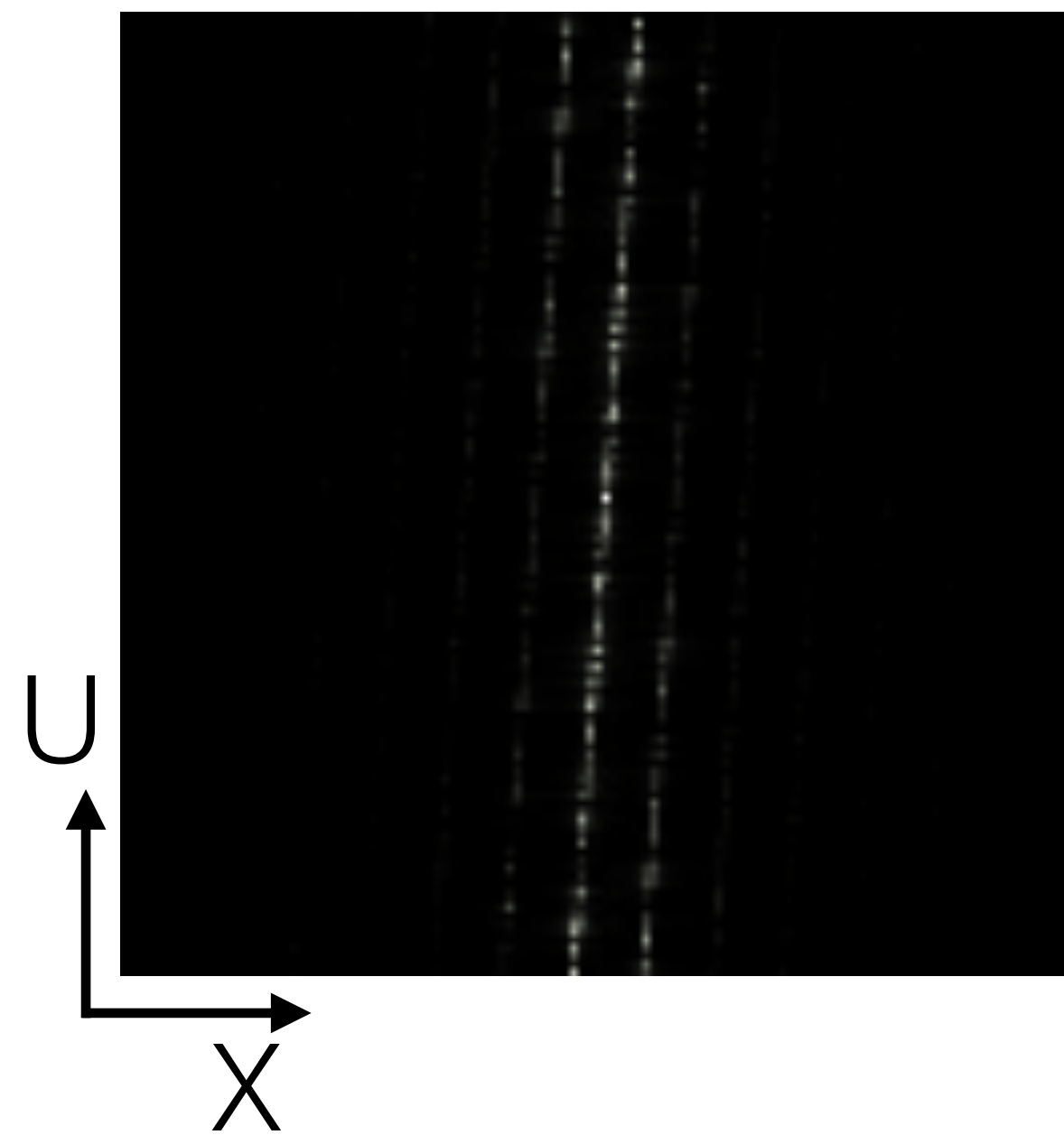
Original Uncorrelated-MultiJittered Samples



