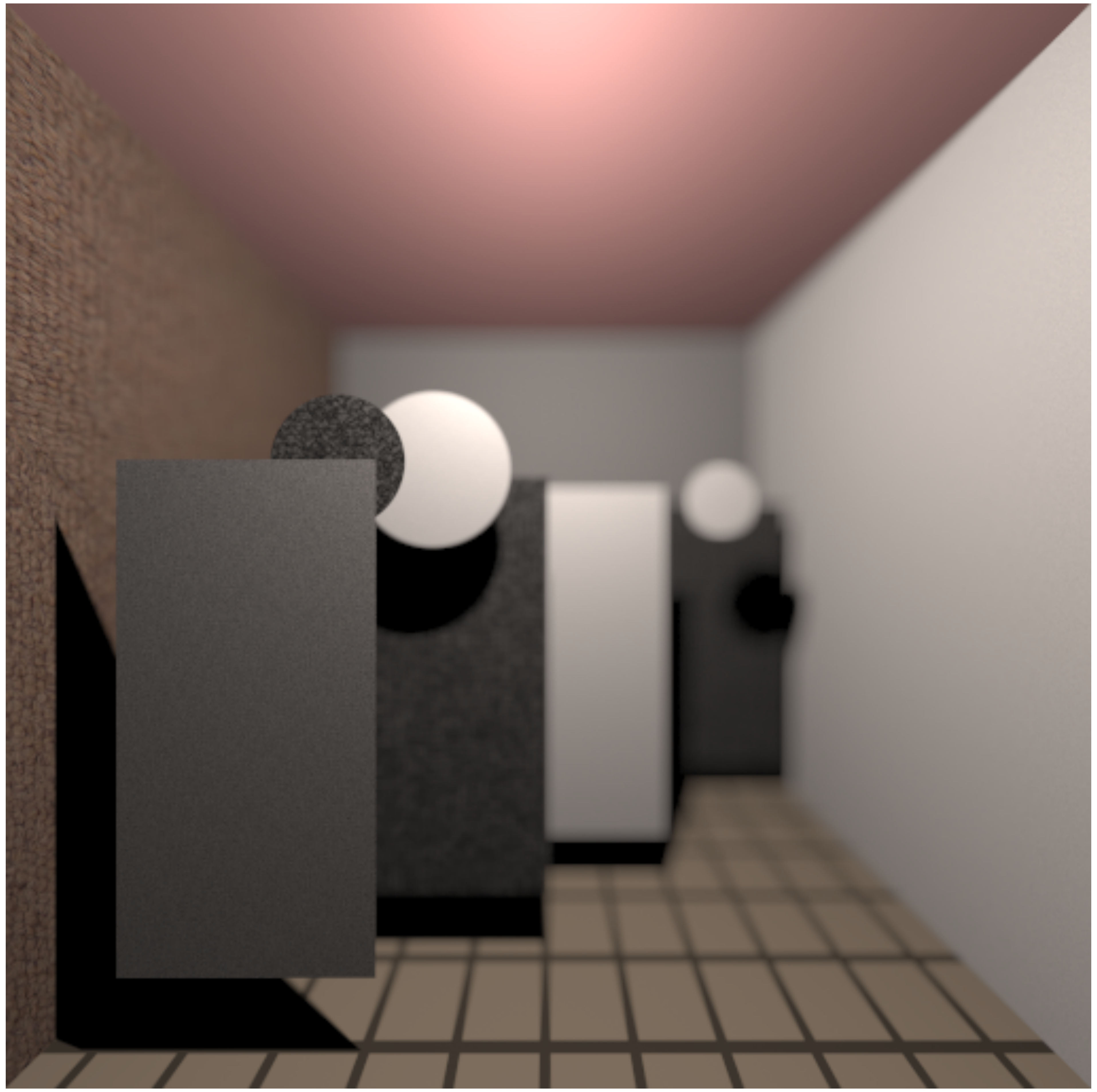
SIGGRAPH2017
AT THE ♥ of COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES

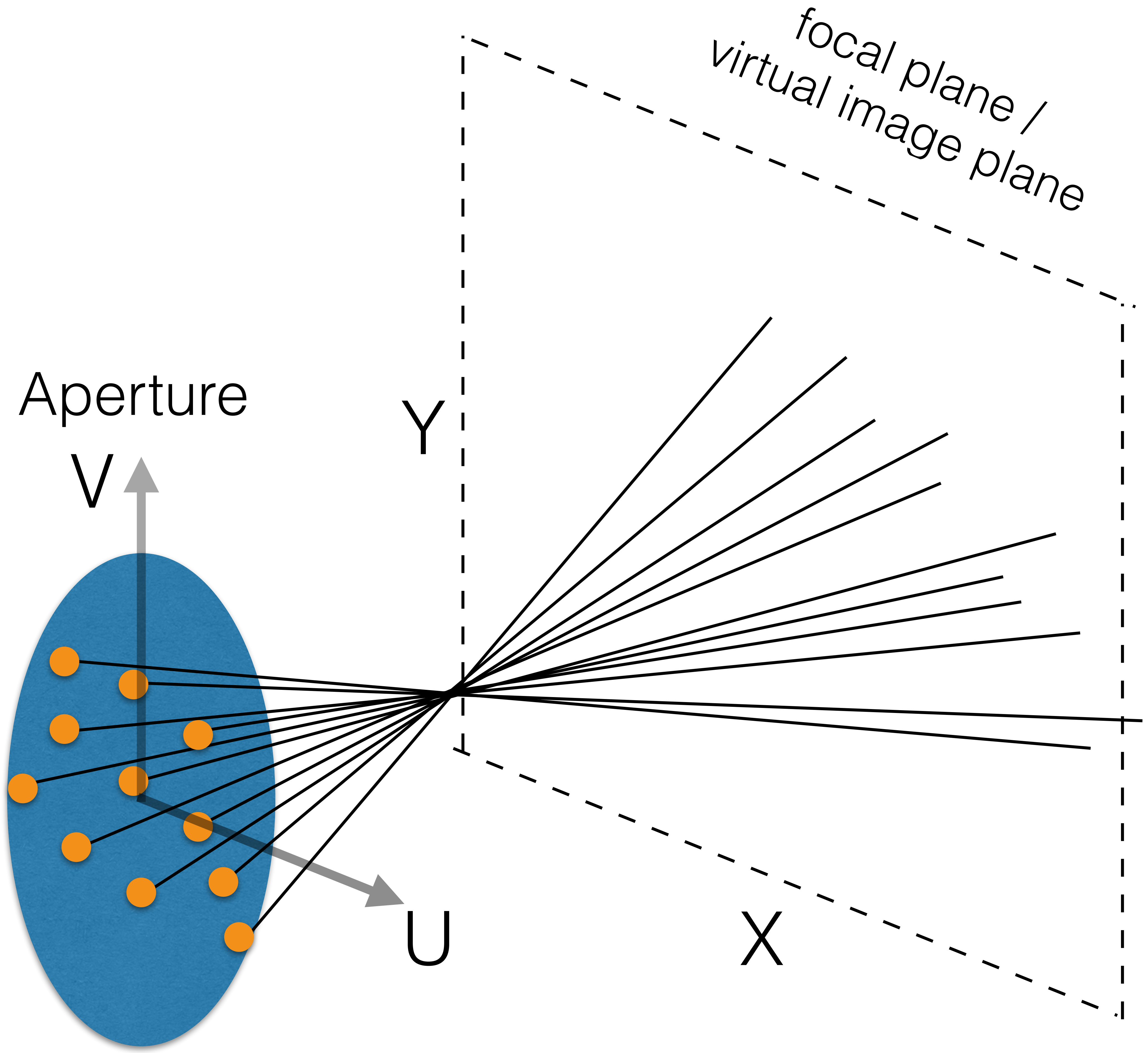
Convergence Analysis for Anisotropic Monte Carlo Sampling Spectra

