

# Deep Point Correlation Design: Supplemental Material

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## 1 OVERVIEW

This document contains additional results to supplement explanations and experiments of the main paper. Sec. 2 provides more details on the variance reduction for gradients in the proposed network architecture. In Sec. 3, we first reproduce full-size power spectra (Fig. 1) for rank-1 and Fibonacci lattices from Fig. 13 of the main paper that only shows the cropped versions. Additional results on novel sampling patterns are shown in the following figures that were optimized for different class of functions in multiple dimensions. Fig. 5 shows improvement when a 3D sampler is optimized for 2D projections vs. full-dimensional optimization.

## 2 VARIANCE REDUCTION

The estimation of point correlations by means of a single point set realization is inherently noisy. To aid stability during training we

average the correlations of multiple realizations to reduce their variance. Consider point pattern realizations  $\{X_i\}_{i=0}^{n-1}$  resulting from our network, and a target spectrum  $S$ . A typical loss in our framework is defined as

$$\mathcal{L}(X) = \left\| \left( \frac{1}{n} \sum_{i=0}^{n-1} \text{spec}(X_i) \right) - S \right\|.$$

This is in contrast to typical mini-batching, which requires a loss function that linearly adds terms corresponding to the training data, i. e.,

$$\mathcal{L}(X) = \frac{1}{n} \sum_{i=0}^{n-1} \|\text{spec}(X_i) - S\|.$$

In our work, we enforce convex combinations of  $\mathcal{L}_1$  and  $\mathcal{L}_2$ , so that the expected value of the stochastic gradient is equal to the true gradient, which would require an infinite number of point set realizations.

## 3 ADDITIONAL EXPERIMENTS

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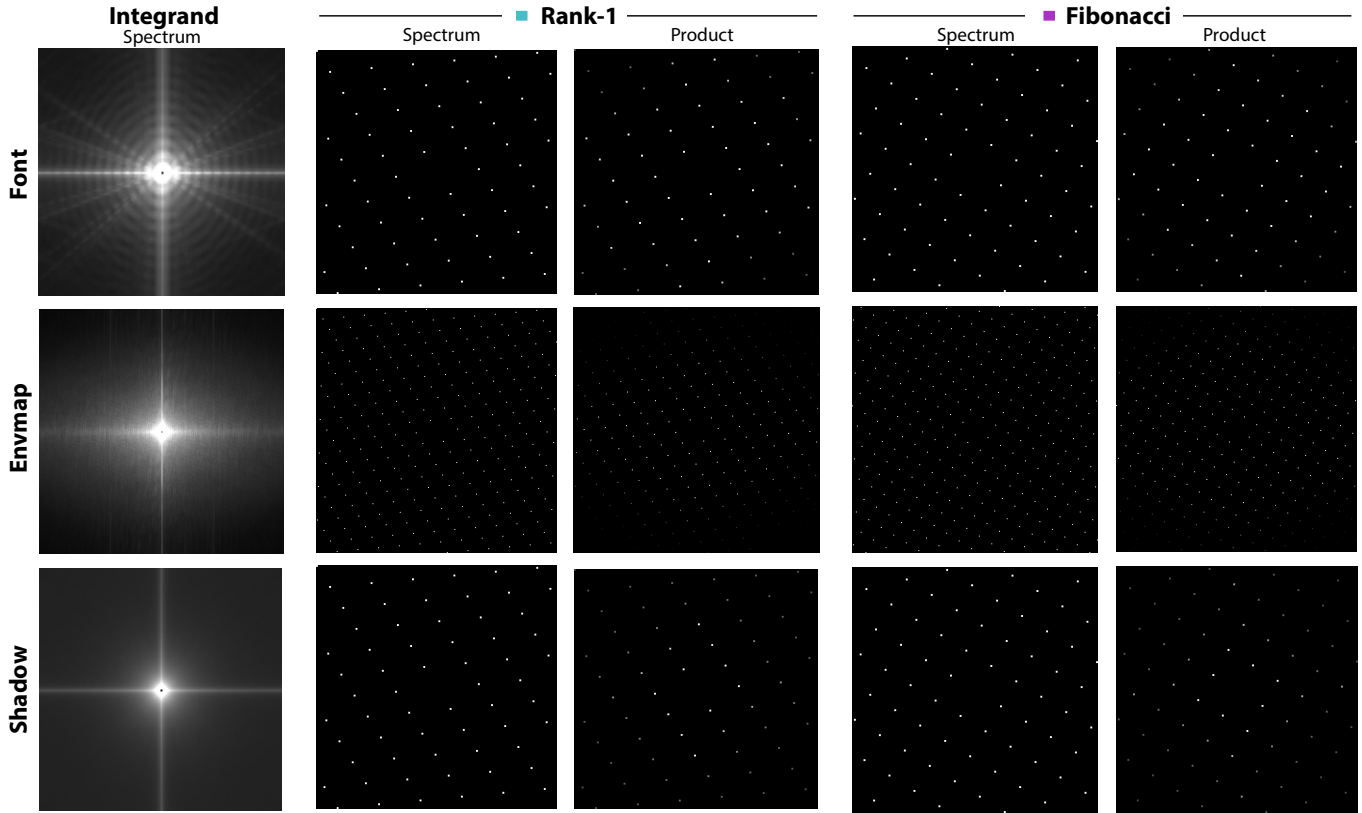