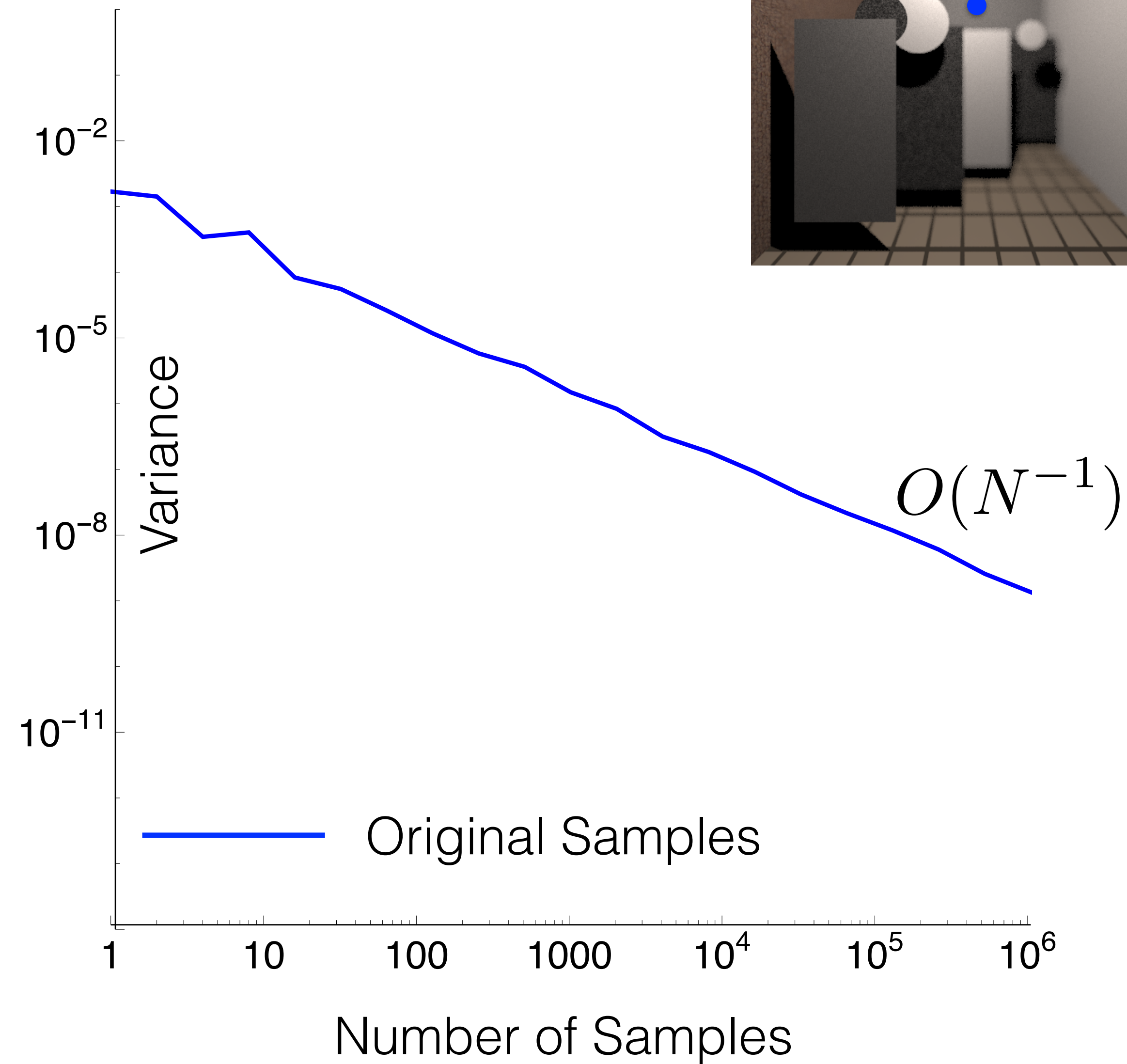
XU Projection



Sampling Spectrum

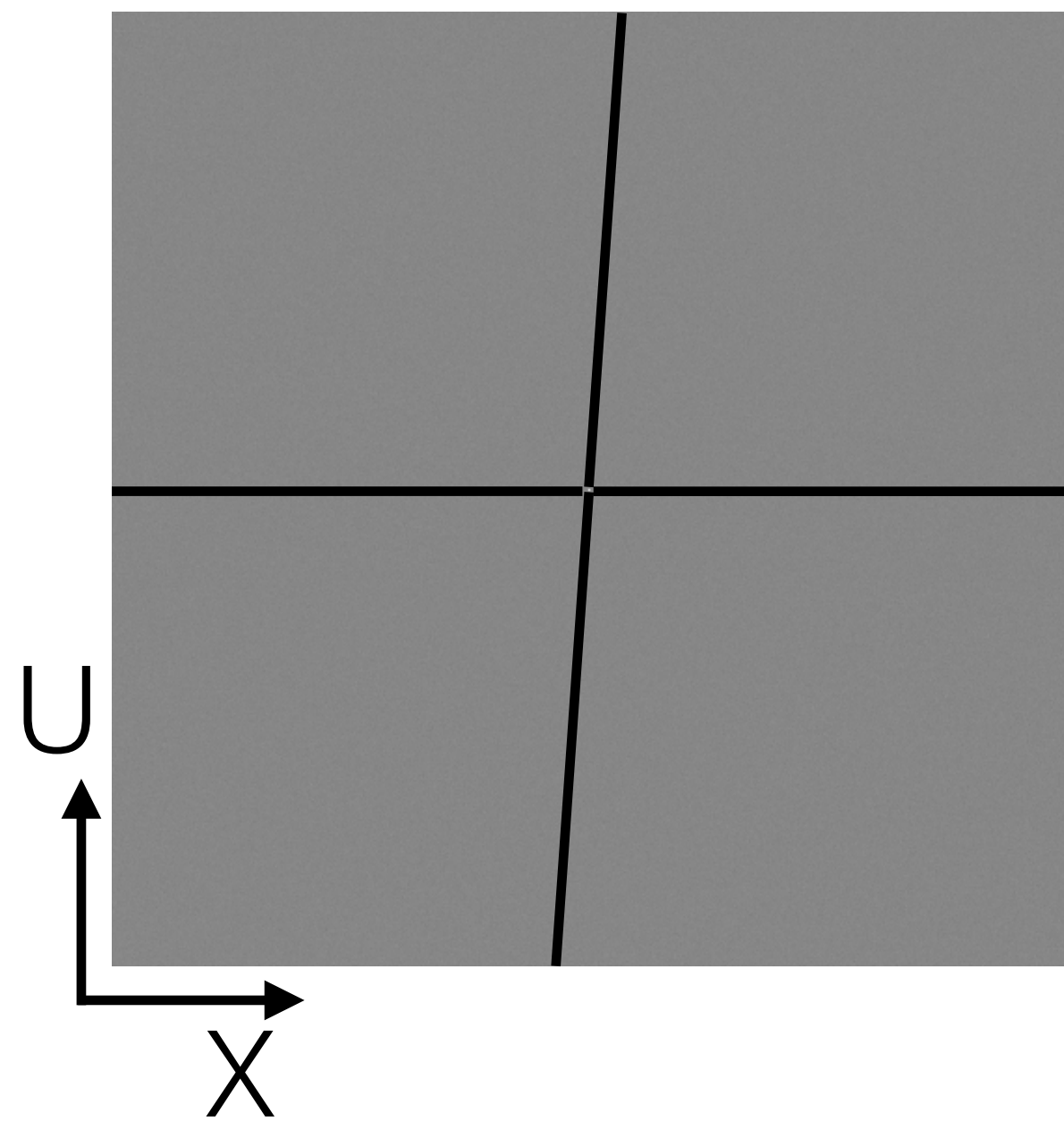


Integrand Spectrum

