Supplementary Information: A Perception-driven Hybrid Decomposition for Multi-layer Accommodative Displays

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A The retinal optimization and light field synthesis

Equation (1) can be reformulated by explicitly writing v as a function of the depth position z_L of the light field plane, keeping in mind that the target light field plane can change along the depth similar to refocusing,

$$\underbrace{\begin{bmatrix} \mathbf{L}(v(z_L),u_1) \\ \mathbf{L}(v(z_L),u_2) \\ \vdots \\ \mathbf{L}(v(z_L),u_K) \end{bmatrix}}_{(KN)\times 1} = \underbrace{\begin{bmatrix} \mathbf{P}_{11}(z_L) & \mathbf{P}_{12}(z_L) & \dots & \mathbf{P}_{1D}(z_L) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{P}_{K1}(z_L) & \mathbf{P}_{K2}(z_L) & \dots & \mathbf{P}_{KD}(z_L) \end{bmatrix}}_{(KN)\times(DN)} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_D \end{bmatrix}}_{(DN)\times 1}. \tag{S.1}$$

From Eq. S.1, a focal image $\mathbf{F}$ at the depth z_L is computed as

$$\mathbf{F}(z_L) = \sum_{k=1}^{K} \mathbf{L}(v(z_L),u_k). \tag{S.2}$$

Therefore, the focal image can be represented in the following form:

$$\underbrace{\begin{bmatrix}\mathbf{F}(z_L)\end{bmatrix}}_{N\times 1} = \underbrace{\begin{bmatrix}\mathbf{S}_1(z_L) & \mathbf{S}_2(z_L) & \dots & \mathbf{S}_D(z_L)\end{bmatrix}}_{(N)\times(DN)} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_D \end{bmatrix}}_{(DN)\times 1} \tag{S.3}$$

where

$$\mathbf{S}_d(z_L) = \sum_{k=1}^{K} \mathbf{P}_{kd}(z_L). \tag{S.4}$$

If we extend it to a focal stack of F focal images at different depths $z_1, z_2, \dots z_F$, then we obtain the following result,

$$\underbrace{\begin{bmatrix} \mathbf{F}(z_1) \\ \mathbf{F}(z_2) \\ \vdots \\ \mathbf{F}(z_F) \end{bmatrix}}_{(FN)\times 1} = \underbrace{\begin{bmatrix} \mathbf{S}_1(z_1) & \mathbf{S}_2(z_1) & \dots & \mathbf{S}_D(z_1) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{S}_1(z_F) & \mathbf{S}_2(z_F) & \dots & \mathbf{S}_D(z_F) \end{bmatrix}}_{(FN)\times(DN)} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_D \end{bmatrix}}_{(DN)\times 1}. \tag{S.5}$$

This formulation is exactly the same as the optimal multi-plane decompositions in [2,3].

B Simulation of perceived images

We introduce a method to simulate a perceived focal image at the depth z_L when the decomposed images are projected on each layer. In this case, it is assumed that the solution $\mathbf{x}$ is already obtained. Then we first compute the generated light fields $\mathbf{L_p}$ from the decomposed images:

$$\underbrace{\begin{bmatrix} \mathbf{L_p}(v(z_L),u_1) \\ \mathbf{L_p}(v(z_L),u_2) \\ \vdots \\ \mathbf{L_p}(v(z_L),u_K) \end{bmatrix}}_{(KN)\times 1} = \underbrace{\begin{bmatrix} \mathbf{P}_{11}(z_L) & \mathbf{P}_{12}(z_L) & \dots & \mathbf{P}_{1D}(z_L) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{P}_{K1}(z_L) & \mathbf{P}_{K2}(z_L) & \dots & \mathbf{P}_{KD}(z_L) \end{bmatrix}}_{(KN)\times(DN)} \underbrace{\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_D \end{bmatrix}}_{(DN)\times 1} \tag{S.6}$$

Then a perceived focal image $\mathbf{F_p}$ at the depth z_L is computed as

$$\mathbf{F}(z_L) = \sum_{k=1}^{K} \mathbf{L_p}(v(z_L),u_k). \tag{S.7}$$

C A filter-based SART optimization

Here we introduce the formulation of the SART algorithm based on filtering operations. The update rule in the original SART algorithm is given by [1]

$$\mathbf{x}^{(\mathbf{q})} = \mathbf{x}^{(\mathbf{q-1})} + \mathbf{a} \circ (\mathbf{P}^T(\mathbf{b} \circ (\mathbf{L}_t - \mathbf{P}\mathbf{x}^{(\mathbf{q-1})}))). \tag{S.8}$$

$\mathbf{x}^{(\mathbf{q})}$ denotes the decomposed image at the qth iteration and $\circ$ represents the Hadamard product for element-wise multiplication. The elements of $\mathbf{a}$ and $\mathbf{b}$ are given by

$$a_m = \frac{1}{\sum_{n=1}^{N} P_{mn}}, b_n = \frac{1}{\sum_{m=1}^{M} P_{mn}}. \tag{S.9}$$

We can rewrite the above equation in the following form,

$$\mathbf{x}^{(\mathbf{q})} = \mathbf{x}^{(\mathbf{q-1})} + \mathbf{a} \circ \mathbf{P}^T(\mathbf{b} \circ \mathbf{L}_t) - \mathbf{a} \circ \mathbf{P}^T(\mathbf{b} \circ \mathbf{P}\mathbf{x}^{(\mathbf{q-1})}). \tag{S.10}$$

Since $\mathbf{a}$ and $\mathbf{b}$ are uniform constant vector, we can simply write the equation

$$\mathbf{x}^{(\mathbf{q})} = \mathbf{x}^{(\mathbf{q-1})} + \mathbf{c} \circ \mathbf{P}^T\mathbf{L}_t - \mathbf{c} \circ \mathbf{P}^T\mathbf{P}\mathbf{x}^{(\mathbf{q-1})}. \tag{S.11}$$

Here,

$$c_i = a_1 \times b_1. \tag{S.12}$$

Compared to the original formulation, the modified formulation can provide three advantages. Firstly, the second term needs to be computed only once. Secondly, since the projection matrix $\mathbf{P}$ is sparse, $\mathbf{P}^T\mathbf{P}$ is also a sparse matrix. We found that this operation is similar to convolution with a simple discrete kernel. Once we precalculate the kernels, the whole computation can be replaced with the filtering operation,

$$\mathbf{x}^{(\mathbf{q})} = \mathbf{x}^{(\mathbf{q-1})} + \mathbf{c} \circ K_1 * \mathbf{L}_t - \mathbf{c} \circ K_2 * \mathbf{x}^{(\mathbf{q-1})}. \tag{S.13}$$

Thirdly, the third term only depends on the number of displays, rather the number of viewpoints. This algorithm is easily implementable in CUDA and we observed 10x enhanced performance over the vertex-shader-based implementation using OpenGL [1].

REFERENCES

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