Fig. 1. Full-size spectra of Rank-1 and Fibonacci samplers from Fig.13 of the main paper and their corresponding product with the averaged integrand spectra is shown along each row. The spectra resolution in the middle-row is 2x higher than top and bottom rows spectra. Please see the main paper for more details.

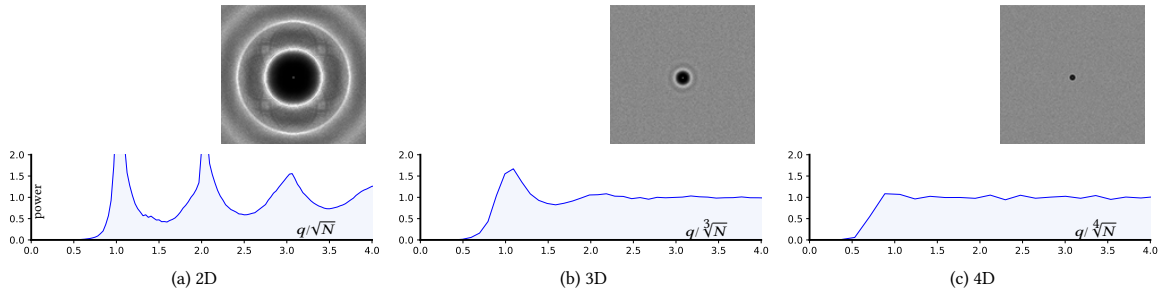


Fig. 2. Radially averaged power spectra are shown for novel sampling patterns in 2D, 3D and 4D. A full-dimensional optimization for  $N=1024$  samples is performed using the variance formulation (Eqn.5) from the main paper. From left-to-right, samples are optimized for a class of functions with a spectral fall-off of: (a)  $q^{-3}$  in 2D, (b)  $q^{-4}$  in 3D and (c)  $q^{-5}$  for 4D. Since the power spectra are isotropic, the insets on top-right in (b) and (c) shows only a single 2D projection of the full-dimensional spectrum.

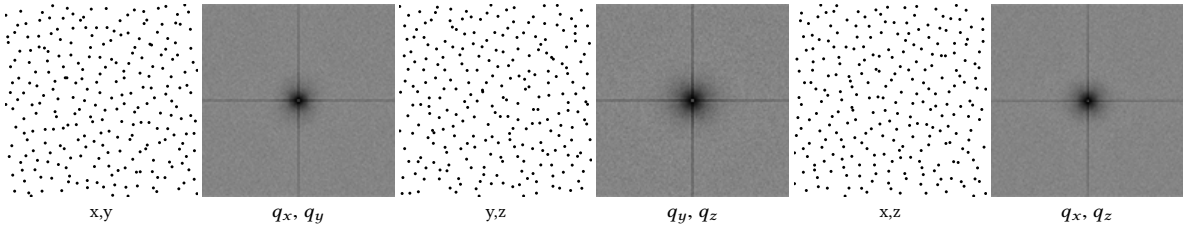


Fig. 3. We optimize for novel 3D point patterns ( $N = 1024$ ) but only in 2D projections. For each 2D projection, the sampling spectra is multiplied with the integrand spectra (2D fonts from Fig.13) which serves as a loss function for that projection. The final loss is the sum over all losses. No target sampling spectra was provided. The resultant spectral properties are novel and optimized w.r.t. the class of functions (fonts here) and is shown for each 2D projection. Note, that our network effortlessly optimizes even when different subspaces have shared coordinate axes.