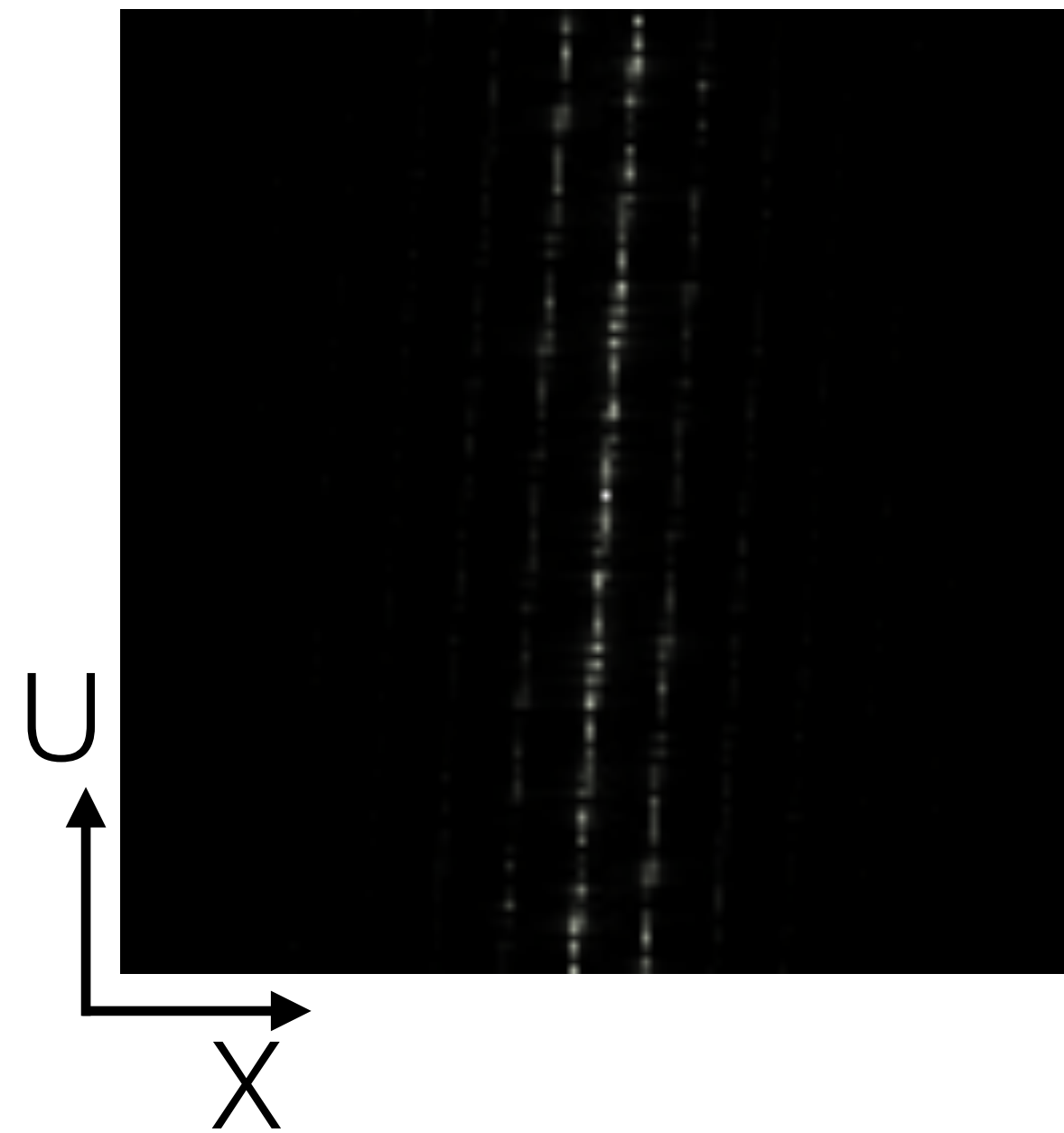


Convergence improvement after Shearing

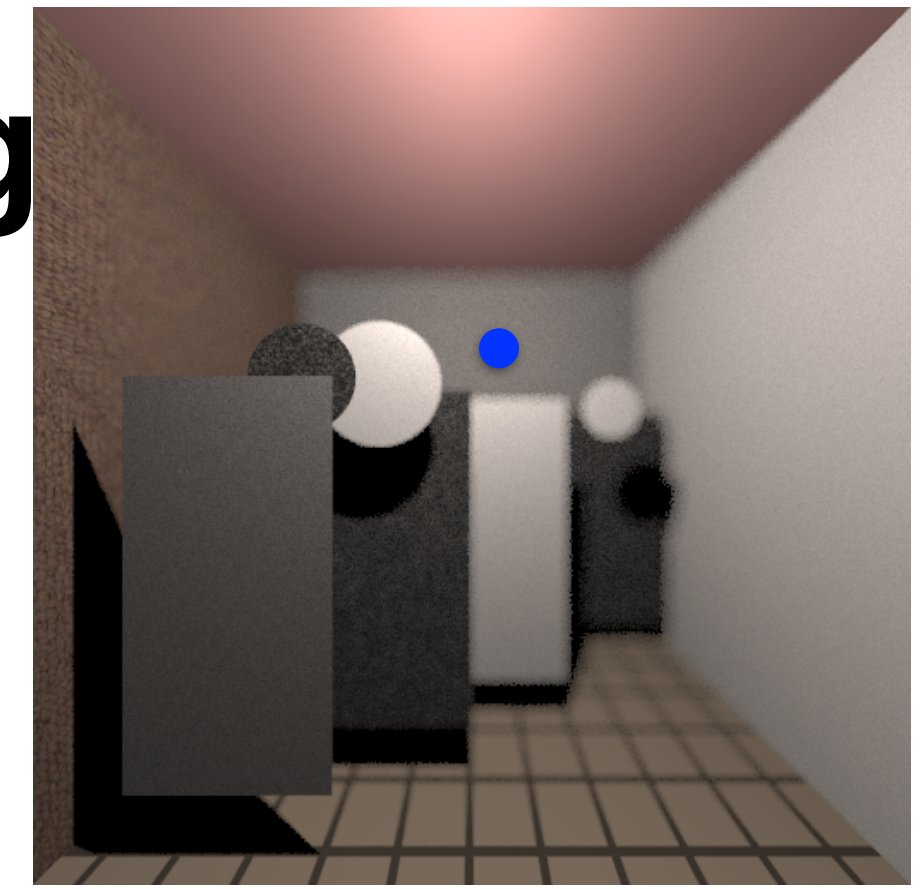
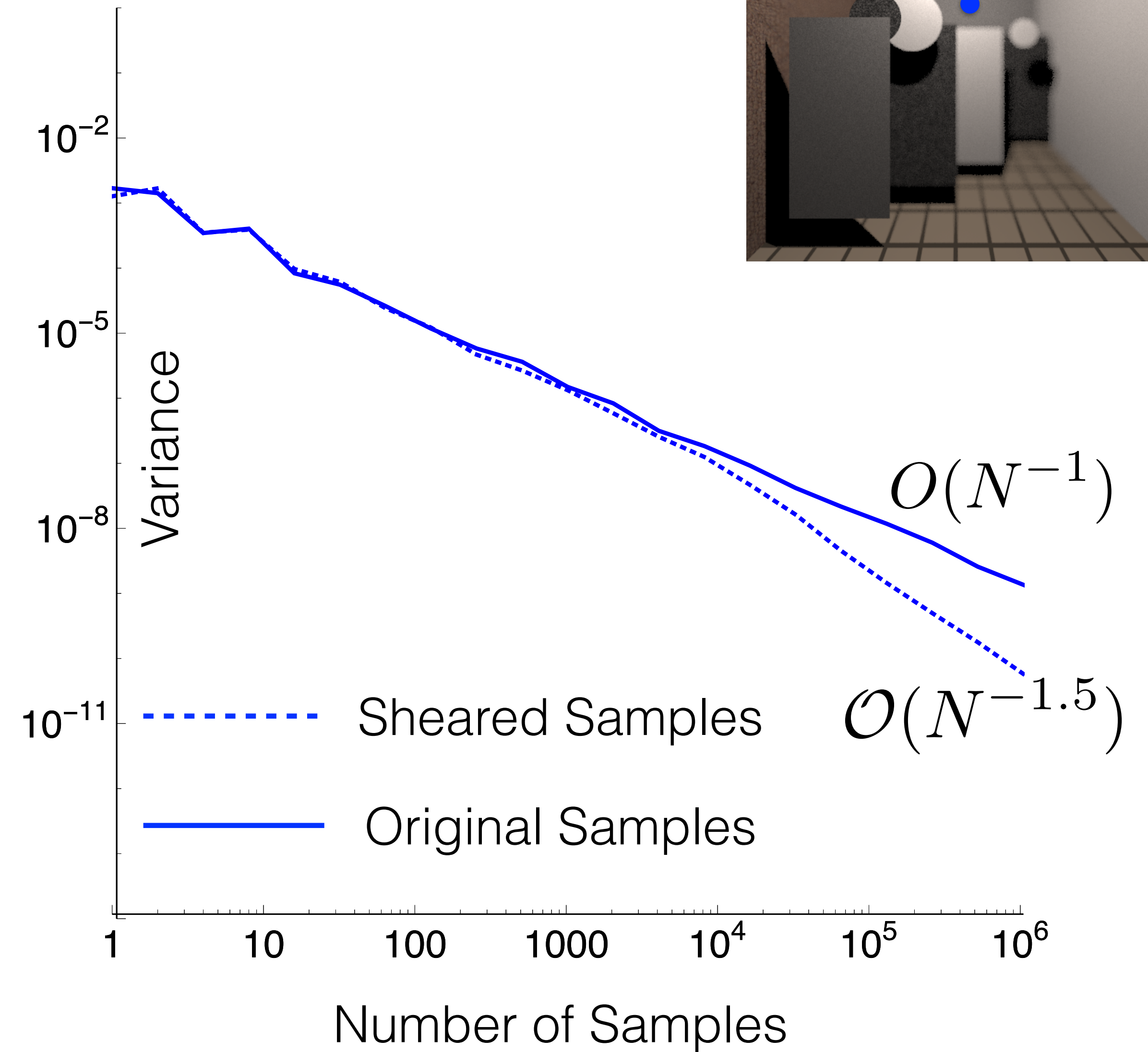
XU Subspace