Gurprit Singh **Wojciech Jarosz**

DARTMOUTH
VISUAL COMPUTING LAB

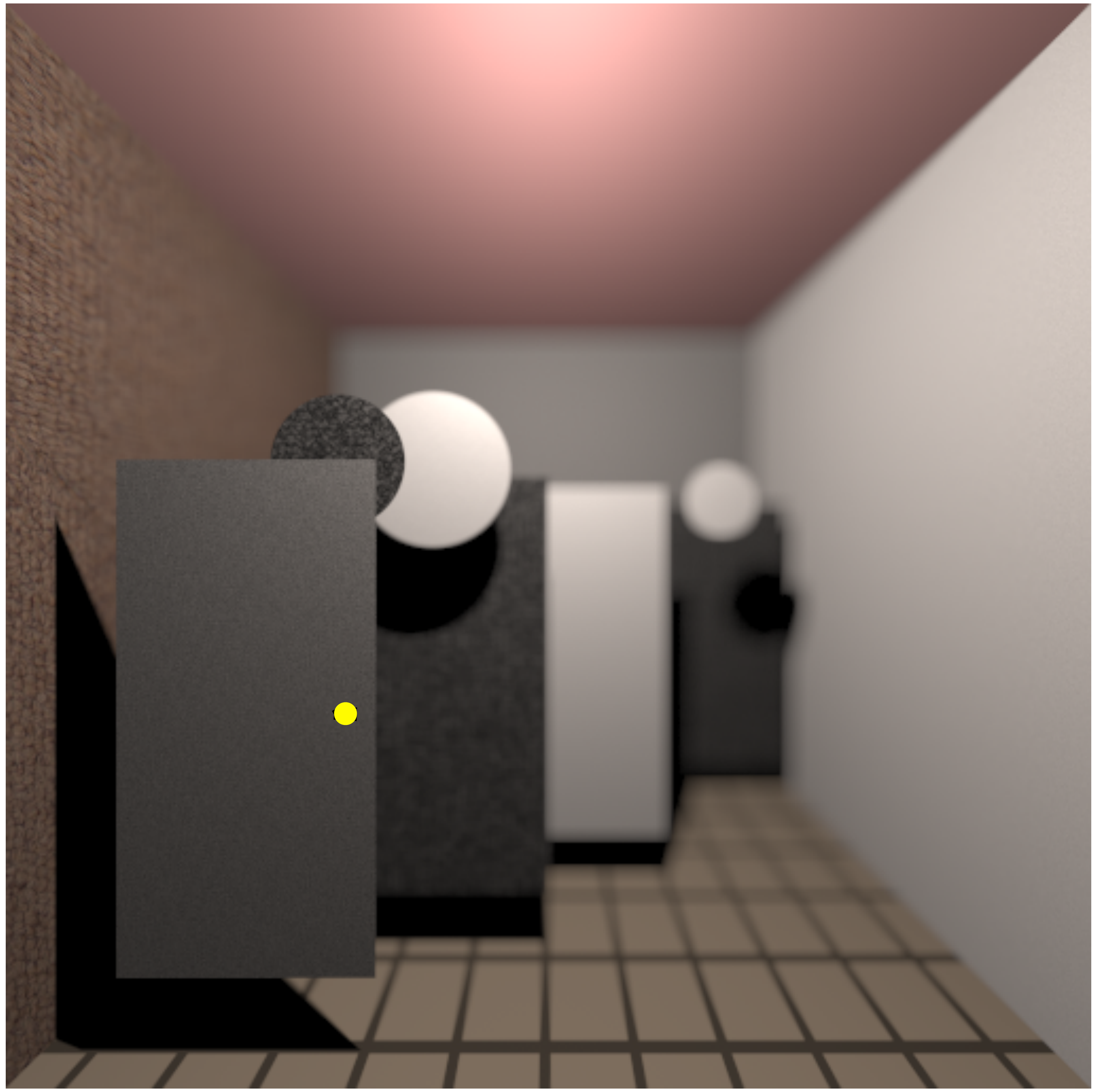
Monte Carlo Integration



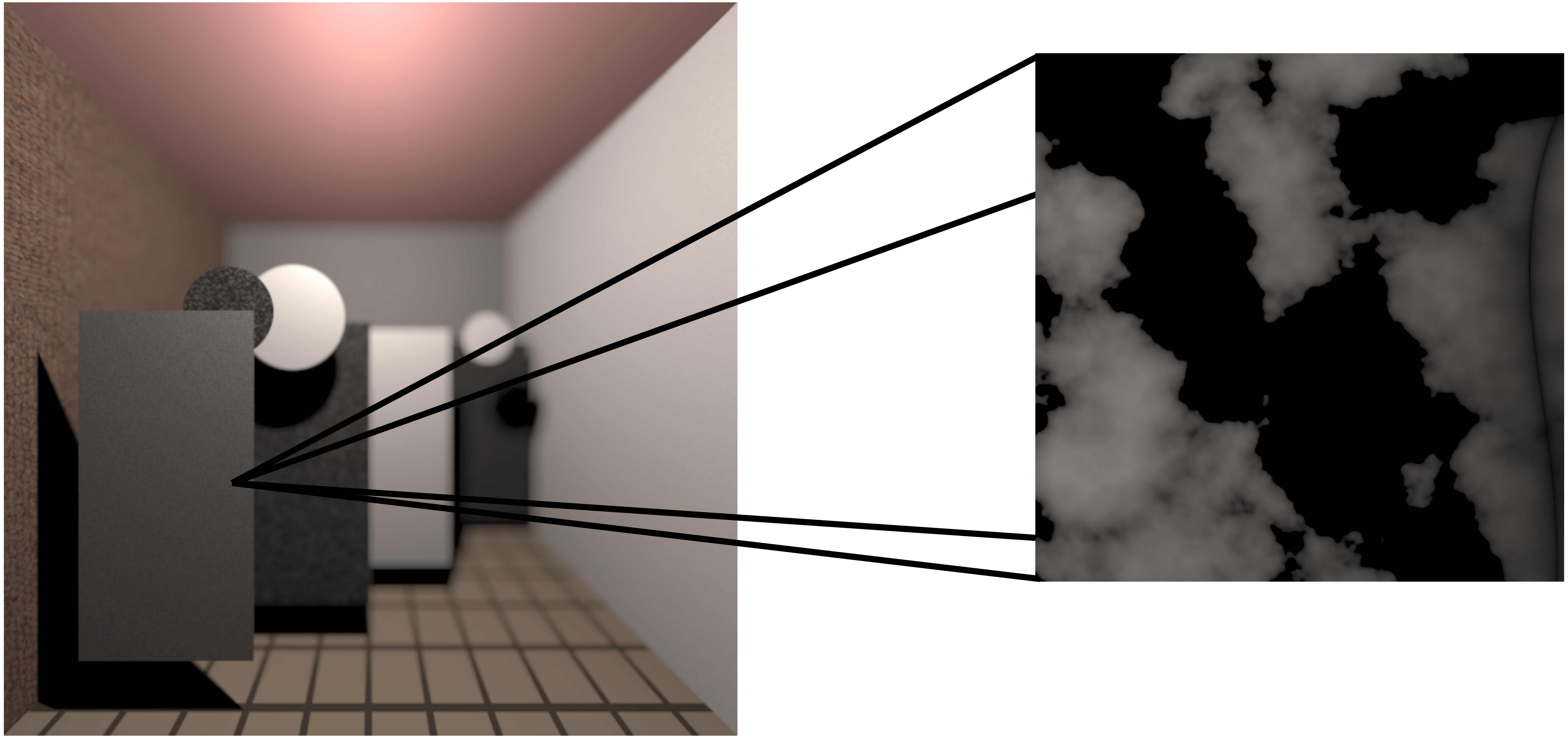


$$\int_x \int_y \int_u \int_v f(x, y, u, v)\, dv\, du\, dy\, dx$$

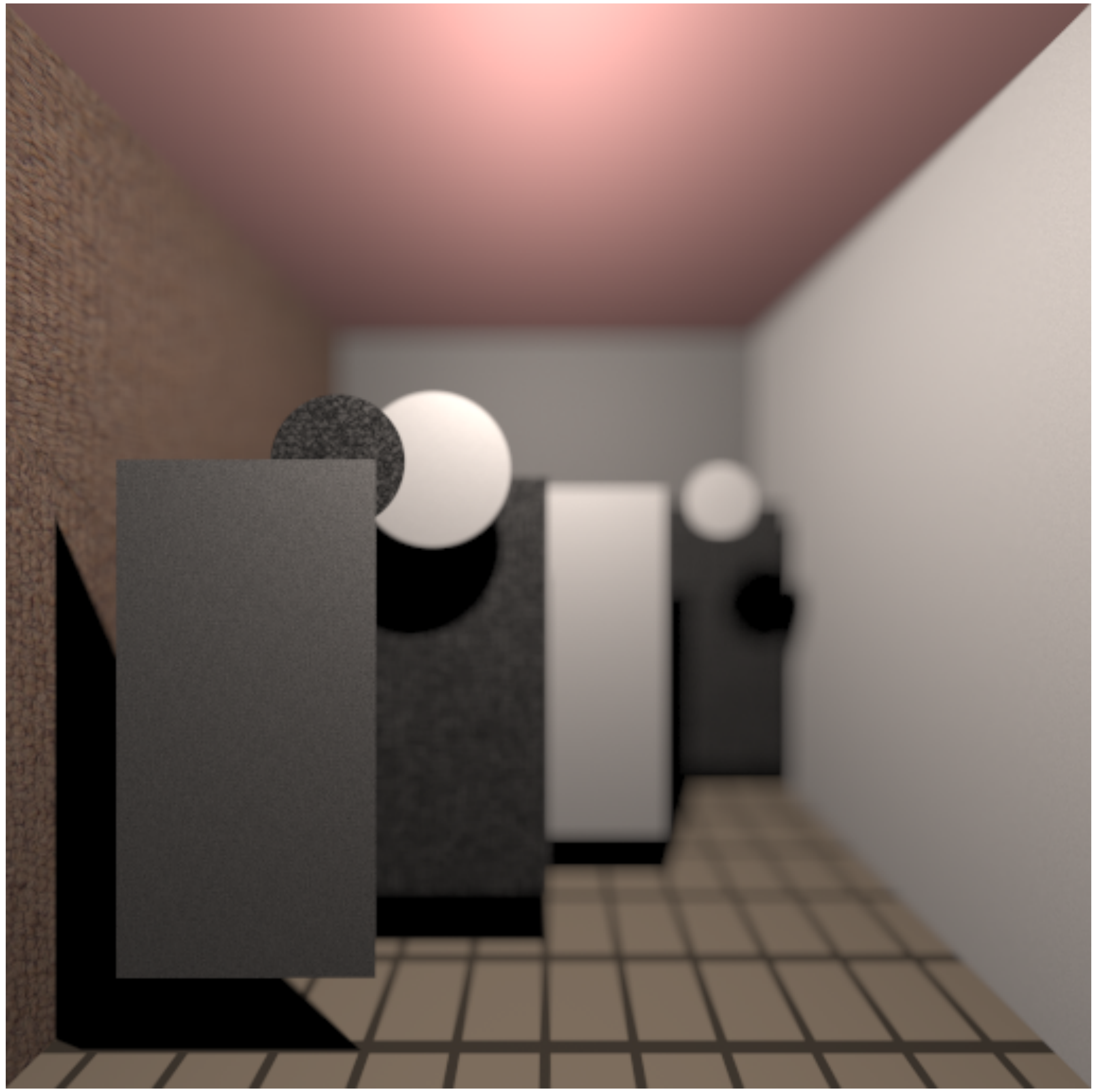
Monte Carlo Integration



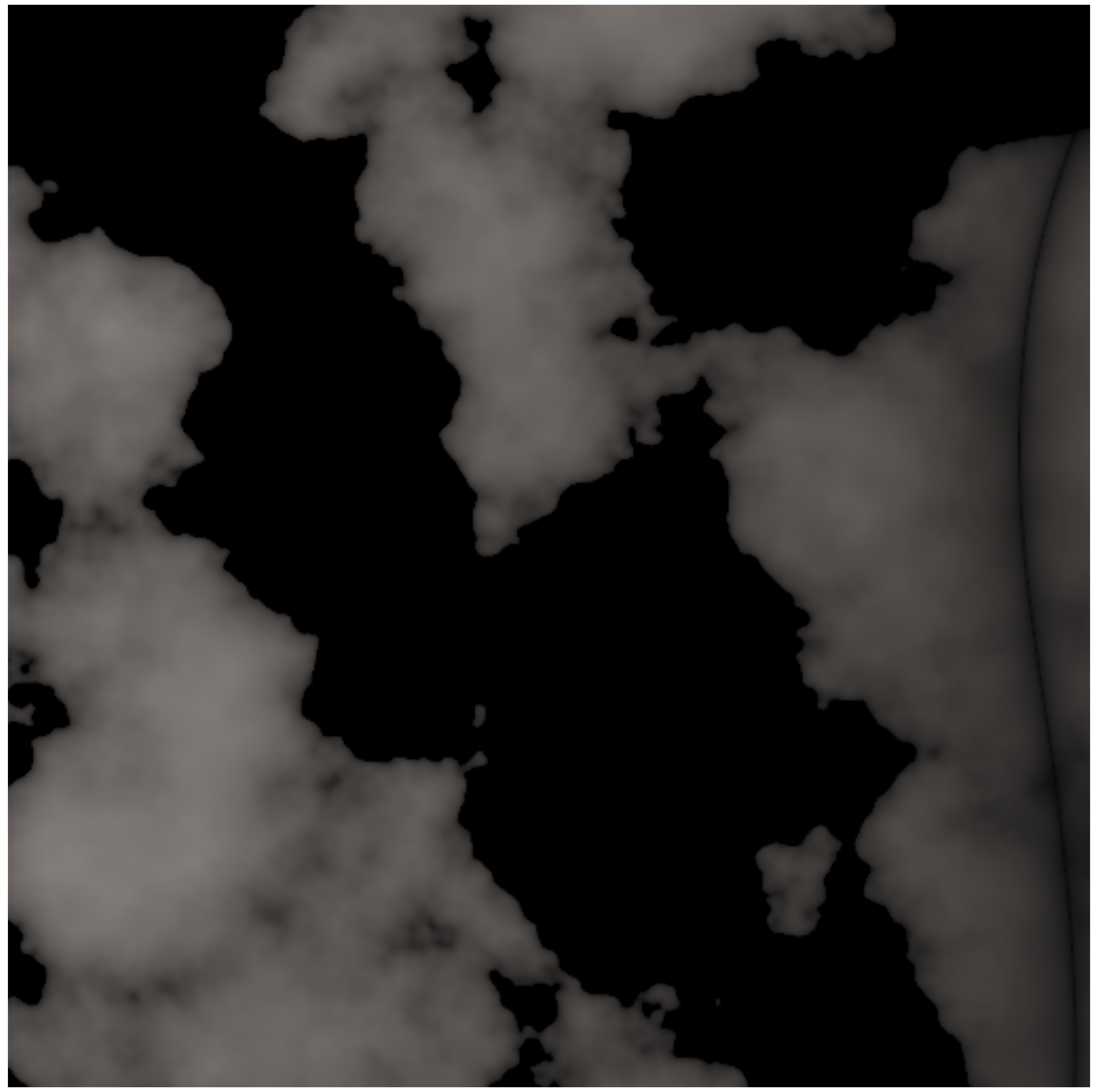
Monte Carlo Integration



Monte Carlo Integration

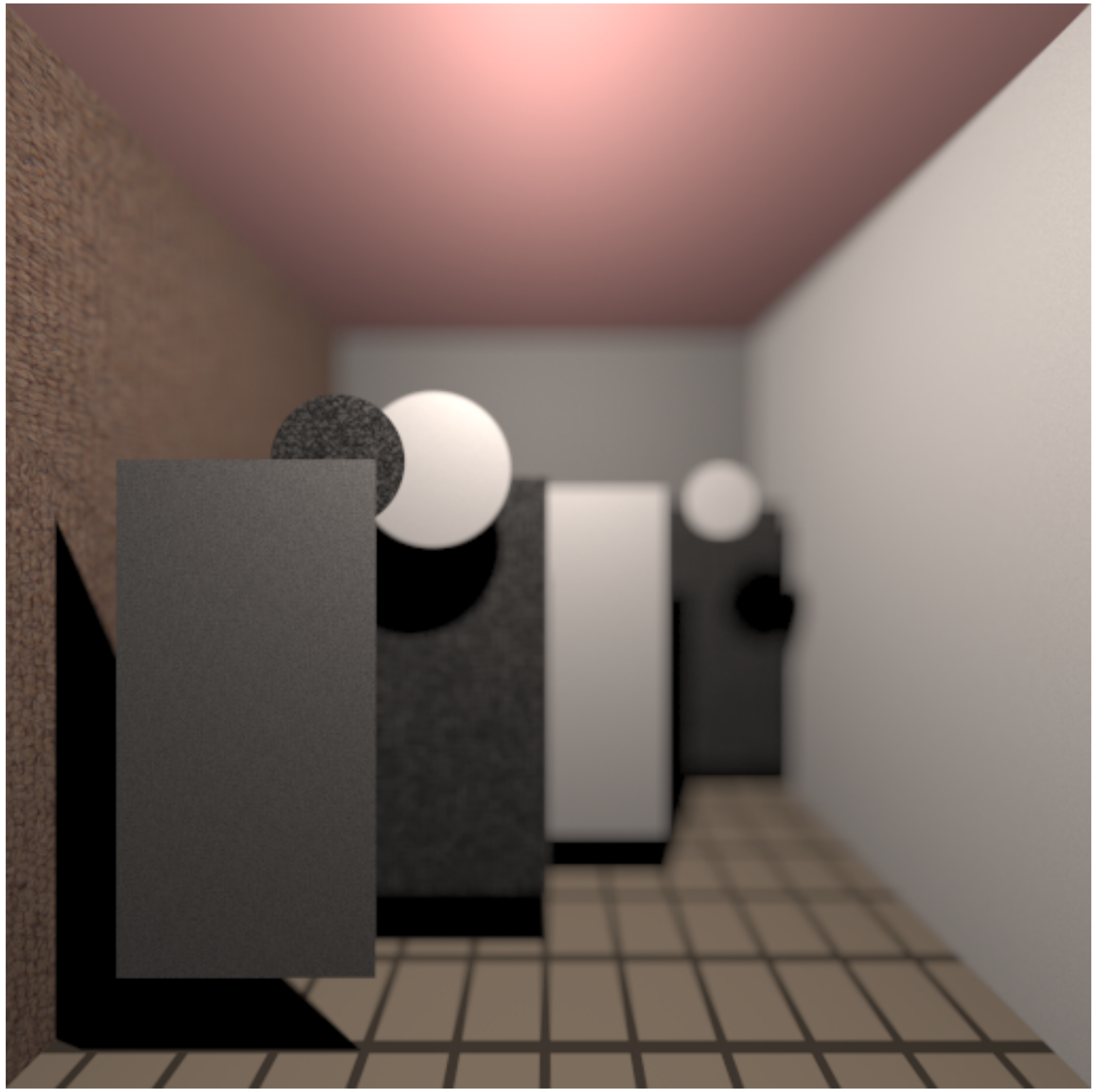


$f(\vec{x})$



$$I = \int_0^1 f(\vec{x})d\vec{x}$$

Monte Carlo Integration



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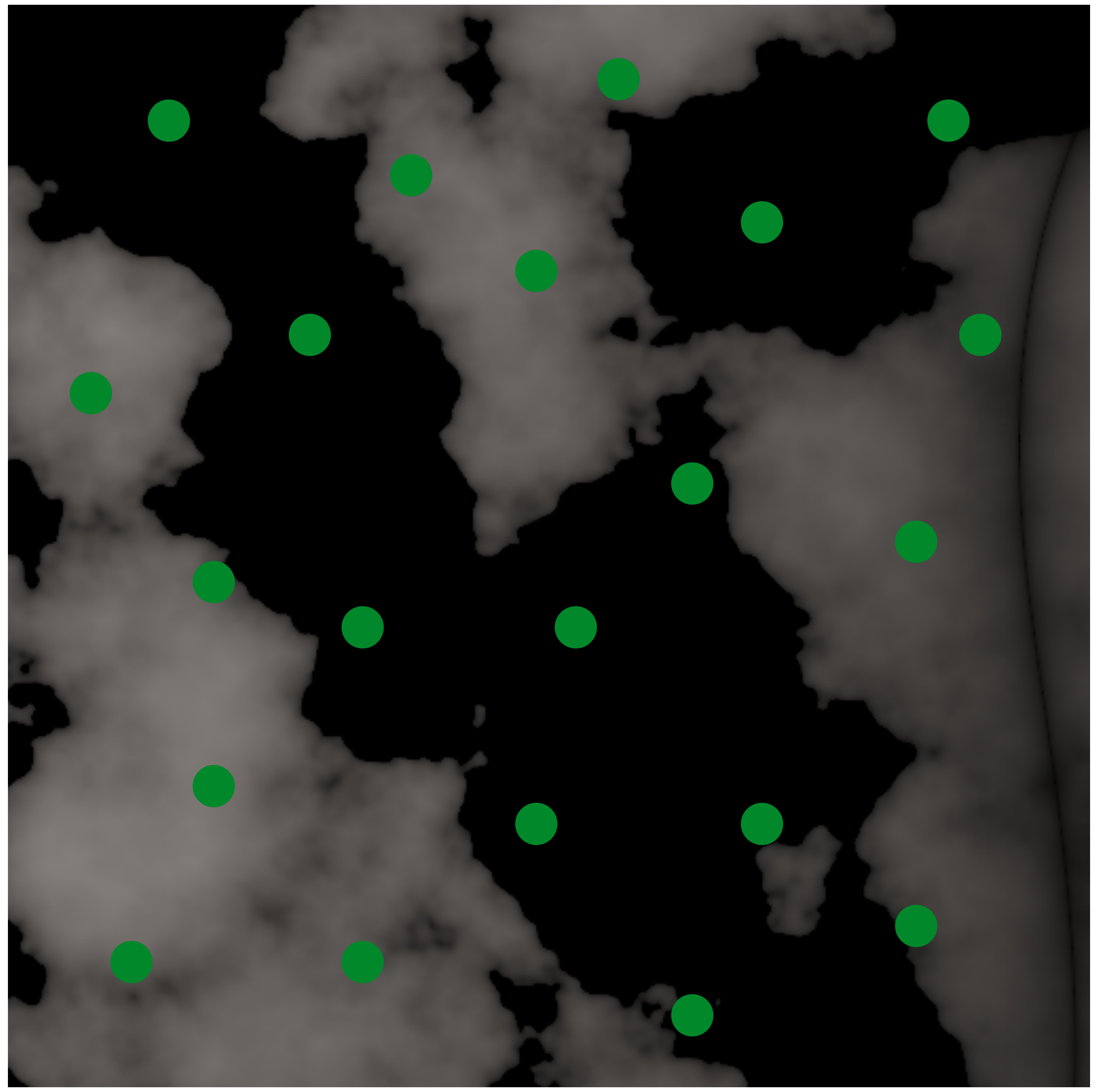