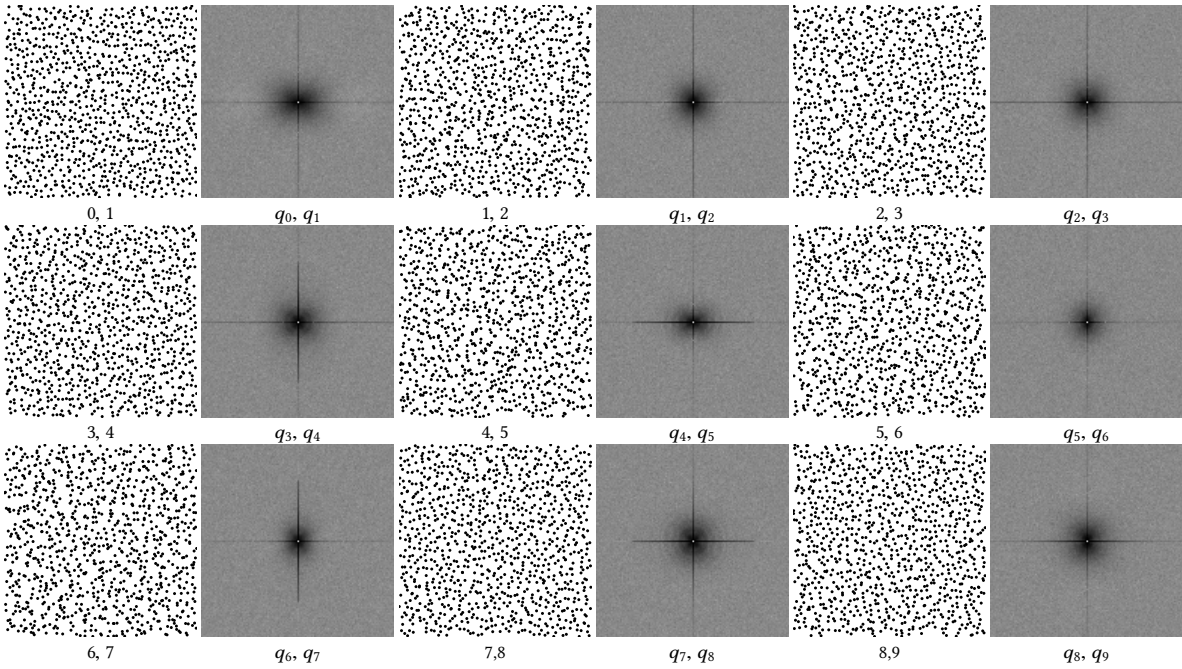


Fig. 4. Similar to Fig. 3, here we optimize for novel 10D point patterns ( $N = 1024$ ) but only in 2D projections. For each 2D projection, the sampling spectra is multiplied with the integrand spectra (2D fonts from Fig.13) which serves as a loss function for that projection. The final loss is the sum over all losses. No target sampling spectra was provided. The resultant spectral properties are novel and optimized w.r.t. the class of functions (fonts here) and is shown for each 2D projection.

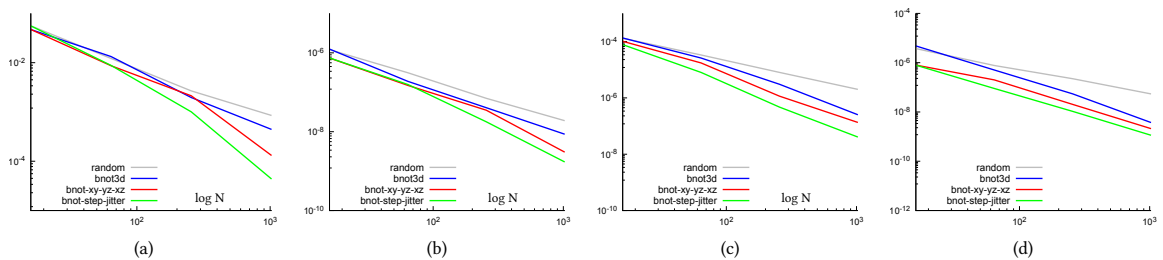


Fig. 5. Comparing samplers optimized in full dimensions (3D) versus their projective counterparts. This analysis shows improvements when optimization is done only for 2D subspaces, without considering the full-dimensional properties.