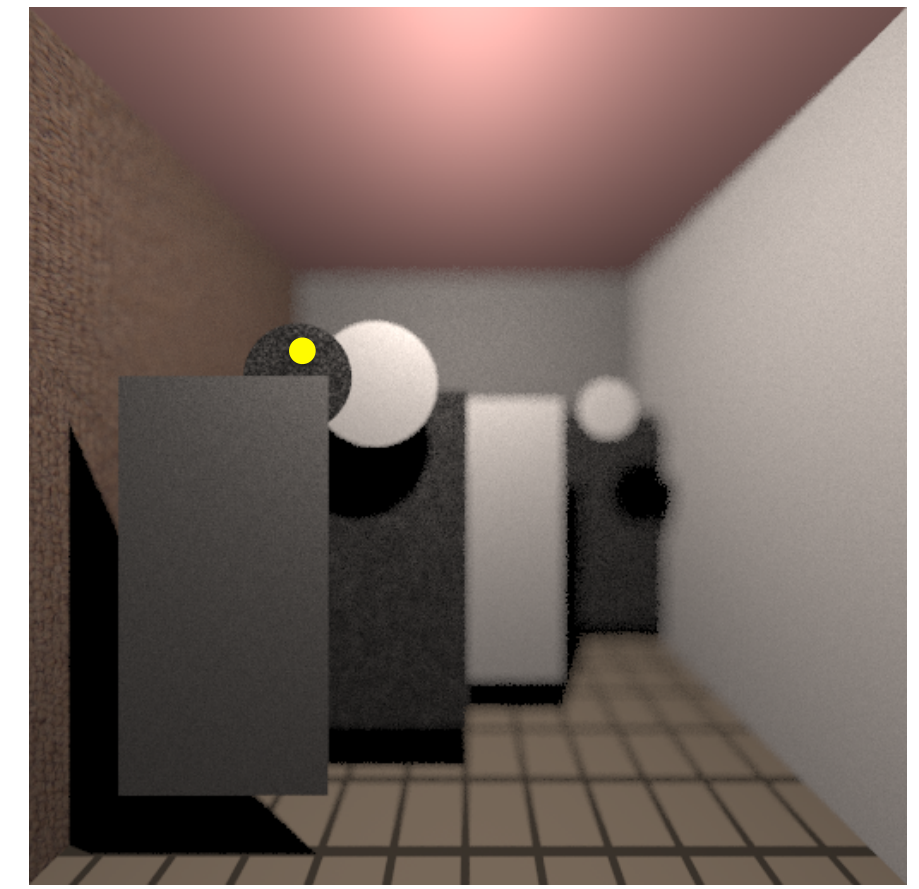
Sampling Spectrum



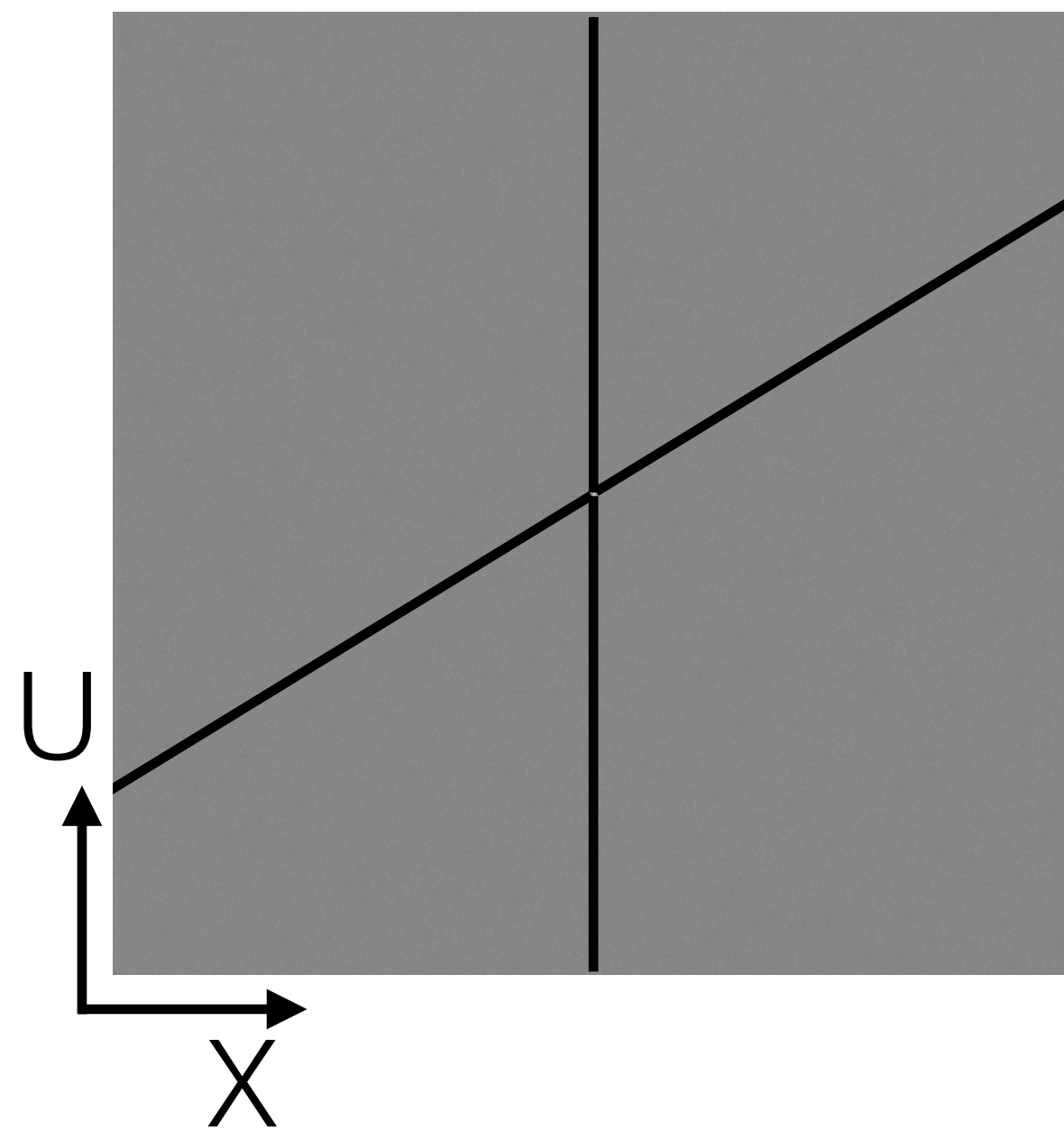
Integrand Spectrum



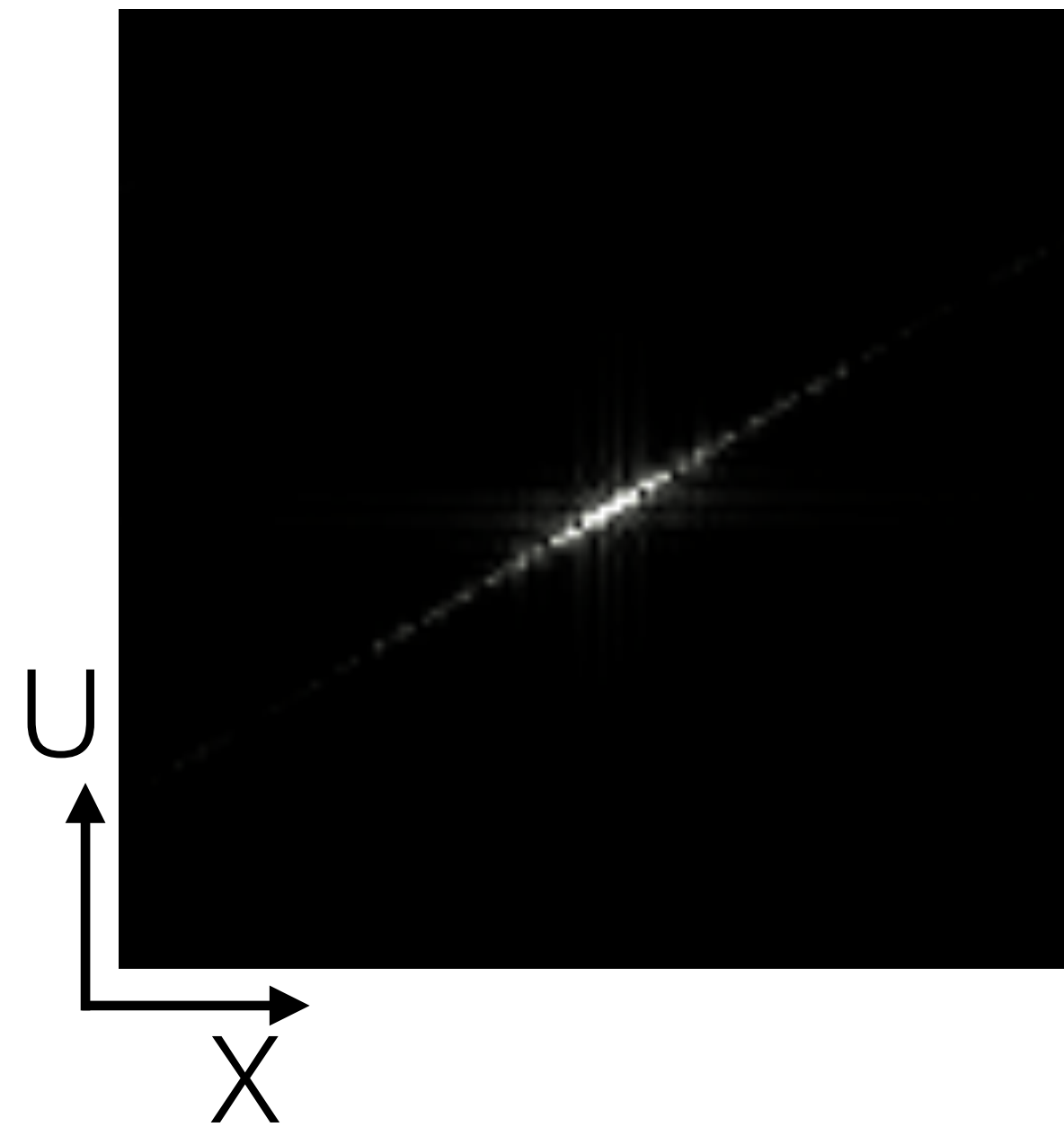
Sheared Uncorrelated Multi-jittered Samples