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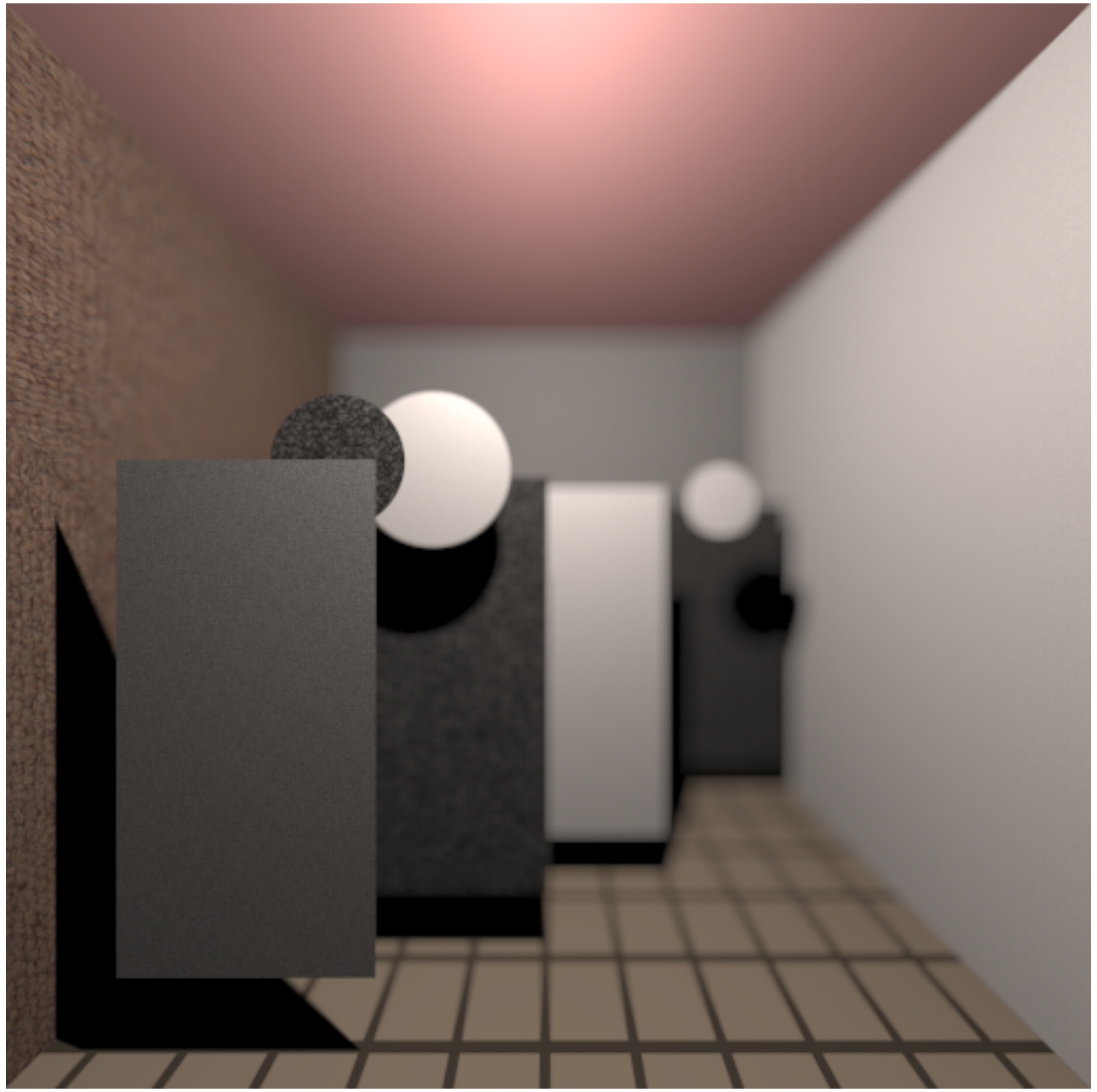
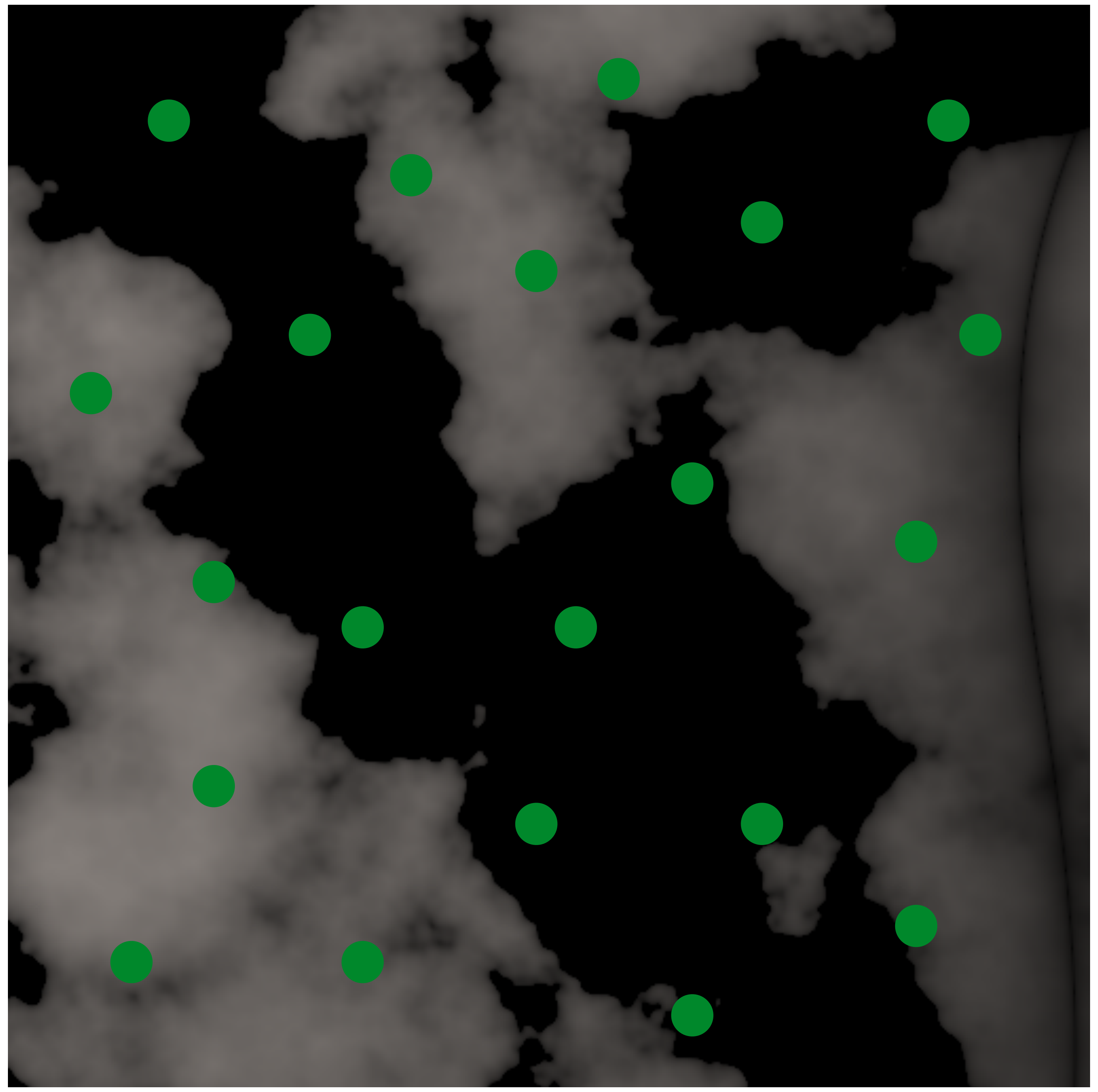
Monte Carlo Integration

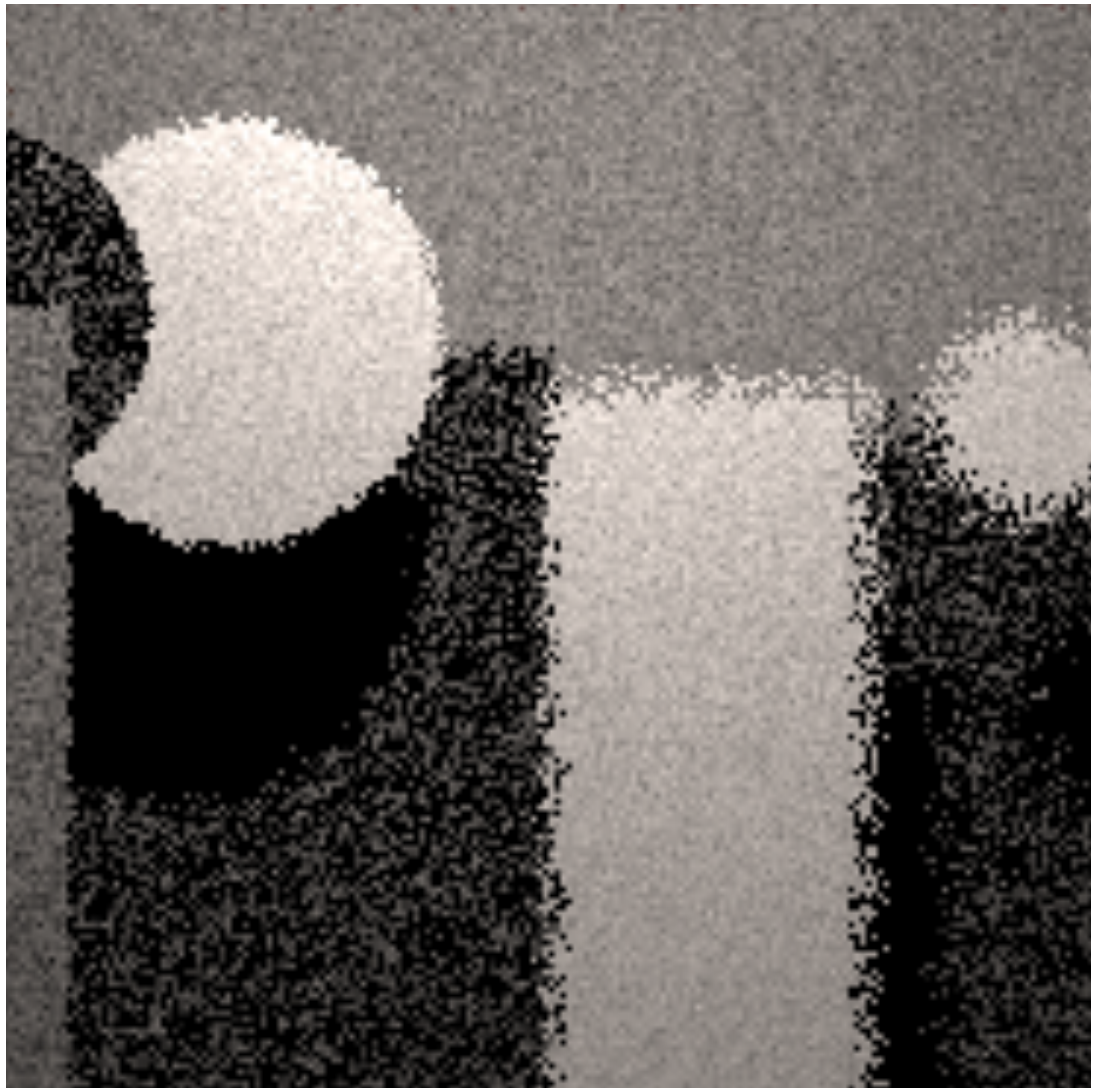


$f(\vec{x})$

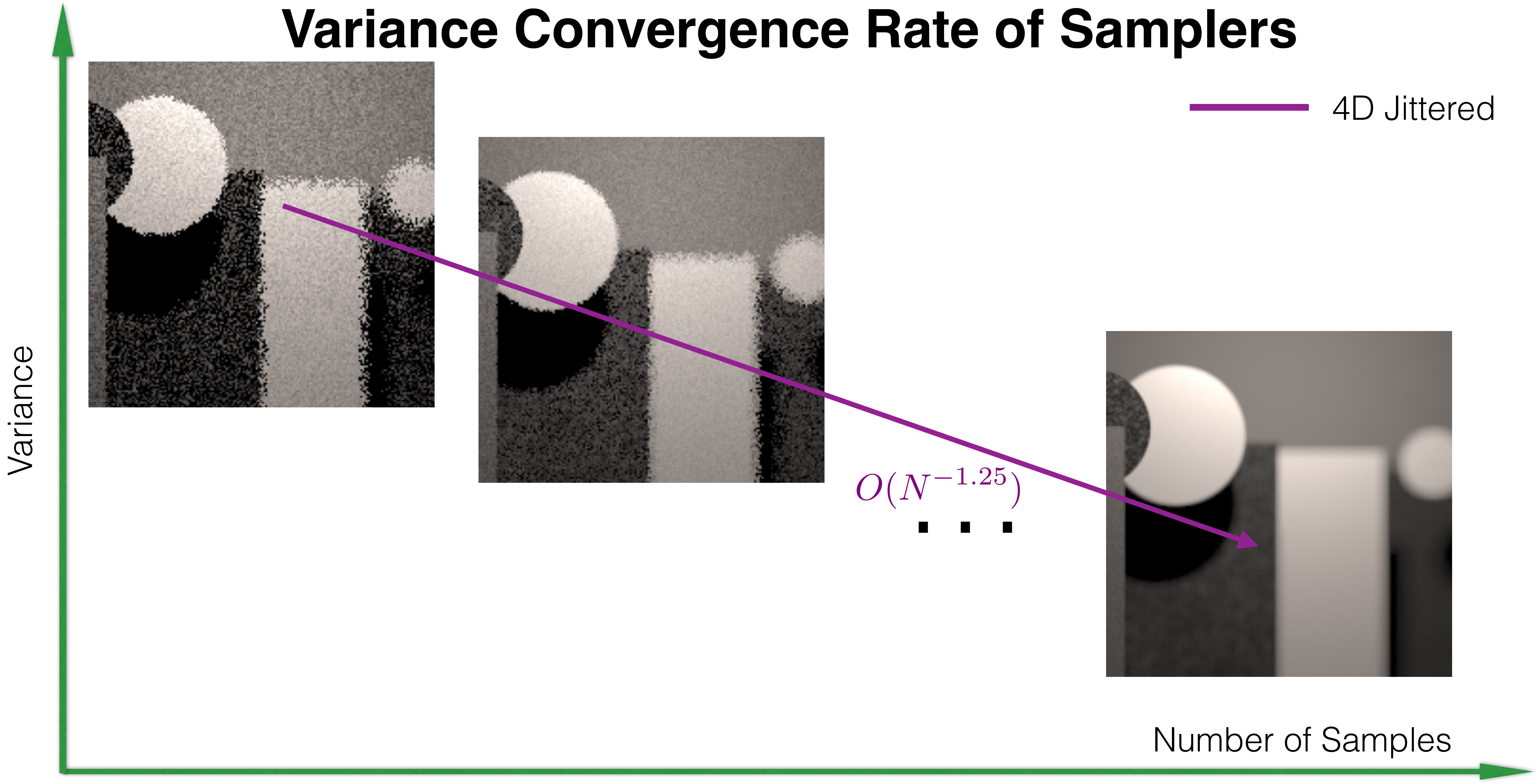


$$I_N = \frac{1}{N} \sum_{k=1}^{N} \frac{f(\vec{x}_k)}{p(\vec{x}_k)}$$

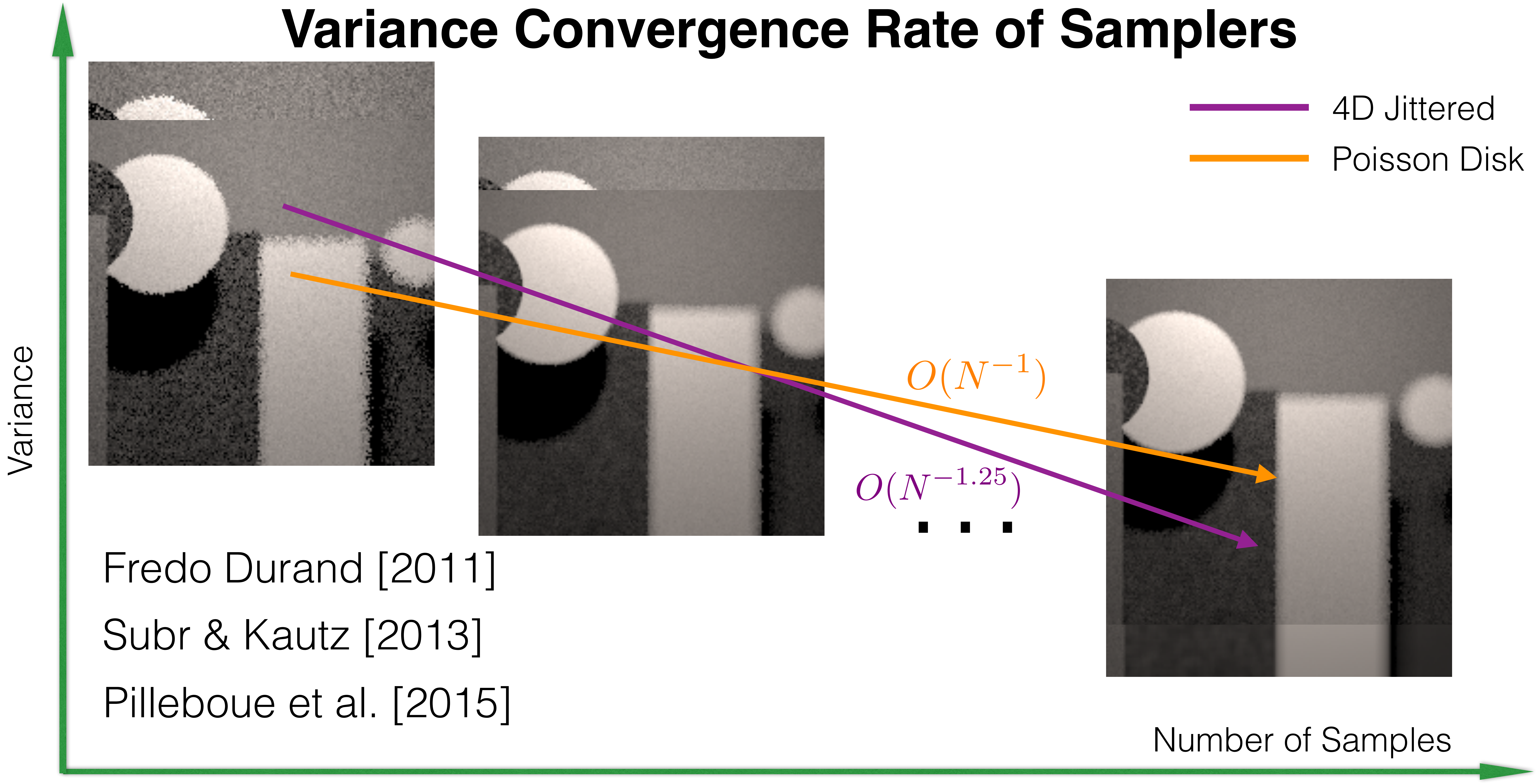
Variance



Variance Convergence Rate of Samplers



Variance Convergence Rate of Samplers



Fredo Durand [2011]