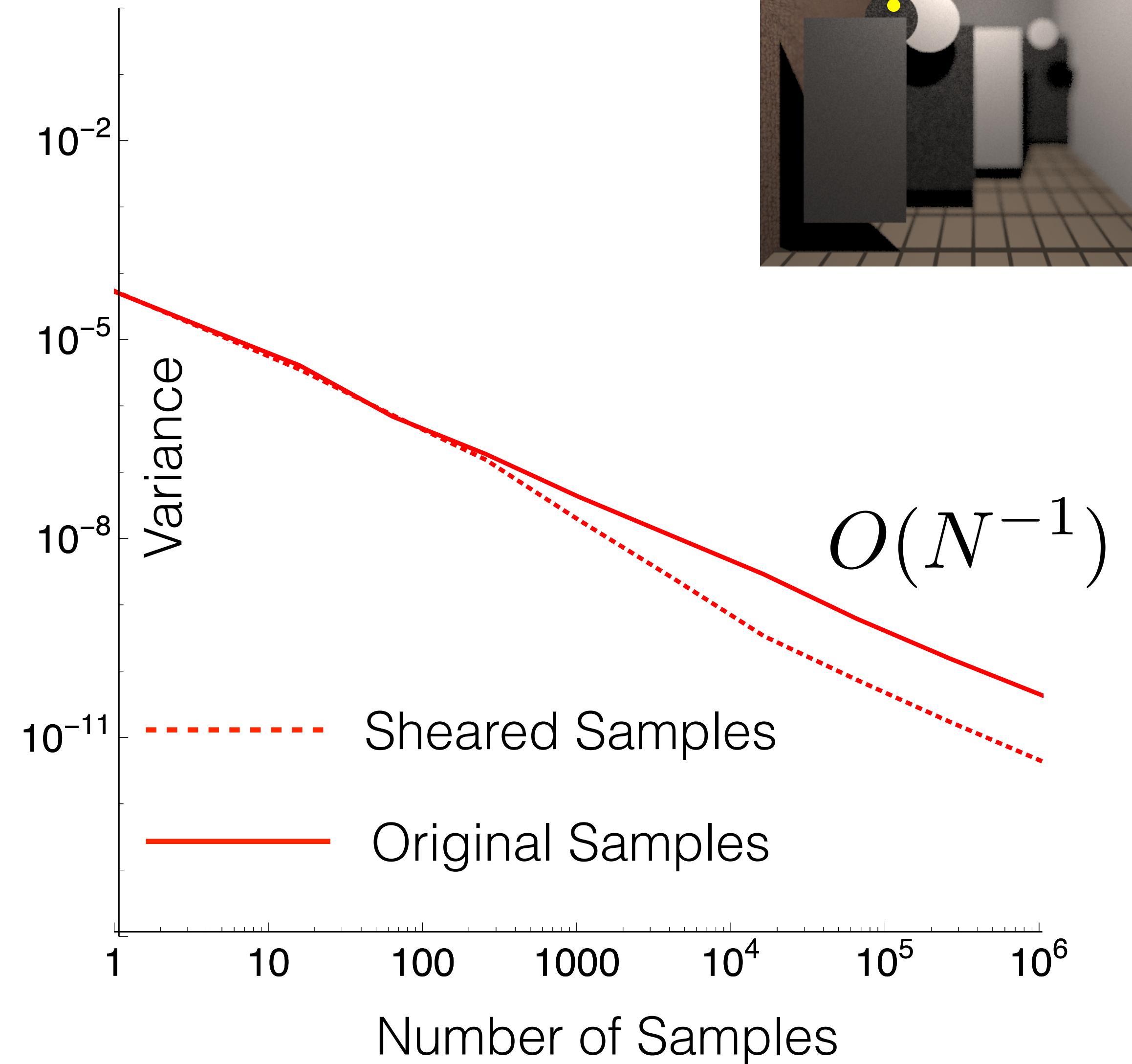
XU Subspace



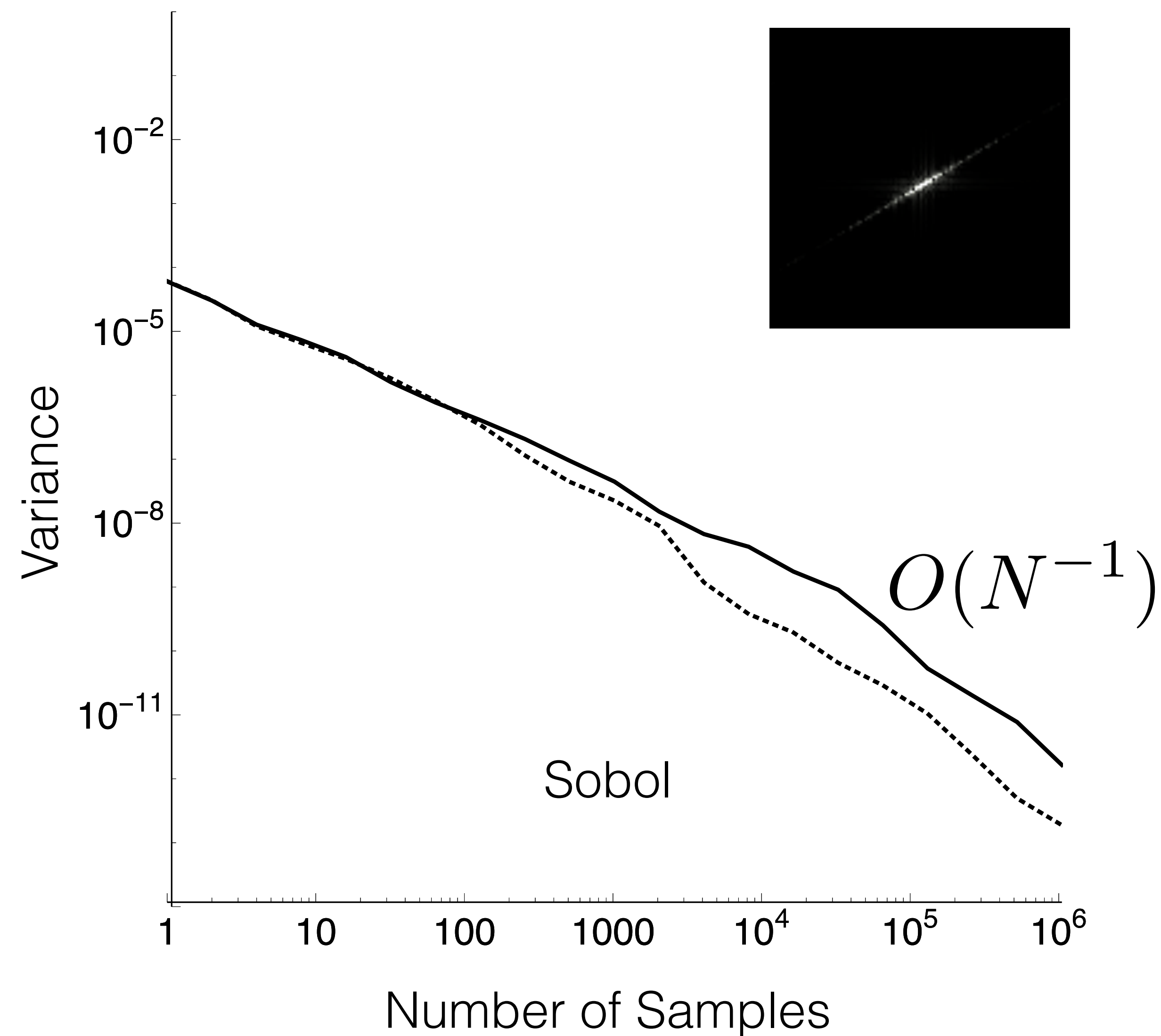
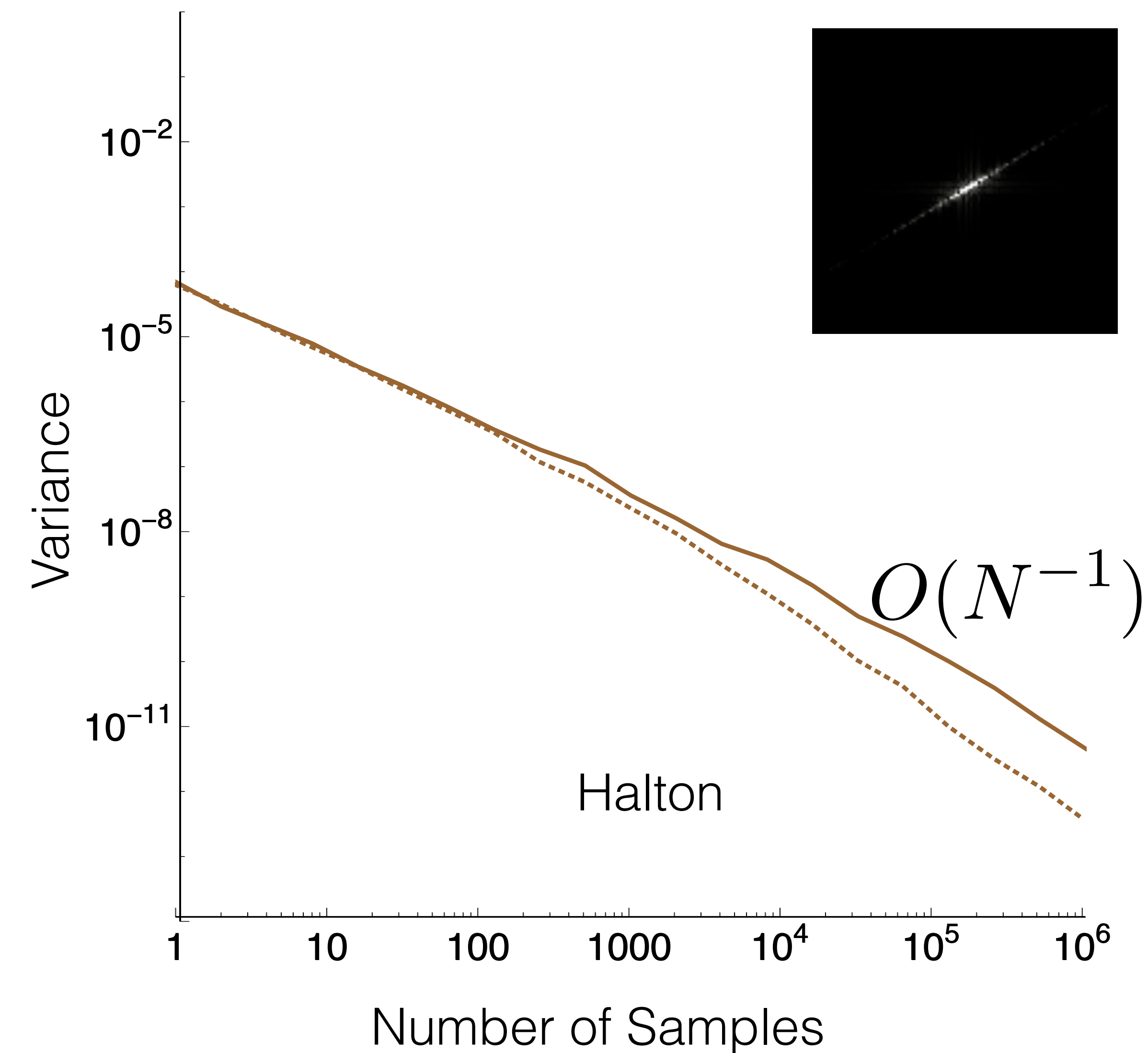
Sampling Spectrum