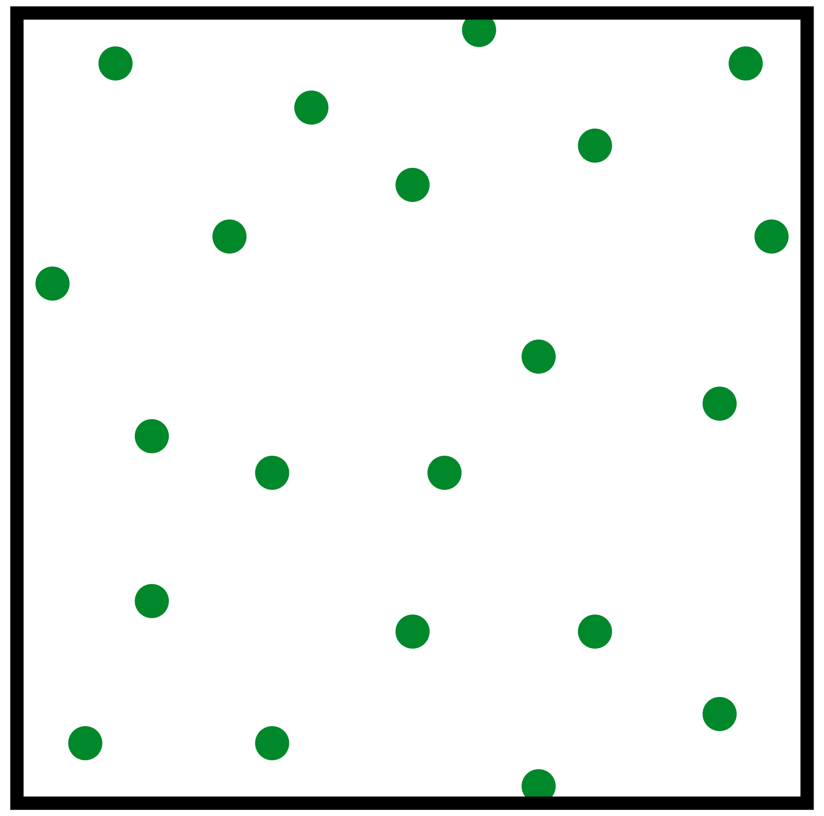
Subr & Kautz [2013]

Pilleboue et al. [2015]

Monte Carlo Estimator

$$I_N = \frac{1}{N}\sum_{k=1}^{N} f(\vec{x}_k) = \int_0^1 \frac{1}{N}\sum_{k=1}^{N}\delta(\vec{x}-\vec{x}_k)\, f(\vec{x})\, d\vec{x} = \int_0^1 S_N(\vec{x})\, f(\vec{x})\, d\vec{x}$$

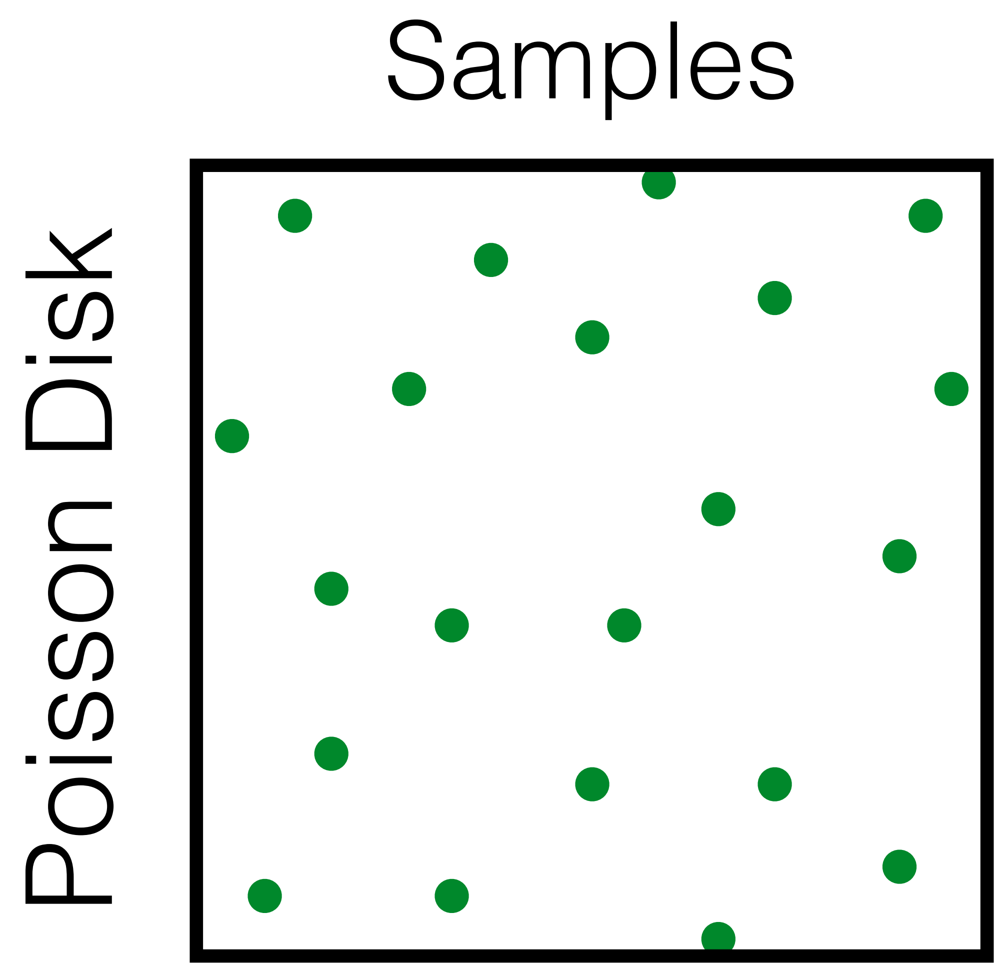
$$S_N(\vec{x}) = \frac{1}{N}\sum_{k=1}^{N}\delta(\vec{x}-\vec{x}_k)$$

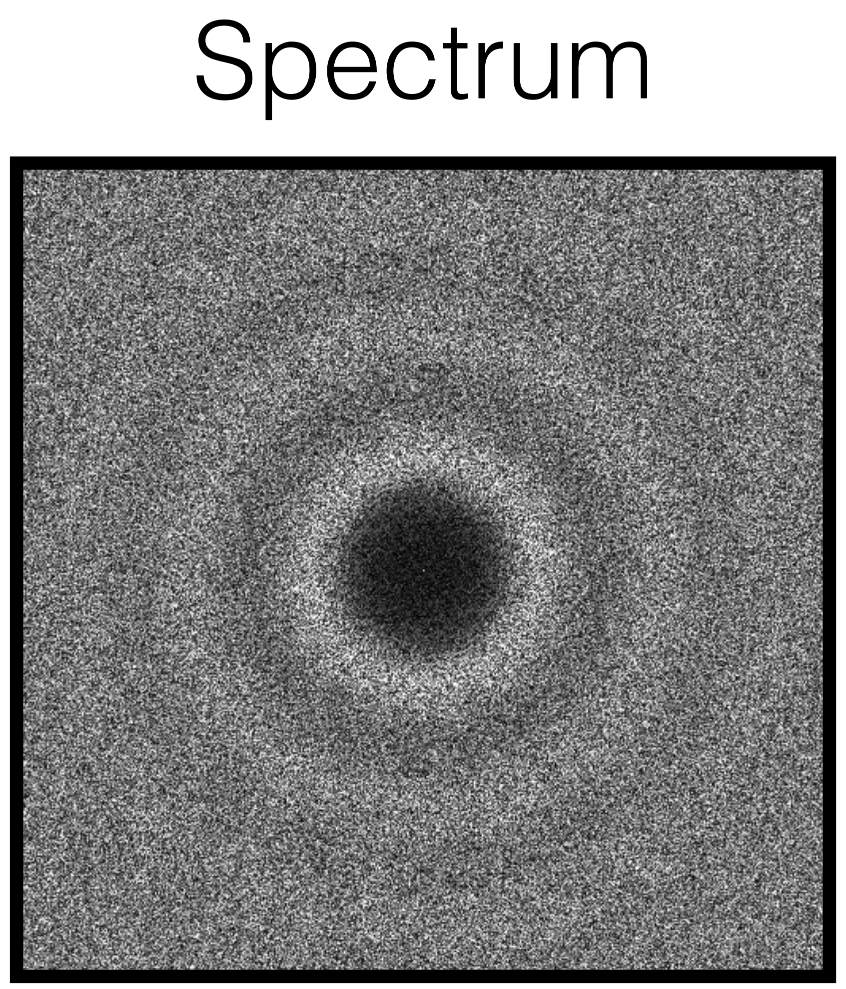


Fredo Durand [2011]

Samples Power Spectrum

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x})\, d\vec{x}$$





$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^{N} \delta(\vec{x} - \vec{x}_k)$$

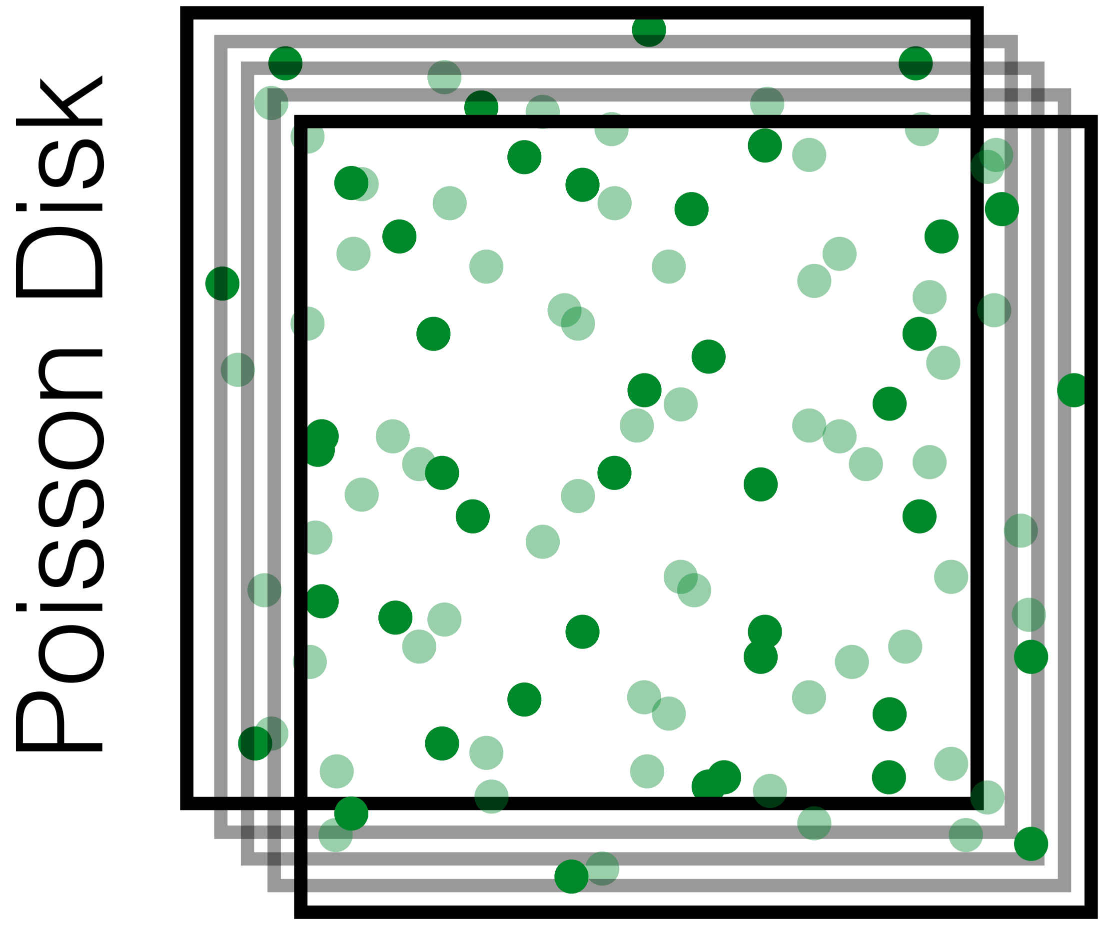
$$\mathcal{P}_{S_N}(\nu) = \left| \frac{1}{N} \sum_{k=1}^{N} e^{-i2\pi\nu\cdot\vec{x}_k} \right|^2$$

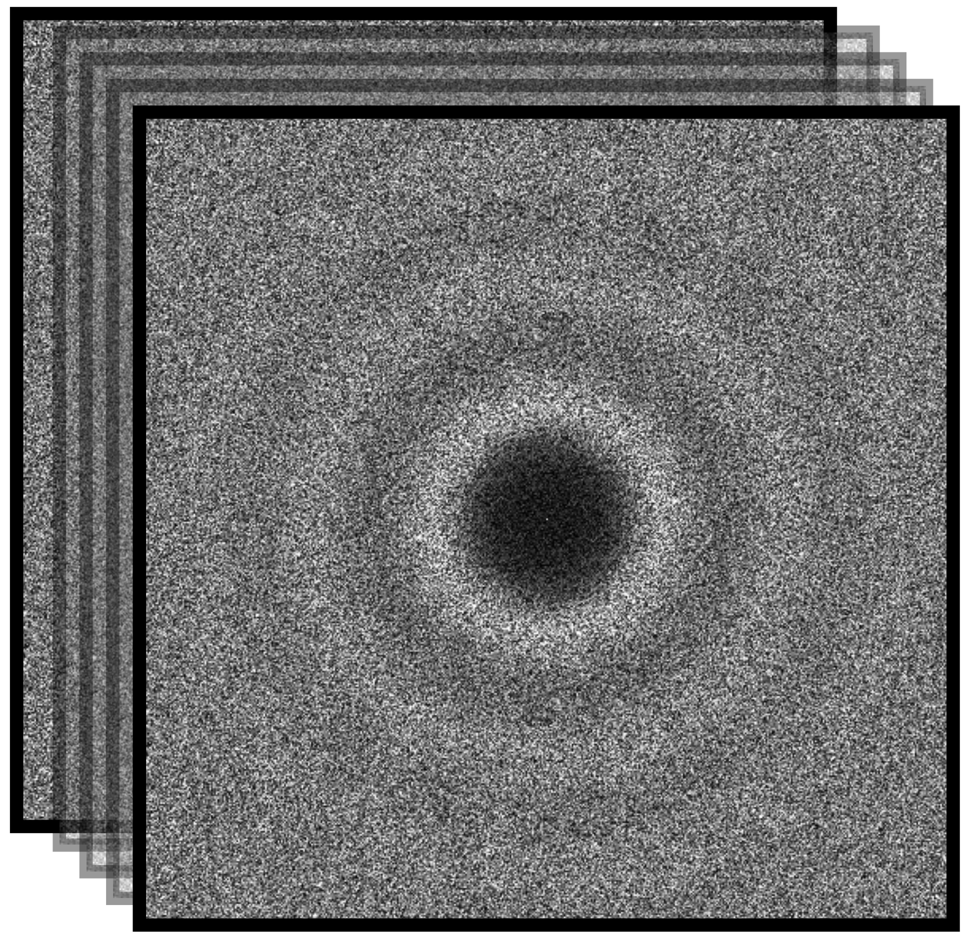
Expected Sampling Power Spectra

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x}) \, d\vec{x}$$

Samples

Spectrum





$$S_N(\vec{x}) = \frac{1}{N} \sum_{k=1}^{N} \delta(\vec{x} - \vec{x}_k)$$

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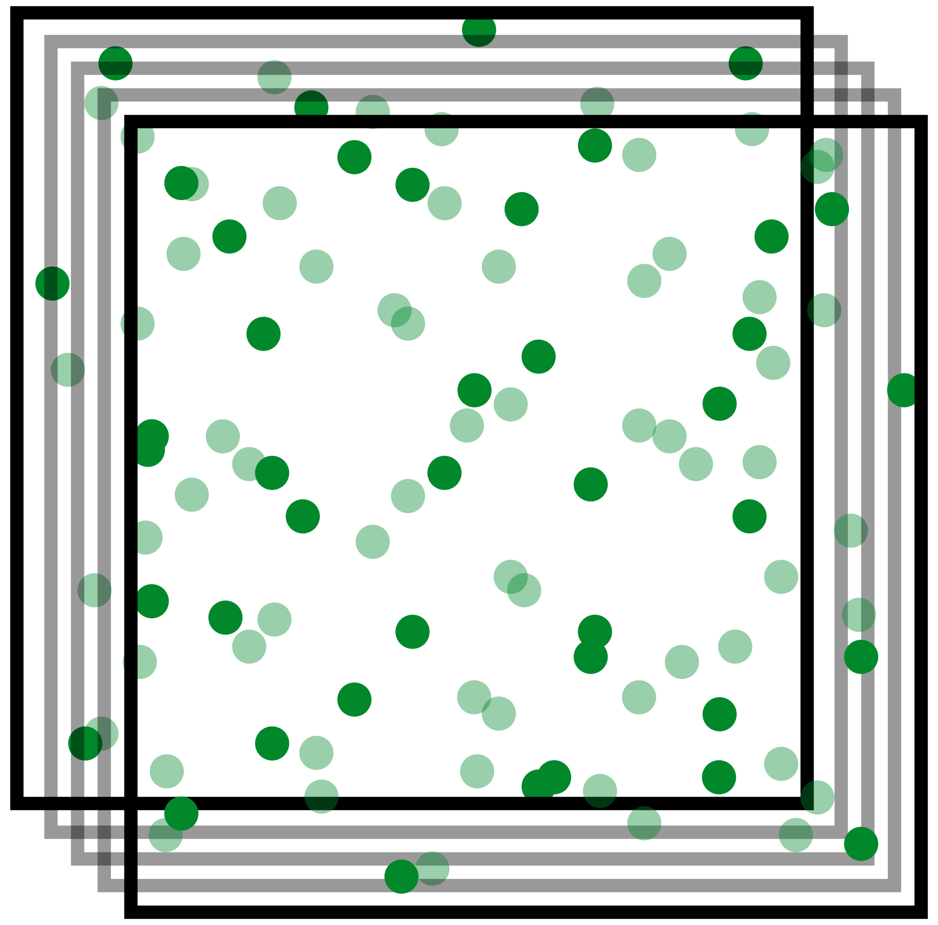
Expected Sampling Power Spectra

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x})\, d\vec{x}$$

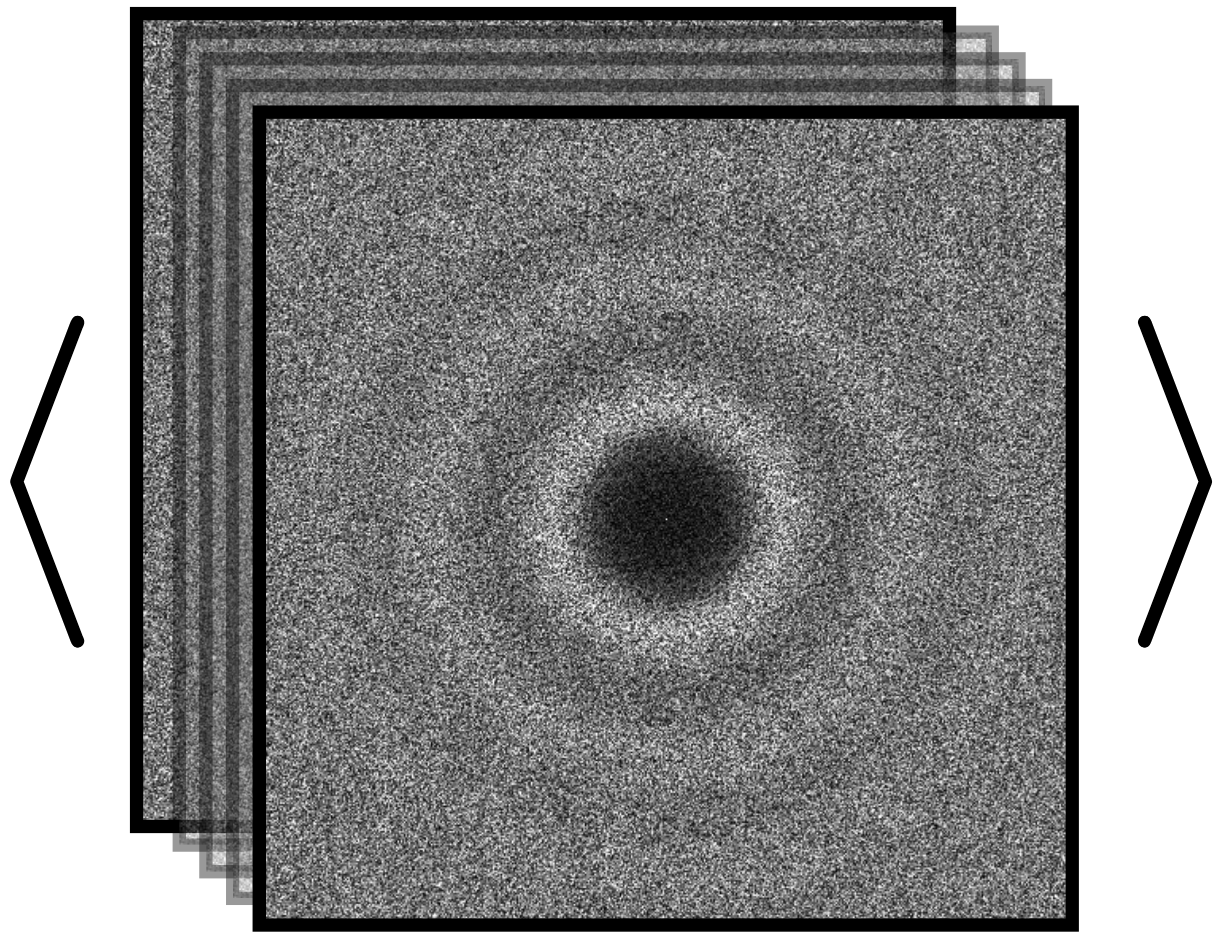
Samples

Poisson Disk



$$S_N(\vec{x}) = \frac{1}{N}\sum_{k=1}^{N} \delta(\vec{x}-\vec{x}_k)$$

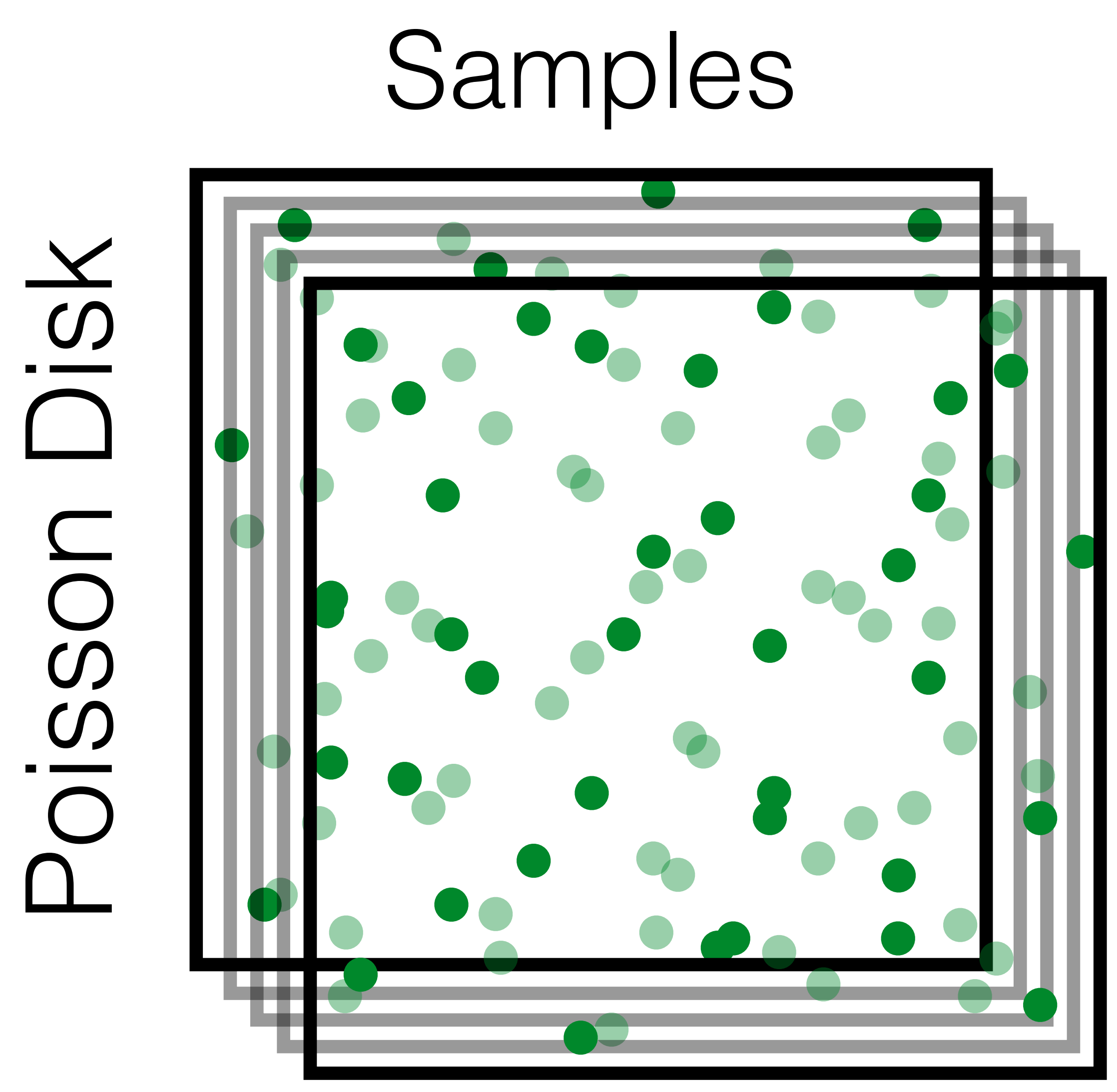
Spectrum

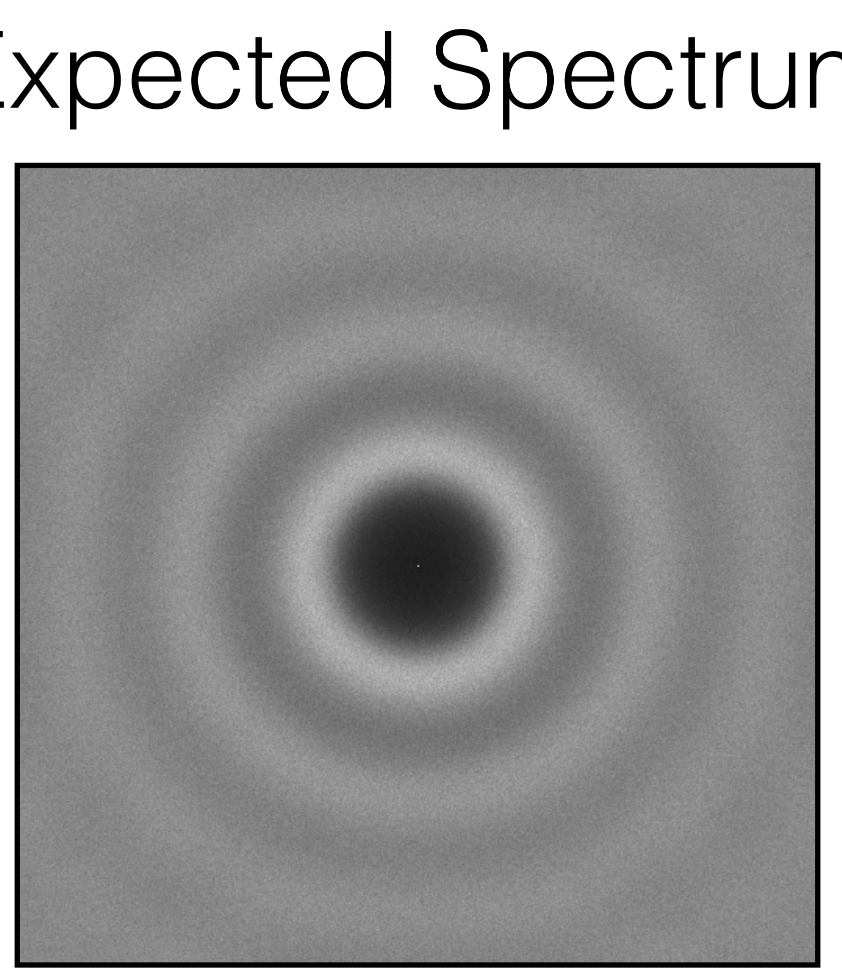


$$\langle \mathcal{P}_{S_N}(\nu)\rangle = \left\langle \left| \frac{1}{N}\sum_{k=1}^{N} e^{-i2\pi\nu\cdot\vec{x}_k}\right|^2 \right\rangle$$

Expected Sampling Power Spectra

$$I_N = \int_0^1 S_N(\vec{x}) f(\vec{x})\, d\vec{x}$$





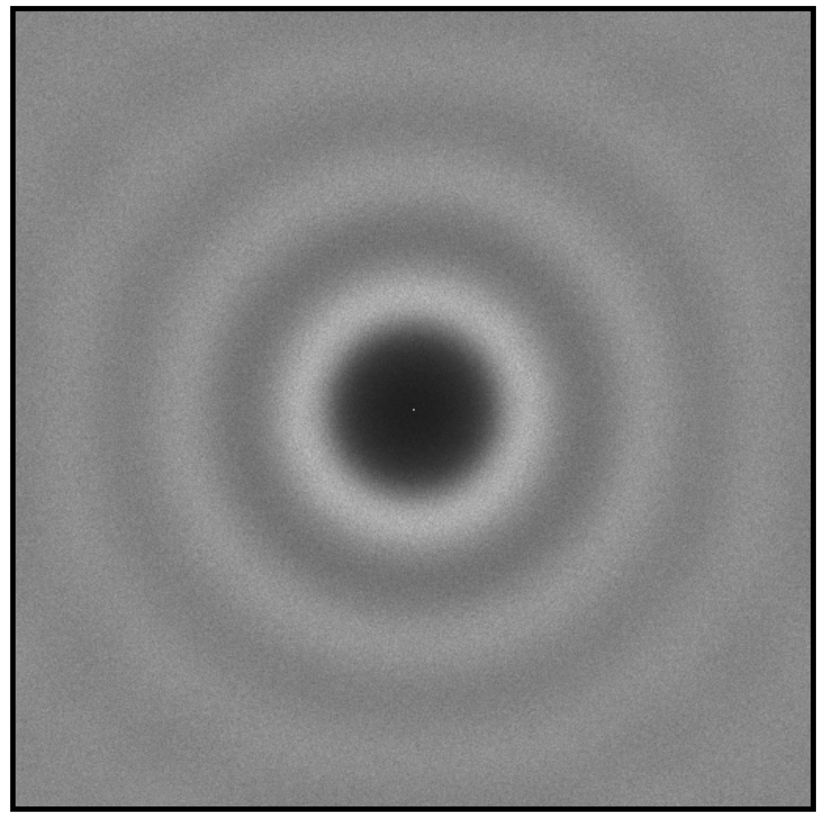
$$S_N(\vec{x}) = \frac{1}{N}\sum_{k=1}^{N} \delta(\vec{x} - \vec{x}_k)$$

$$\langle \mathcal{P}_{S_N}(\nu) \rangle = \left\langle \left| \frac{1}{N} \sum_{k=1}^{N} e^{-i2\pi\nu\cdot\vec{x}_k} \right|^2 \right\rangle$$

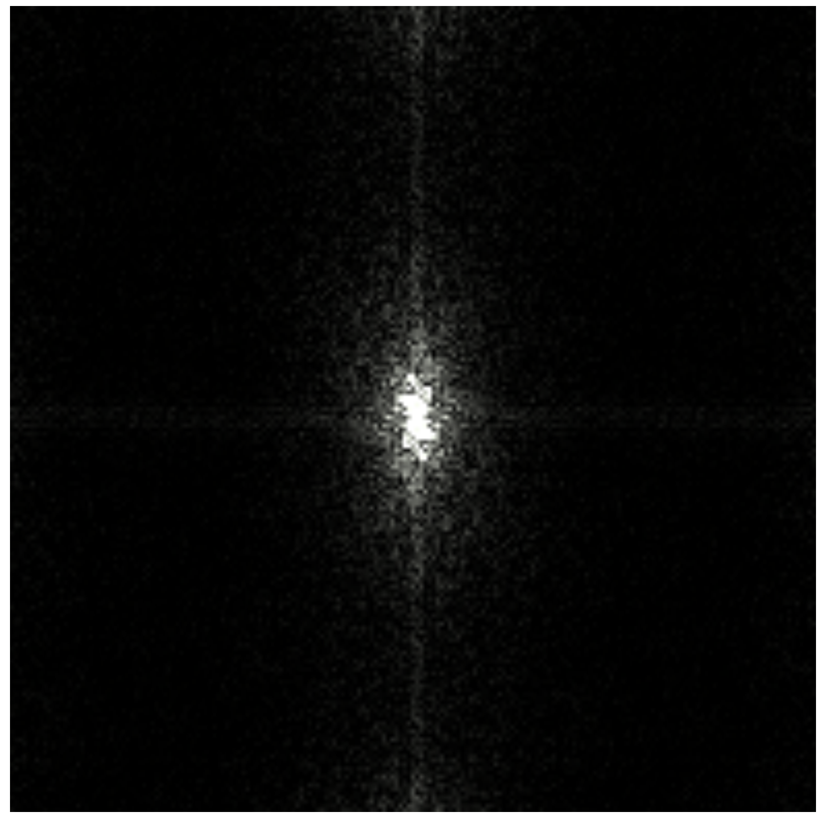
Variance of Monte Carlo Estimator

$$\mathrm{Var}(I_N) = \int_\Omega \langle \mathcal{P}_{S_N}(\nu) \rangle \times \mathcal{P}_f(\nu) \, d\nu$$