Integrand Spectrum

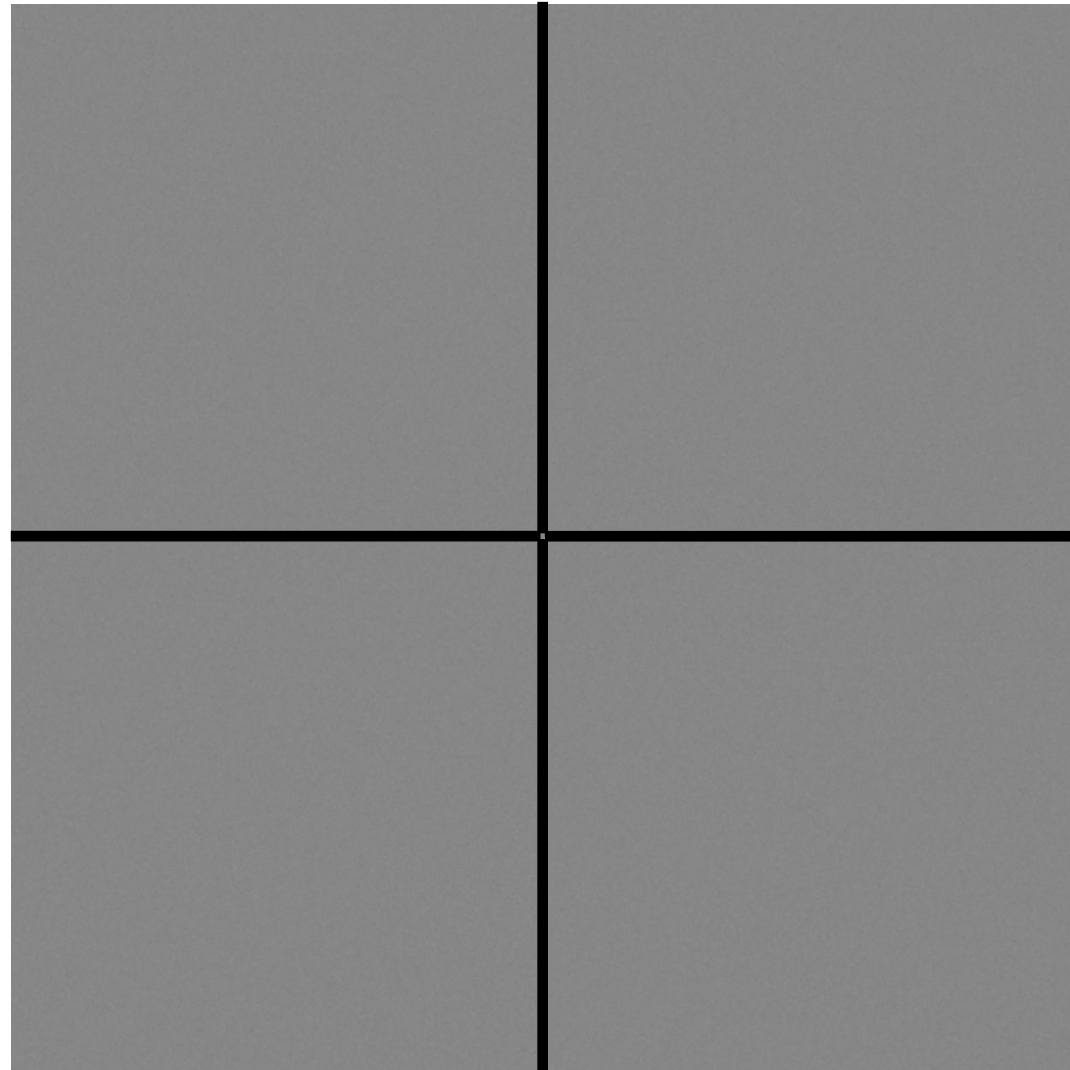


Variance improvement after Shearing

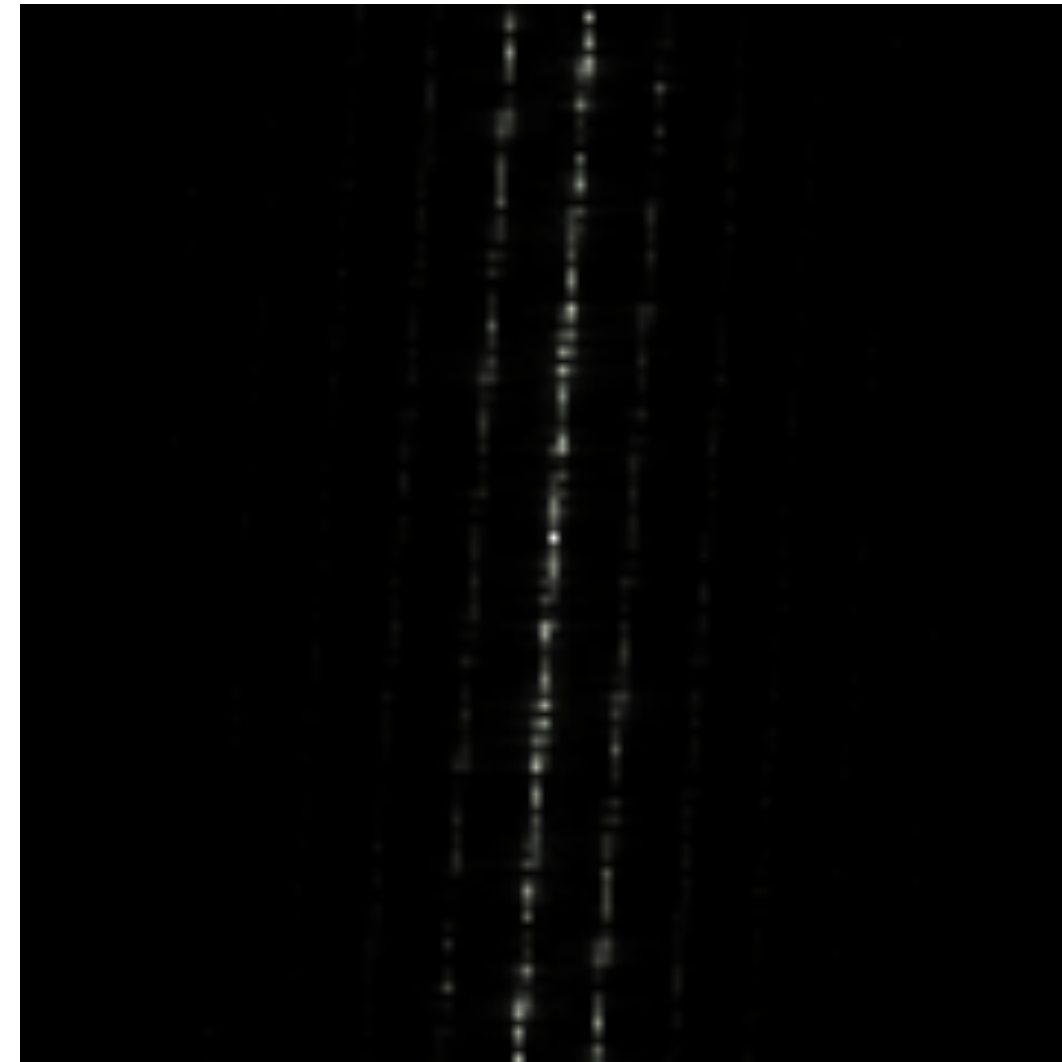


Challenging Cases: XU & YV Projections

Hairline Anisotropy



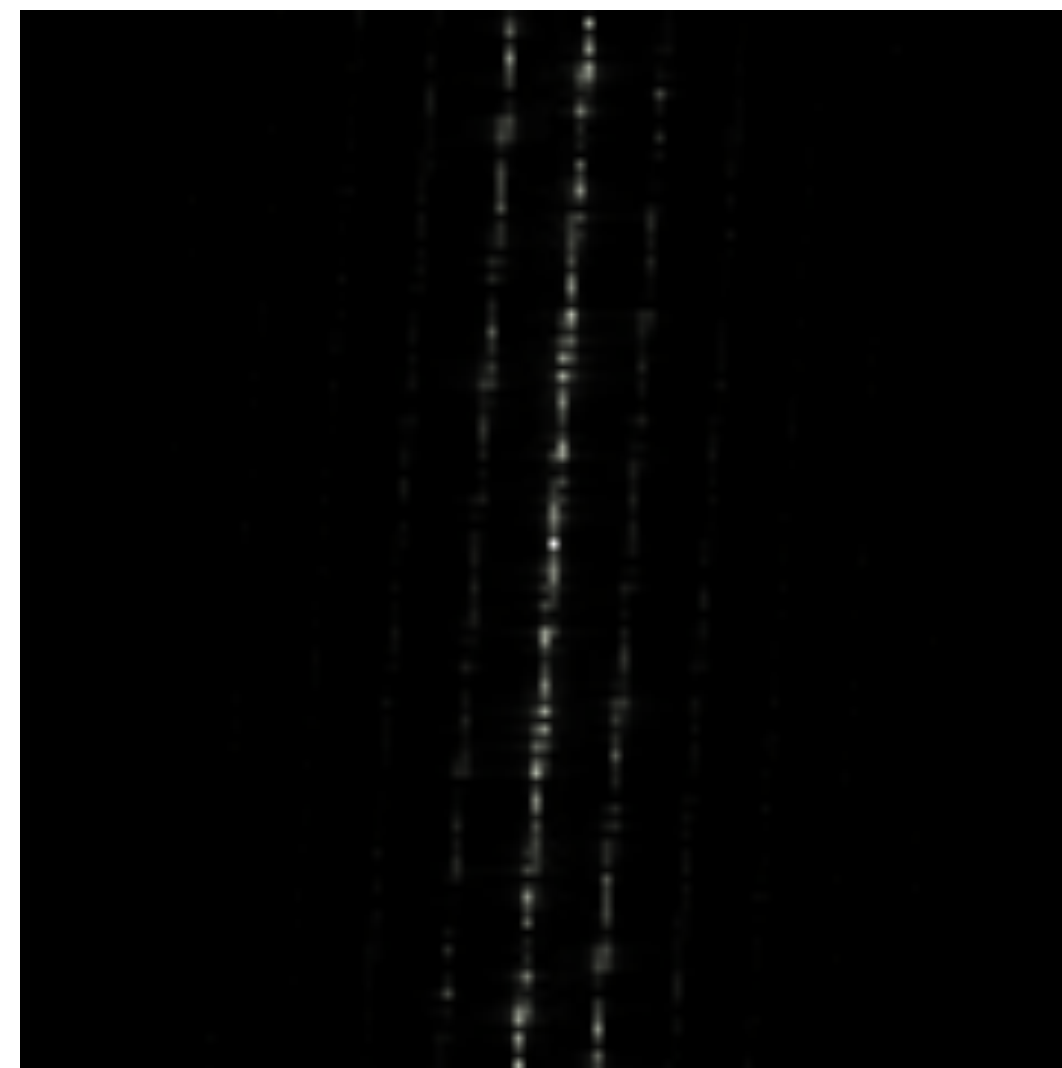
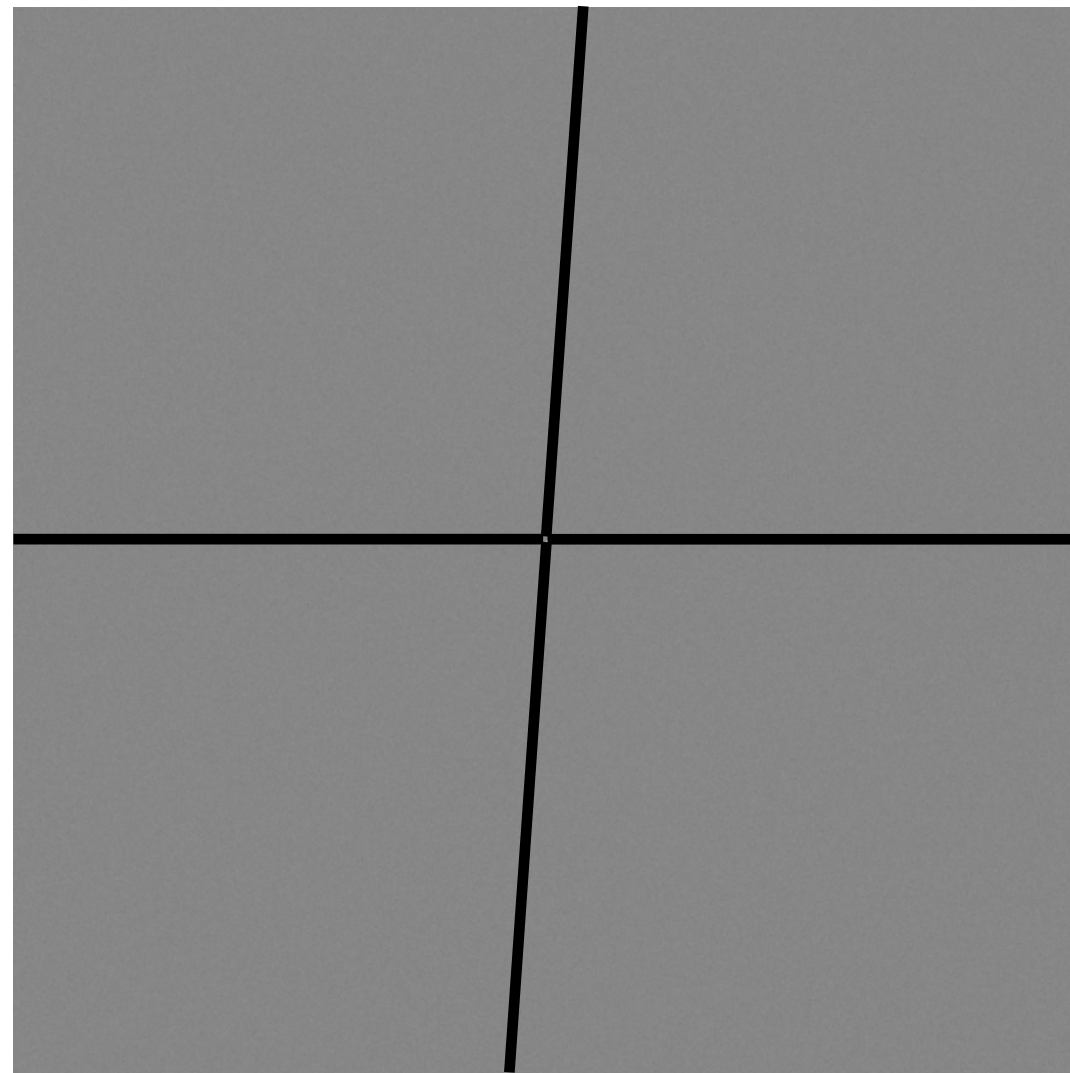
Sampling
XU Spectrum