$\langle \mathcal{P}_{S_N}(\nu) \rangle$

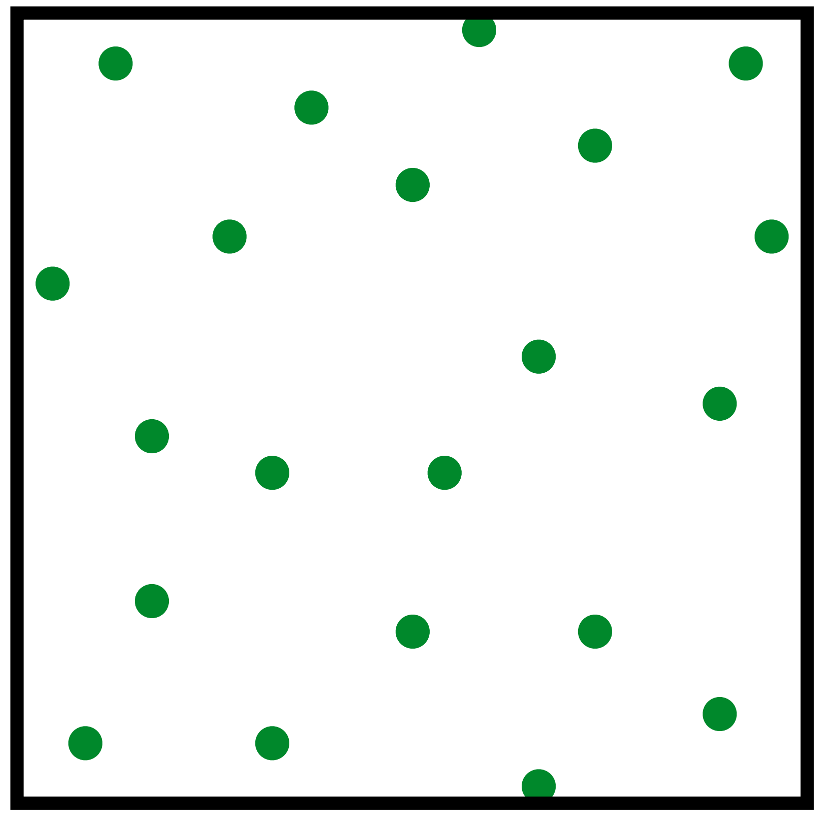


$\mathcal{P}_f(\nu)$

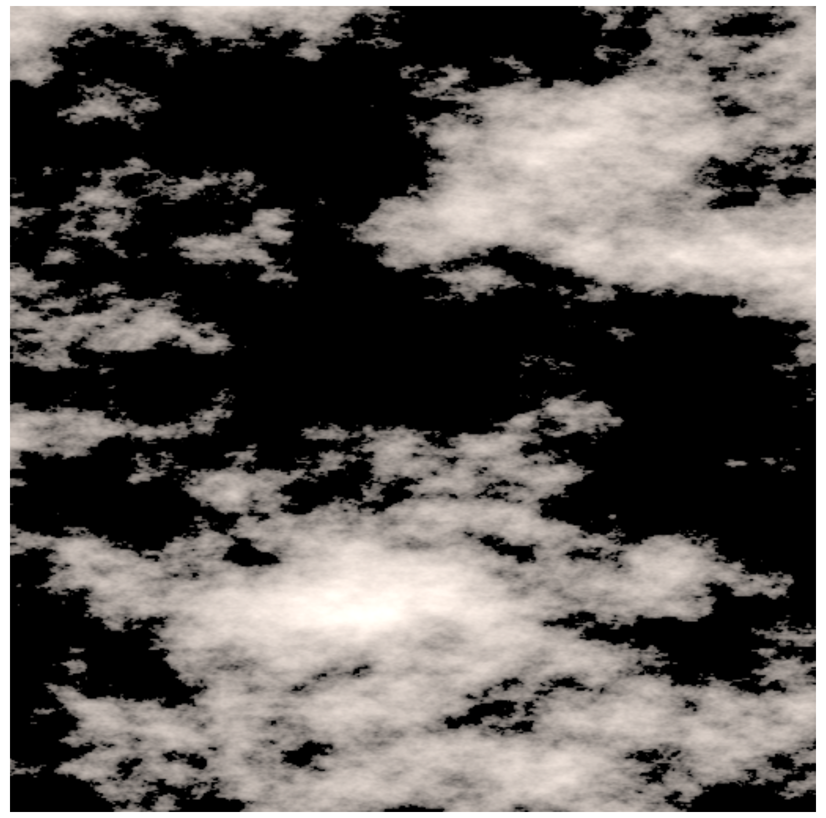


$S_N(\vec{x})$

Poisson Disk



$f(\vec{x})$



Fredo Durand [2011]

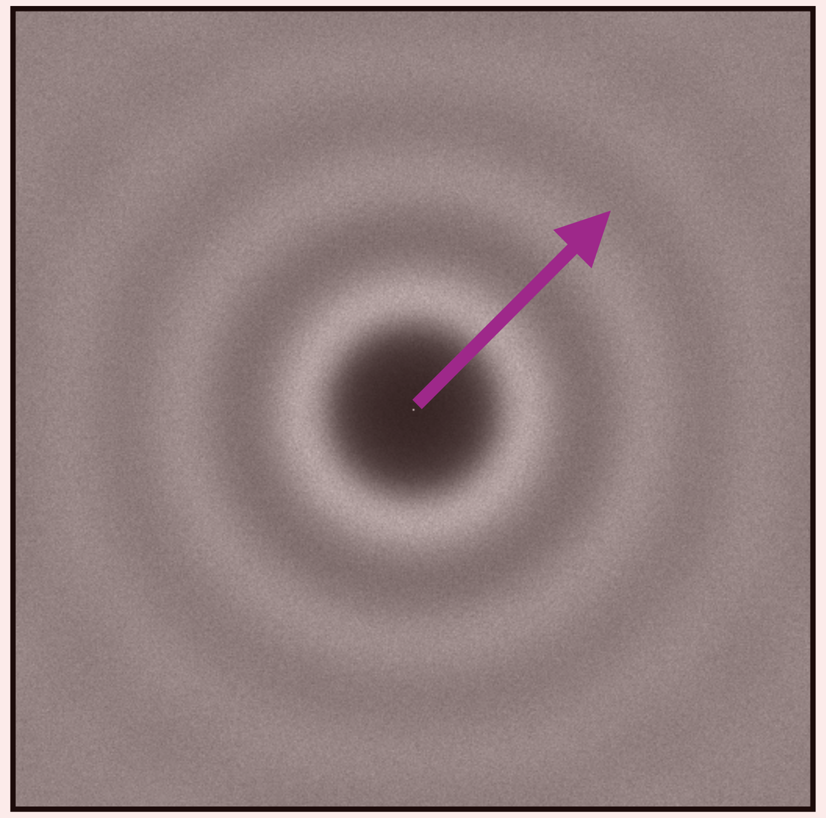
Subr & Kautz [2013]

Pilleboue et al. [2015]

Variance of Monte Carlo Estimator in Polar Coordinates

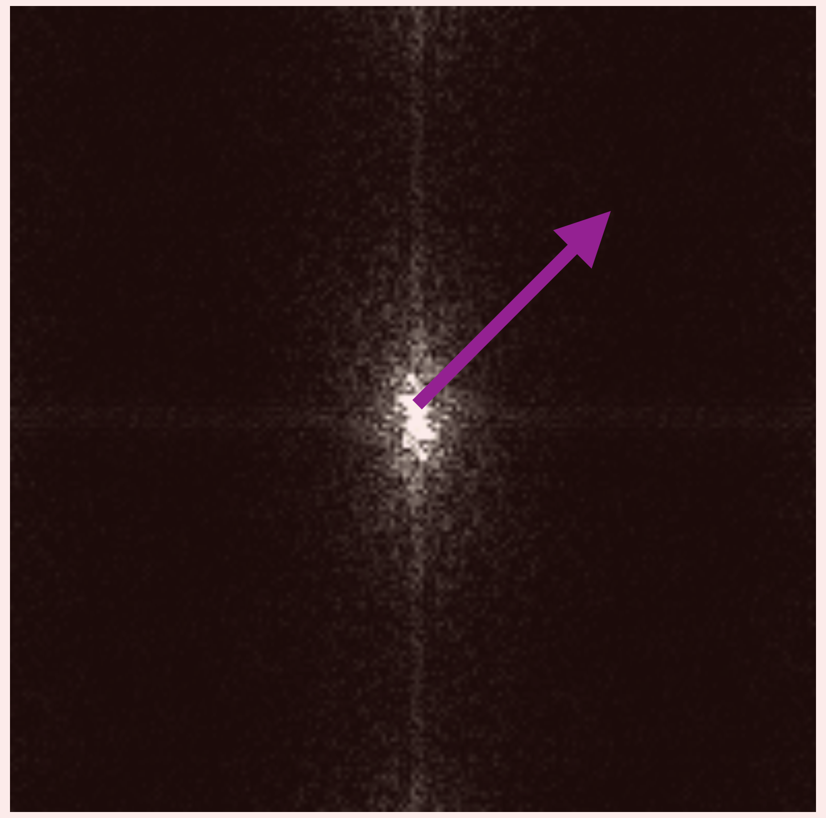
$$\mathrm{Var}(I_N) = \int_0^\infty \rho^{d-1} \int_{\mathcal{S}^{d-1}} \left\langle \tilde{\mathcal{P}}_{S_N}(\rho\mathbf{n}) \right\rangle \times \mathcal{P}_f(\rho\mathbf{n}) \, d\mathbf{n} \, d\rho$$

$\left\langle \tilde{\mathcal{P}}_{S_N}(\rho\mathbf{n}) \right\rangle$



$\times$

$\mathcal{P}_f(\rho\mathbf{n})$

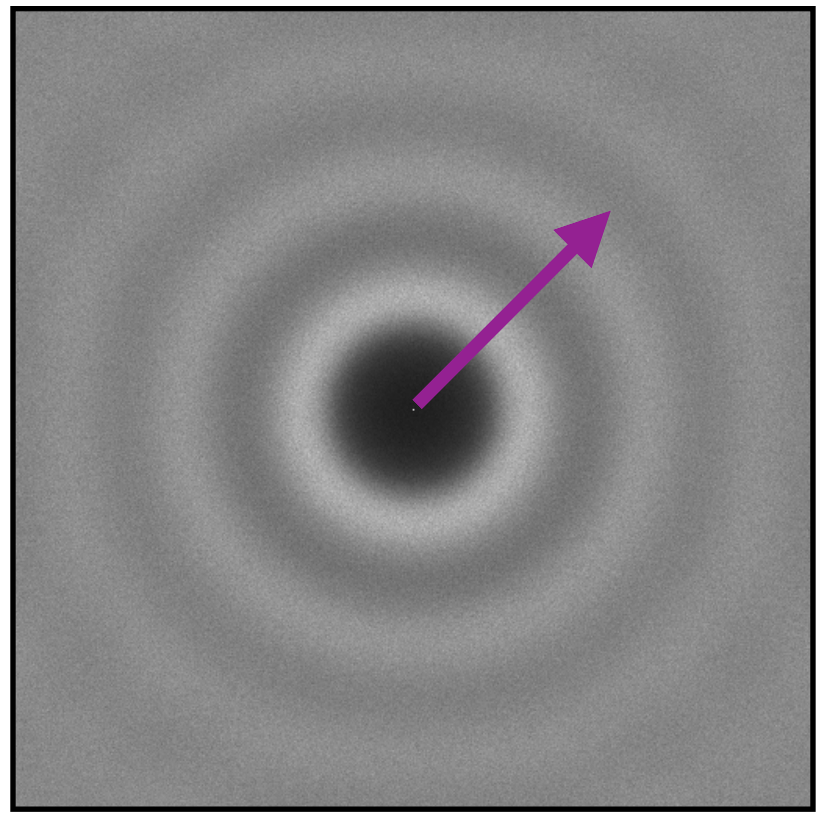


Pilleboue et al. [2015]

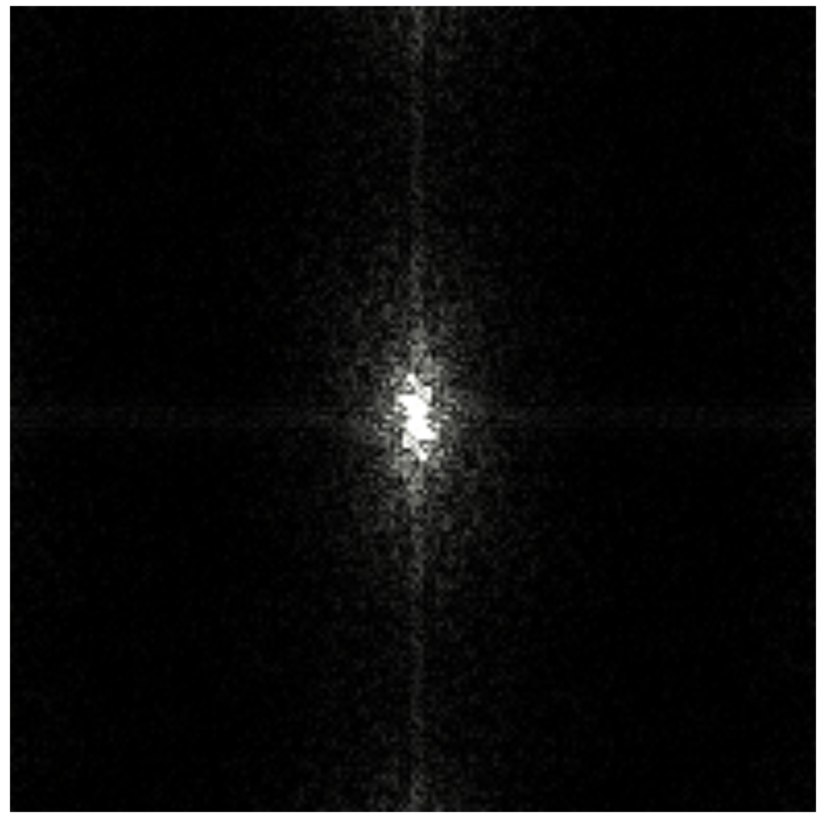
Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

$\left\langle \tilde{\mathcal{P}}_{S_N}(\rho\mathbf{n}) \right\rangle$ $\qquad$ $\mathcal{P}_f(\rho\mathbf{n})$

$$\mathrm{Var}(I_N) = \int_0^\infty \rho^{d-1} \int_{\mathcal{S}^{d-1}}$$



$\times$



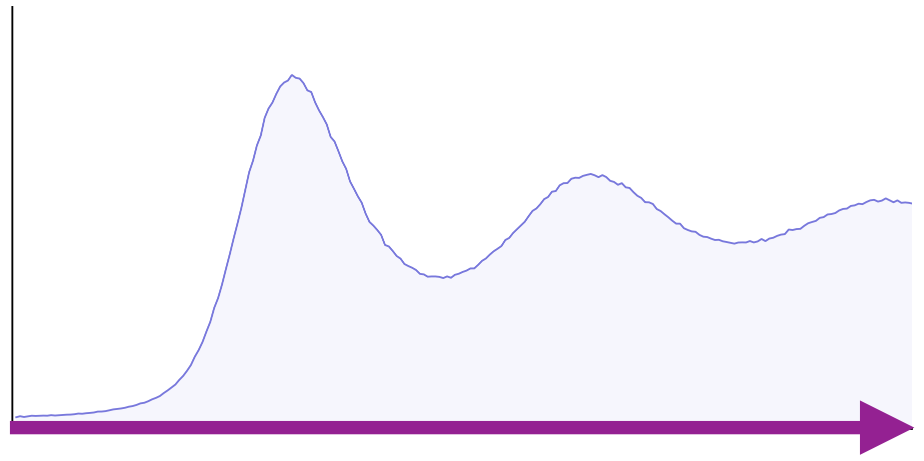
$$d\mathbf{n}\, d\rho$$

Pilleboue et al. [2015]