Pixel A
XU Spectrum

Challenging Cases: XU & YV Projections

Hairline Anisotropy

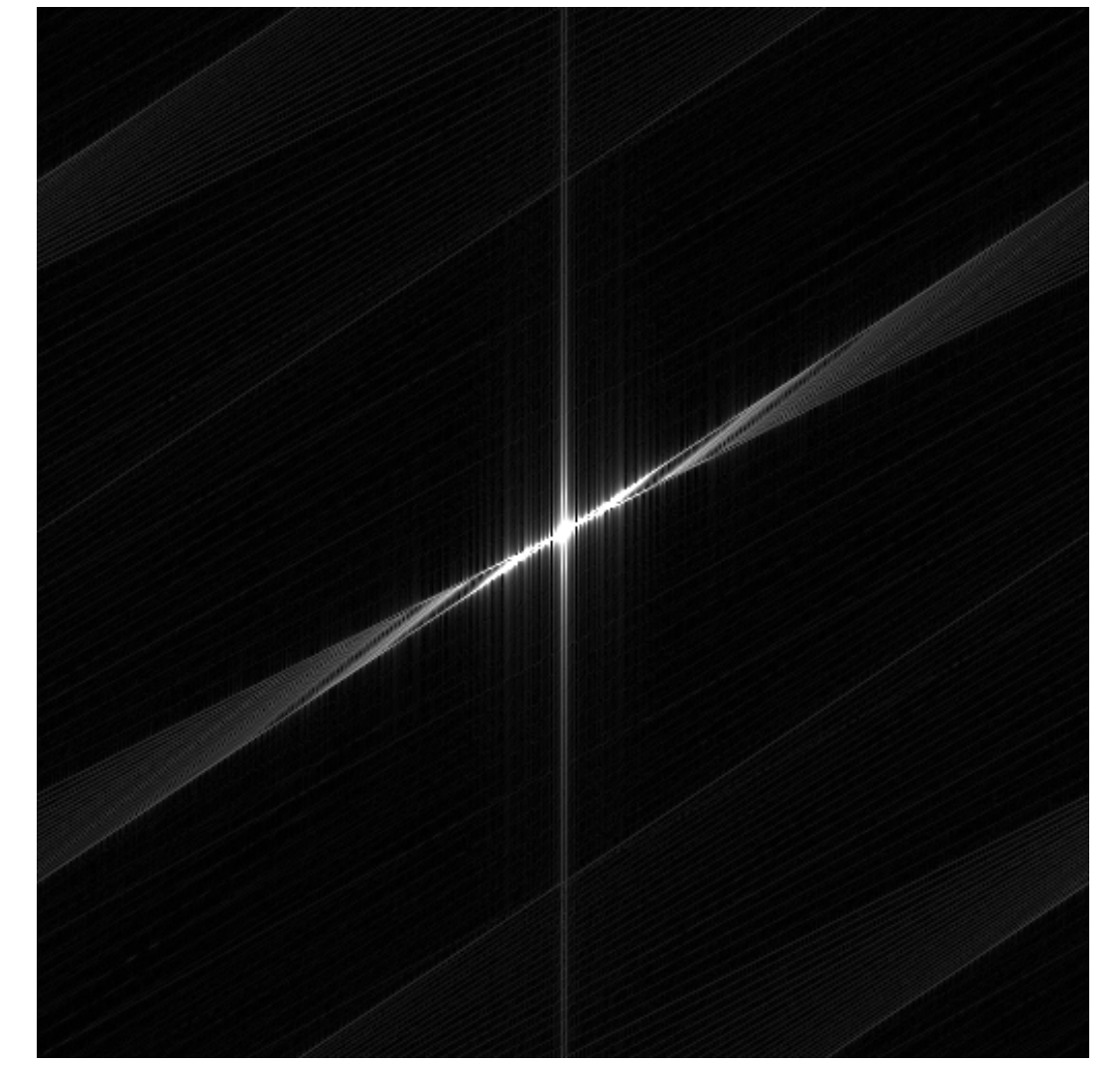
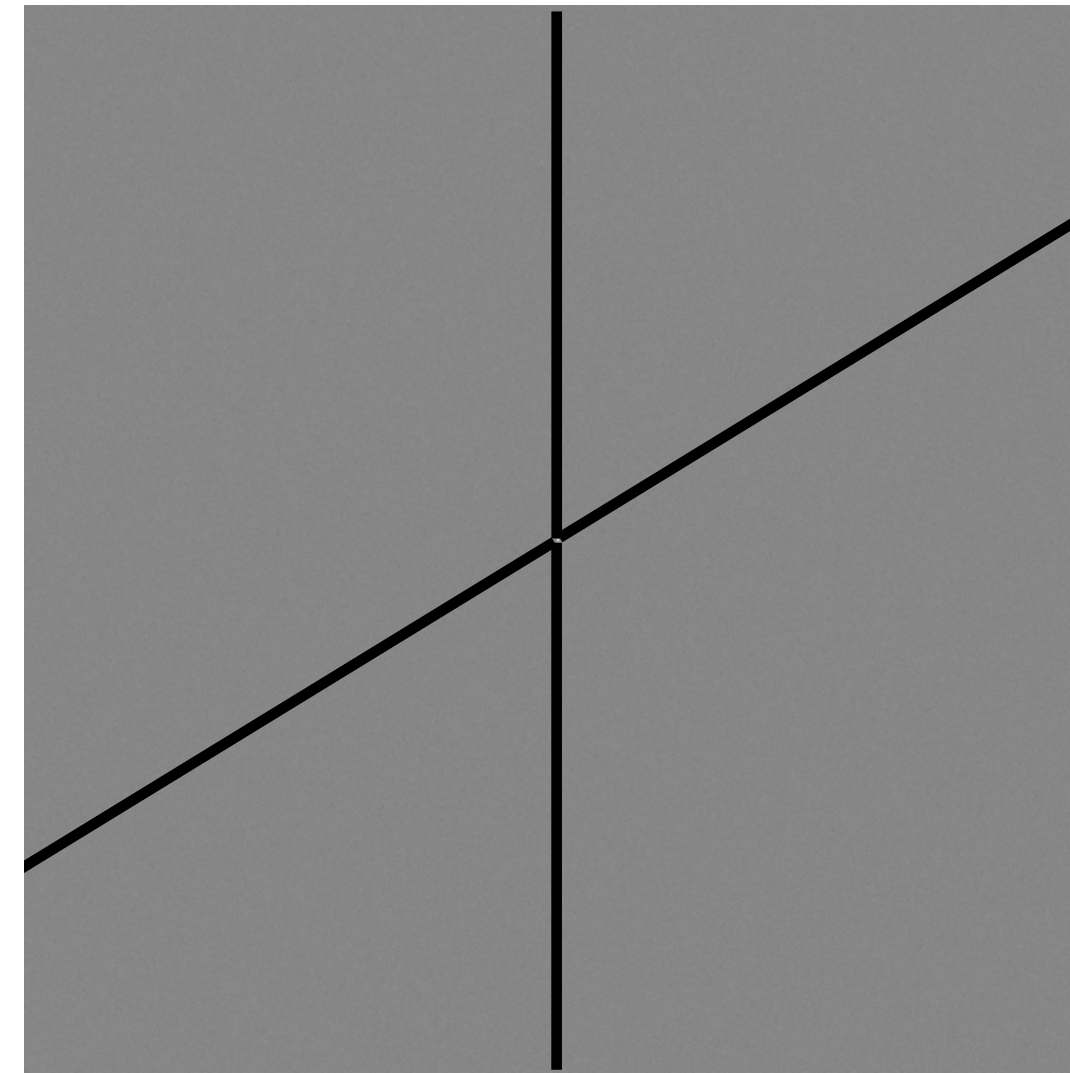


Sampling
XU Spectrum

Pixel A
XU Spectrum

Oracle Accuracy

Double-wedge Anisotropy



Sampling
XU Spectrum

Pixel B
XU Spectrum

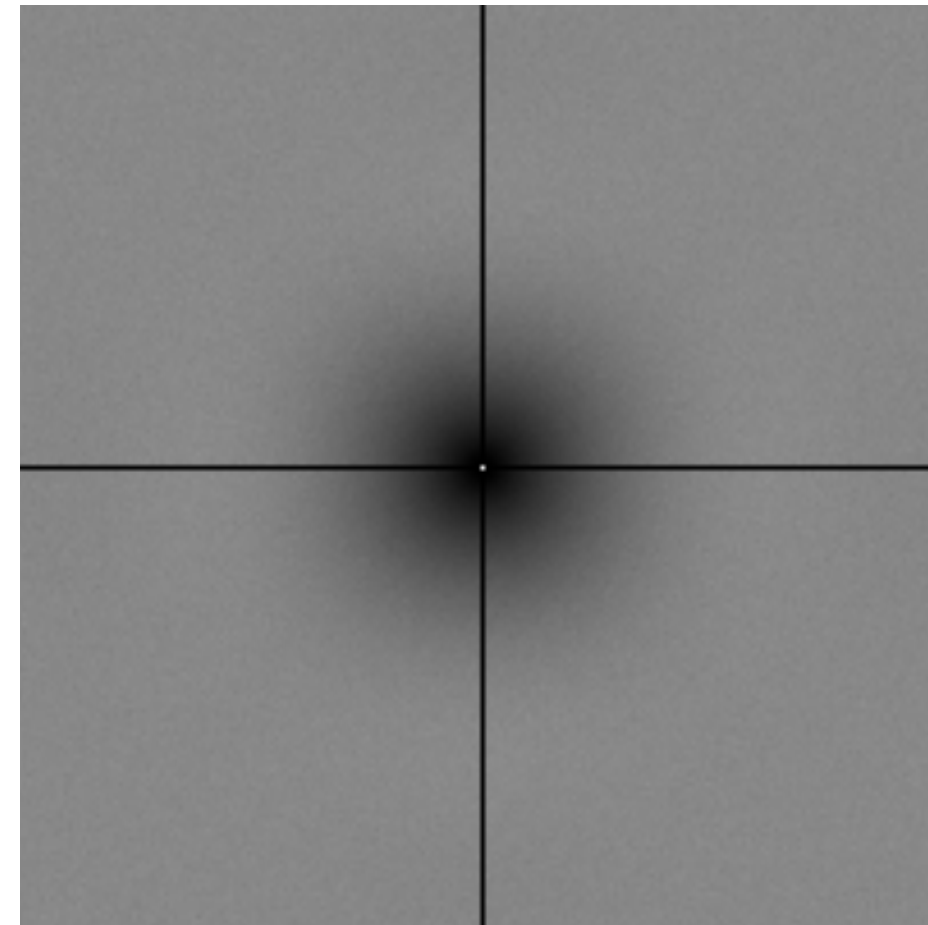
Double-wedge Spectrum

Design Principles for New Sampling Patterns

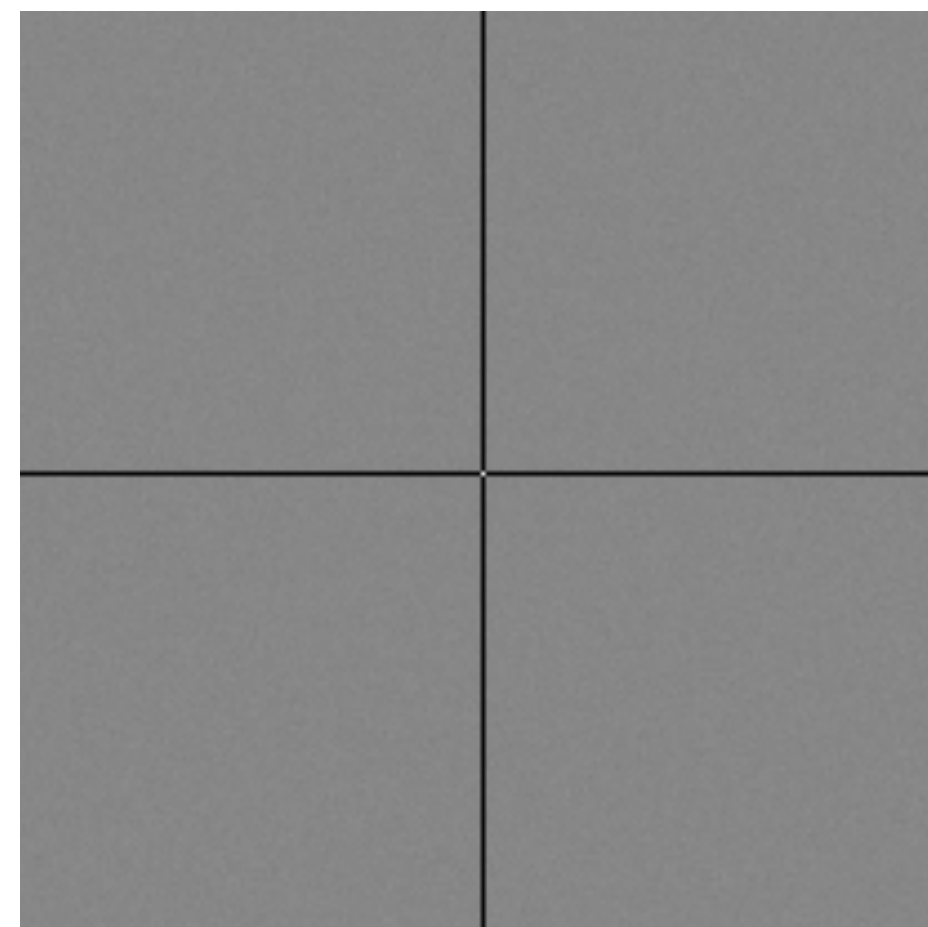
Multi-Jittered Spectra

Desired Sampling Spectra

XY

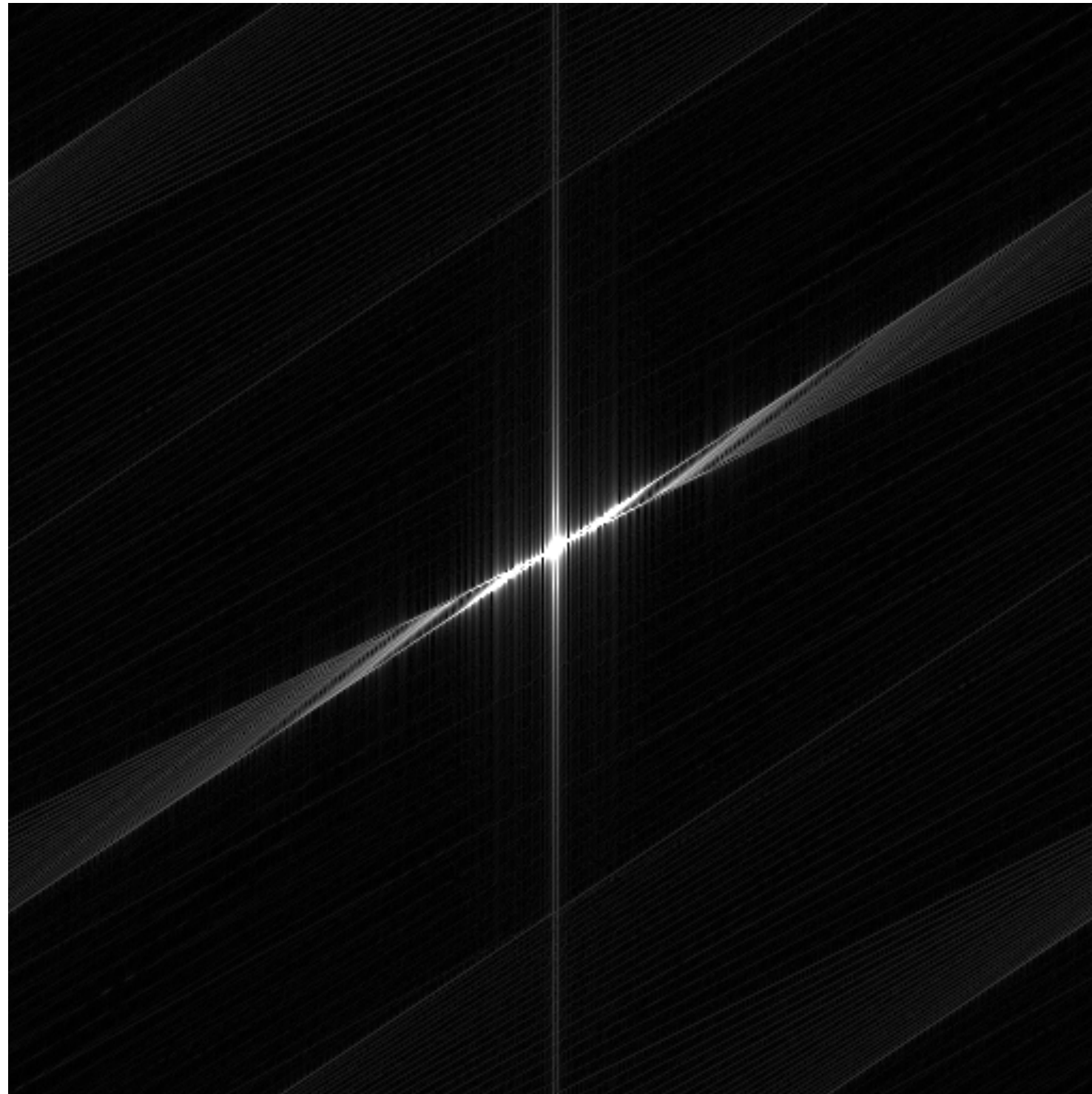


XU



Design Principles for New Sampling Patterns

Integrand Spectrum



Desired Sampling Spectra



In both XU and YV Projections

Thank you for your attention!