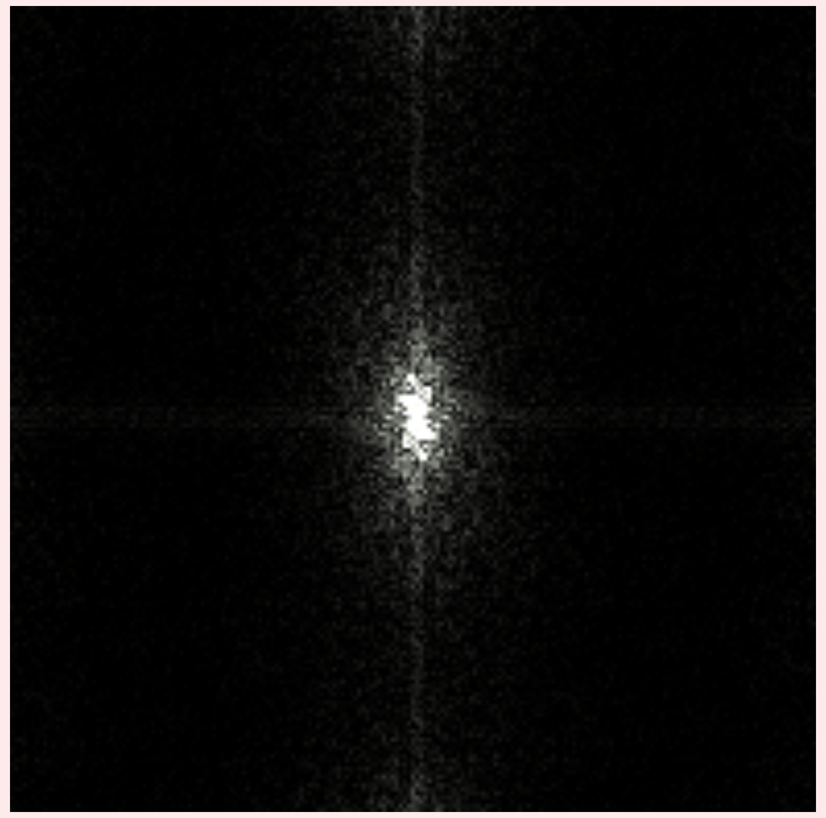
Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

$\tilde{\mathcal{P}}_{S_N}(\rho)$ $\mathcal{P}_f(\rho\mathbf{n})$

$$\mathrm{Var}(I_N) = \int_0^\infty \rho^{d-1}$$



$$\int_{\mathcal{S}^{d-1}}$$



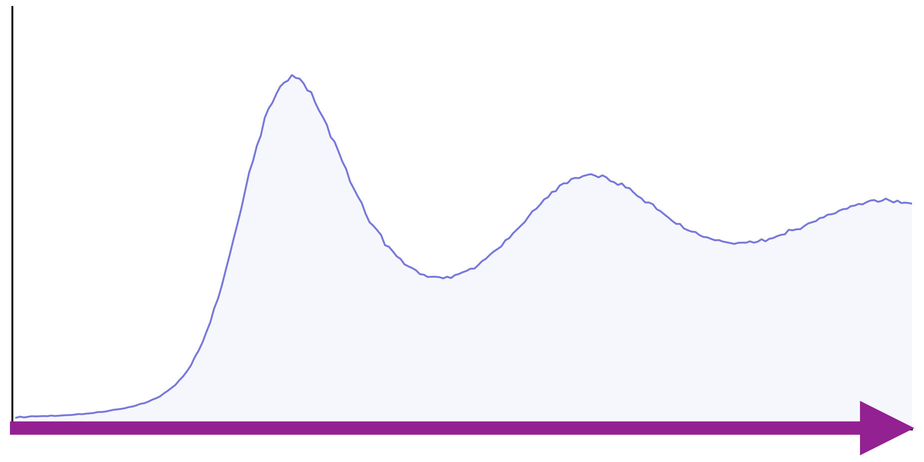
$$d\mathbf{n}\, d\rho$$

Pilleboue et al. [2015]

Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

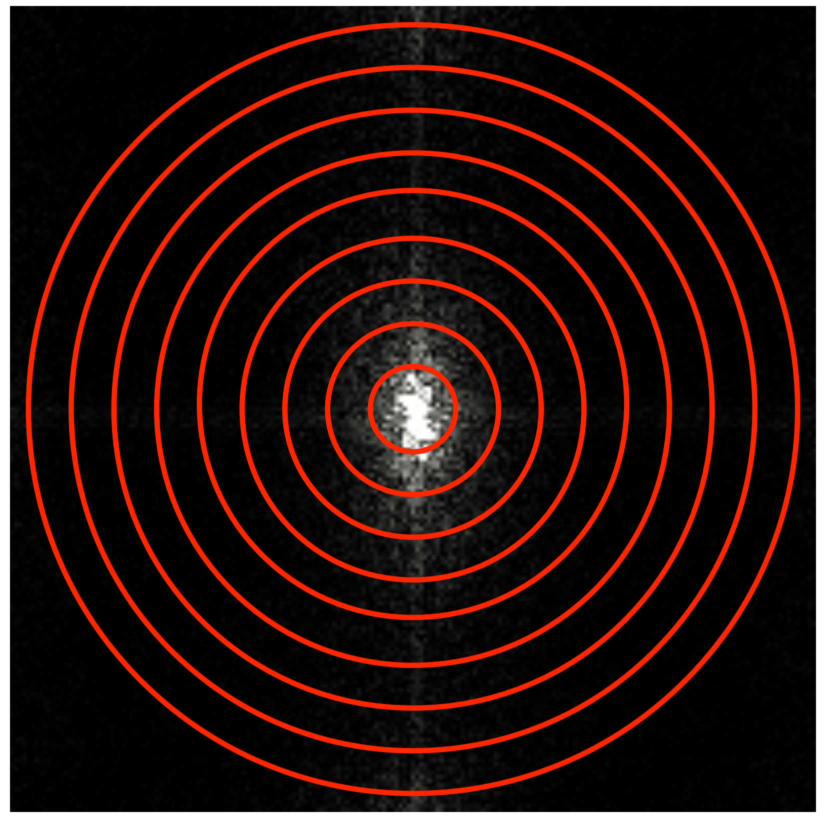
$$\mathrm{Var}(I_N) = \int_0^\infty \rho^{d-1}$$

$\tilde{\mathcal{P}}_{S_N}(\rho)$



$\times$

$\mathcal{P}_f(\rho\mathbf{n})$



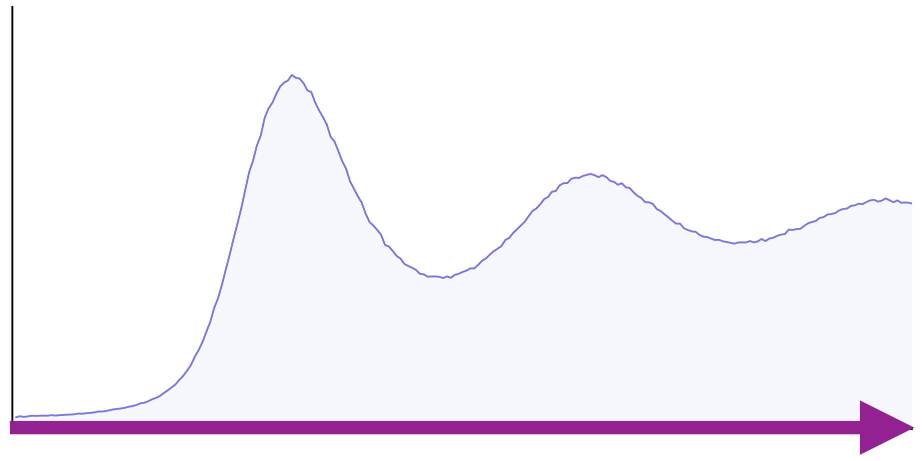
$d\rho$

Pilleboue et al. [2015]

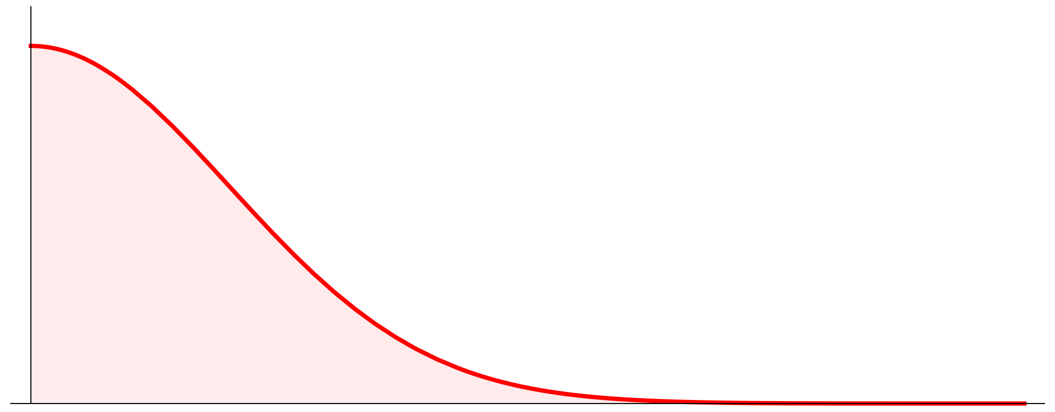
Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

$\tilde{\mathcal{P}}_{S_N}(\rho)$ $\qquad$ $\mathcal{P}_f(\rho)$

$$\mathrm{Var}(I_N) = \int_0^\infty \rho^{d-1}$$



$$\times$$



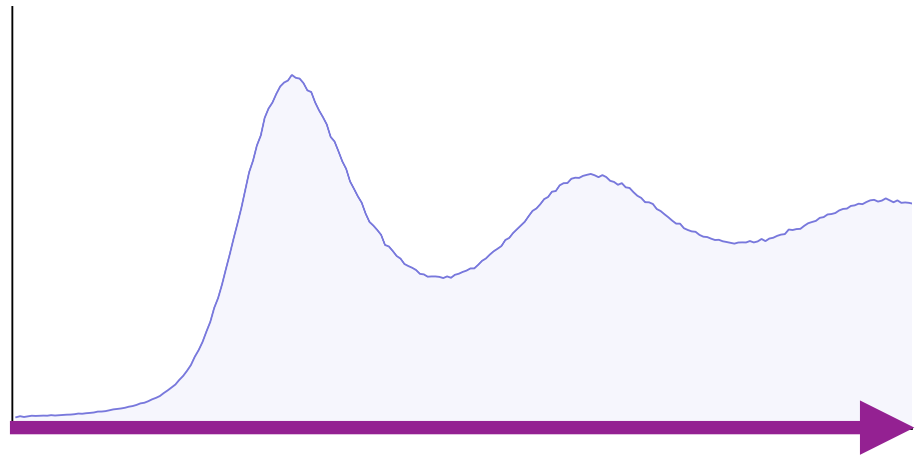
$$d\rho$$

Pilleboue et al. [2015]

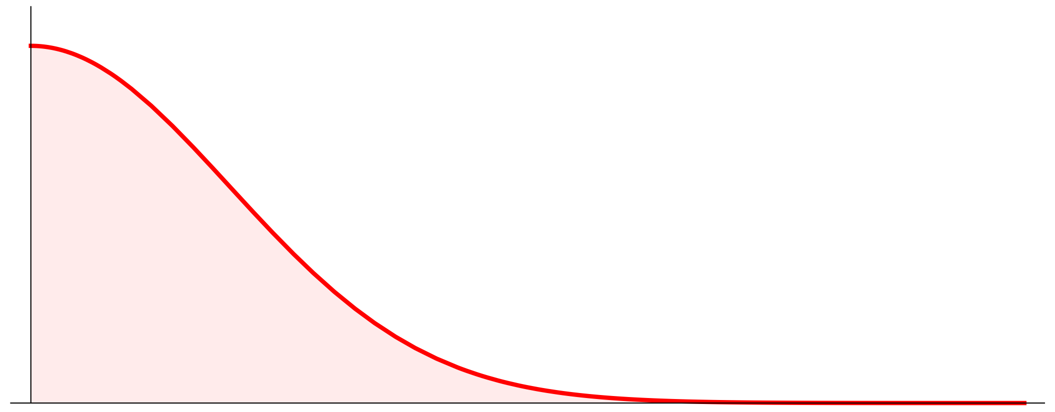
Variance of Monte Carlo Estimator for Isotropic Sampling Spectra

$\tilde{\mathcal{P}}_{S_N}(\rho)$ $\quad\quad$ $\mathcal{P}_f(\rho)$

$$\mathrm{Var}(I_N) = \int_0^\infty \rho^{d-1}$$



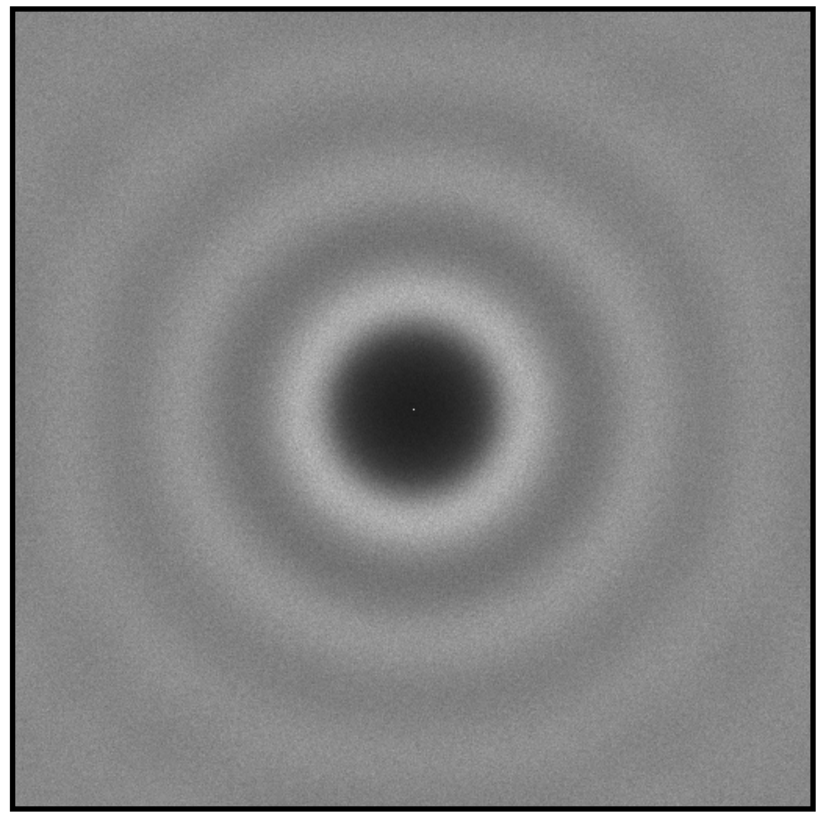
$\times$



$d\rho$

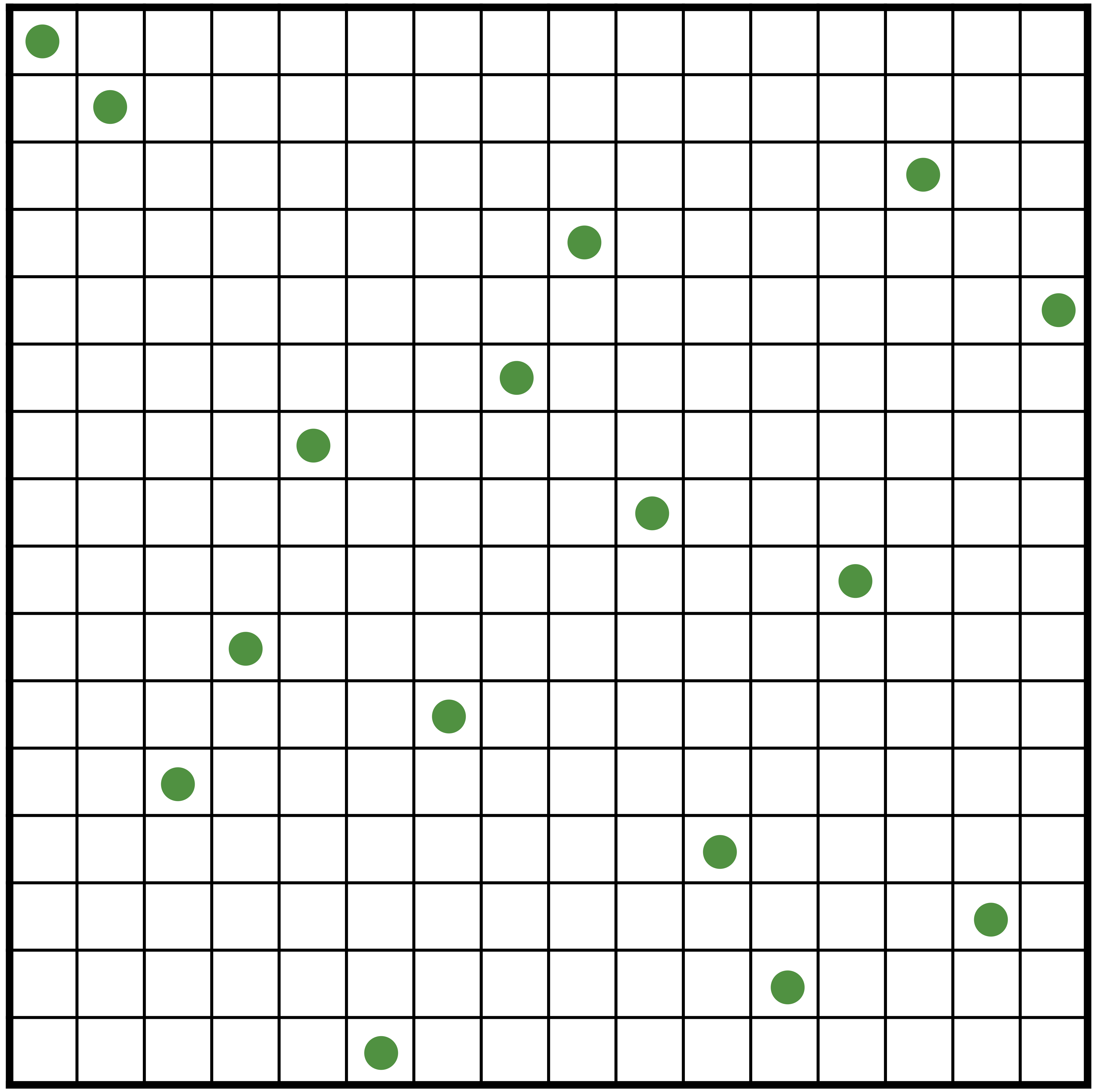
Samplers	Worst Case	Best Case
Random	$\mathcal{O}(N^{-1})$	$\mathcal{O}(N^{-1})$
Poisson Disk	$\mathcal{O}(N^{-1})$	$\mathcal{O}(N^{-1})$
CCVT	$\mathcal{O}(N^{-1.5})$	$\mathcal{O}(N^{-3})$

Isotropic Spectrum
Poisson Disk



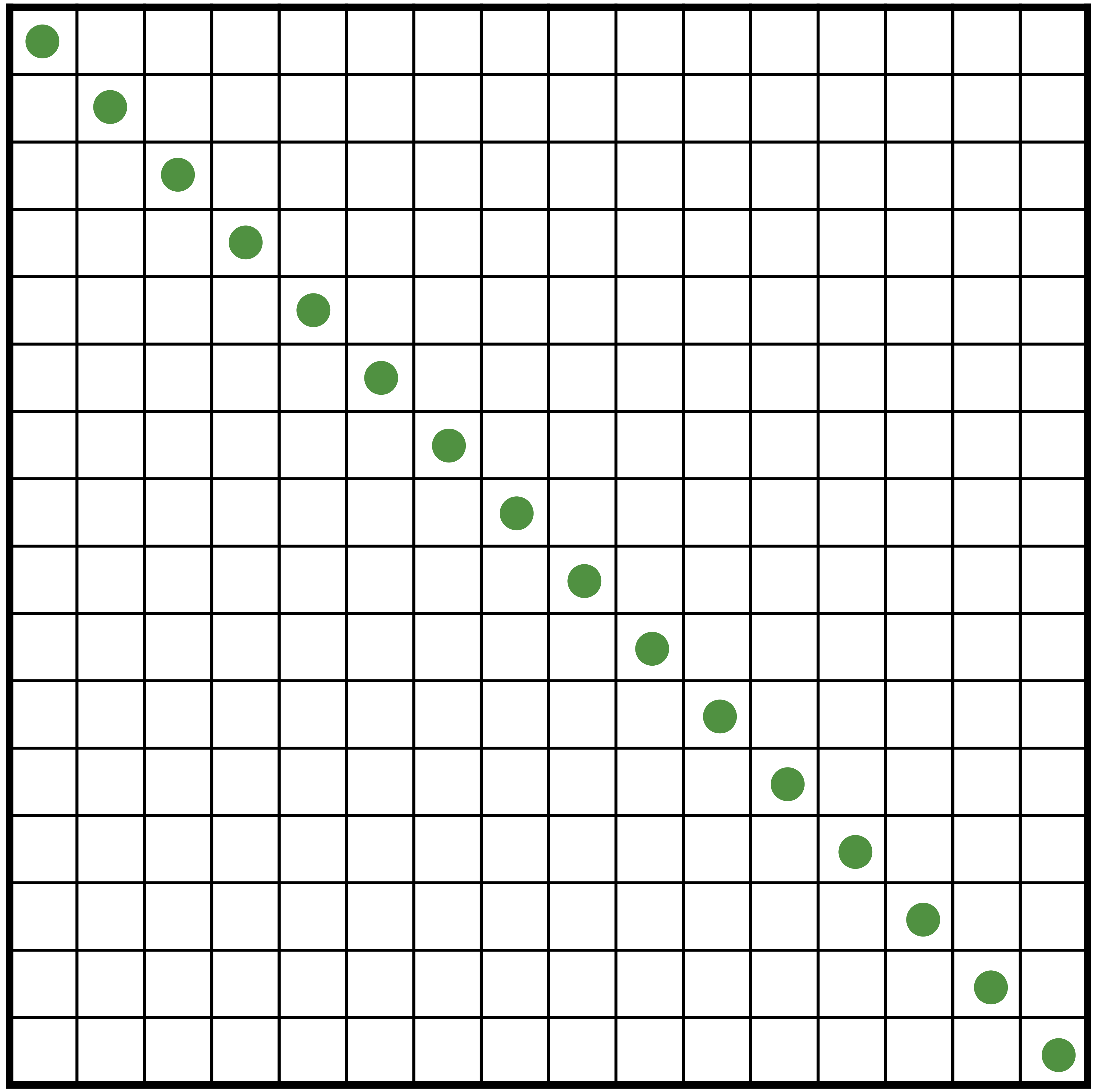
Pilleboue et al. [2015]

Latin Hypercube Sampler (N-rooks)



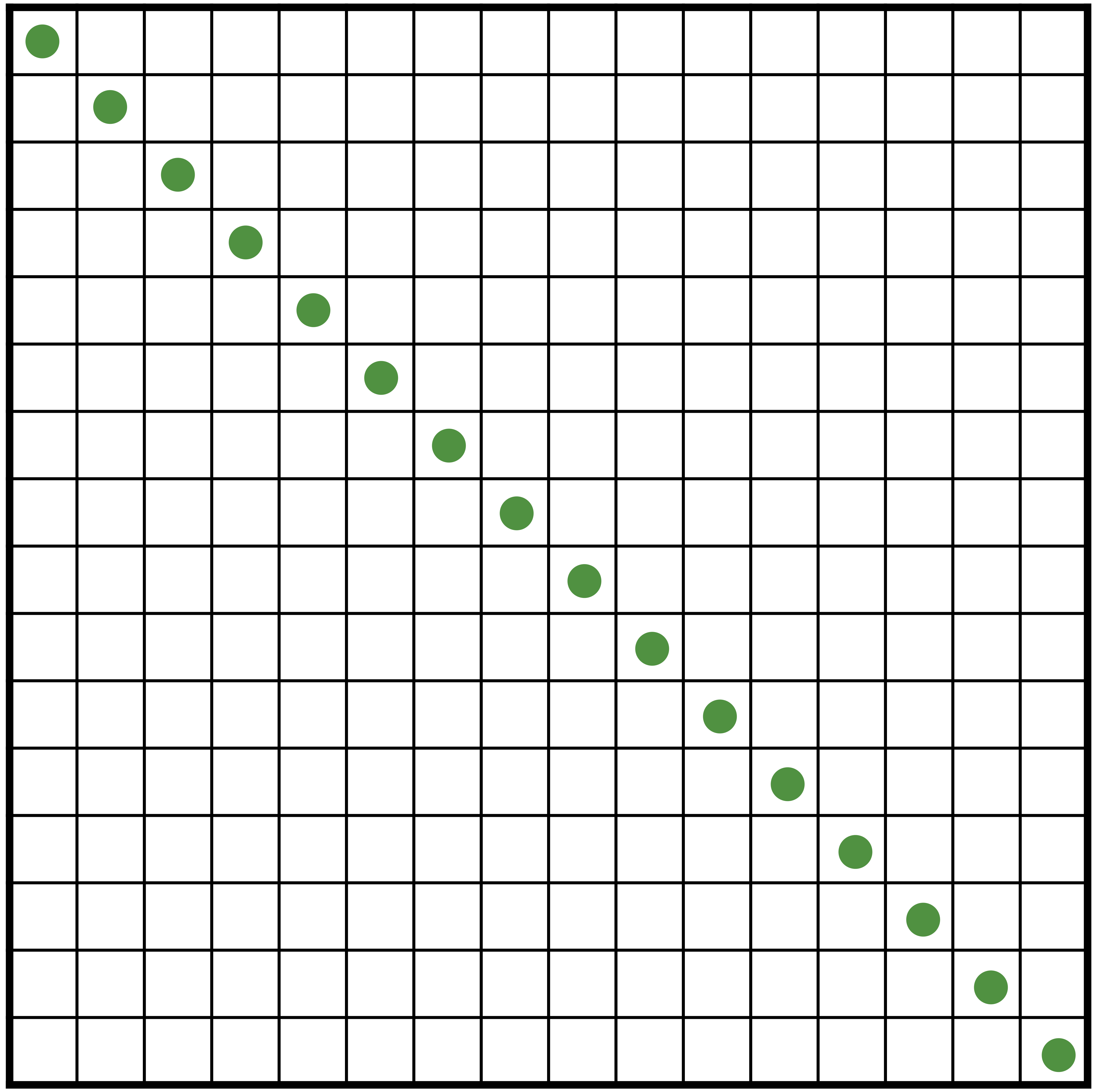
Latin Hypercube Sampler (N-rooks)

Initialize

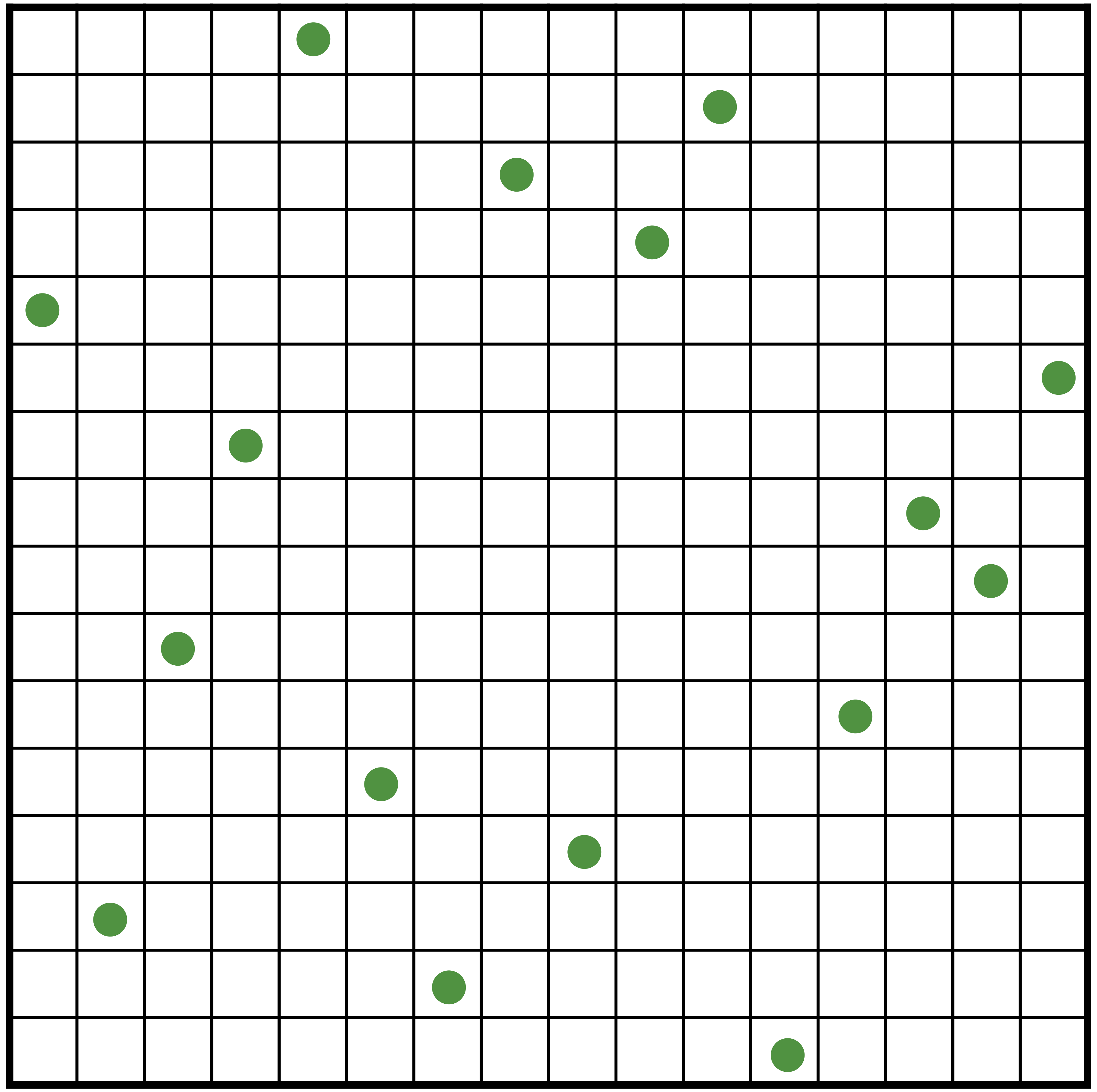


Latin Hypercube Sampler (N-rooks)

Shuffle rows

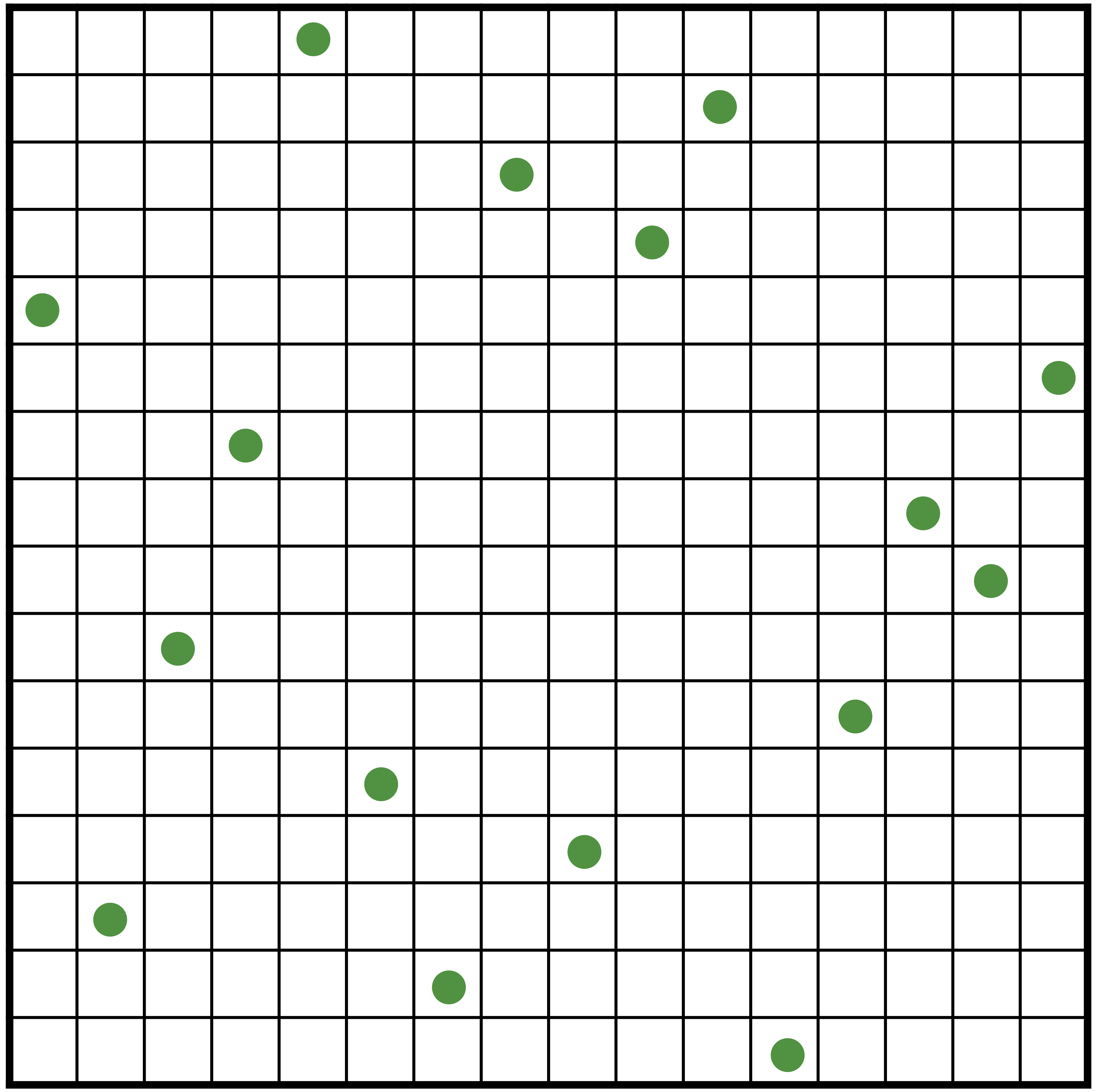


Latin Hypercube Sampler (N-rooks)

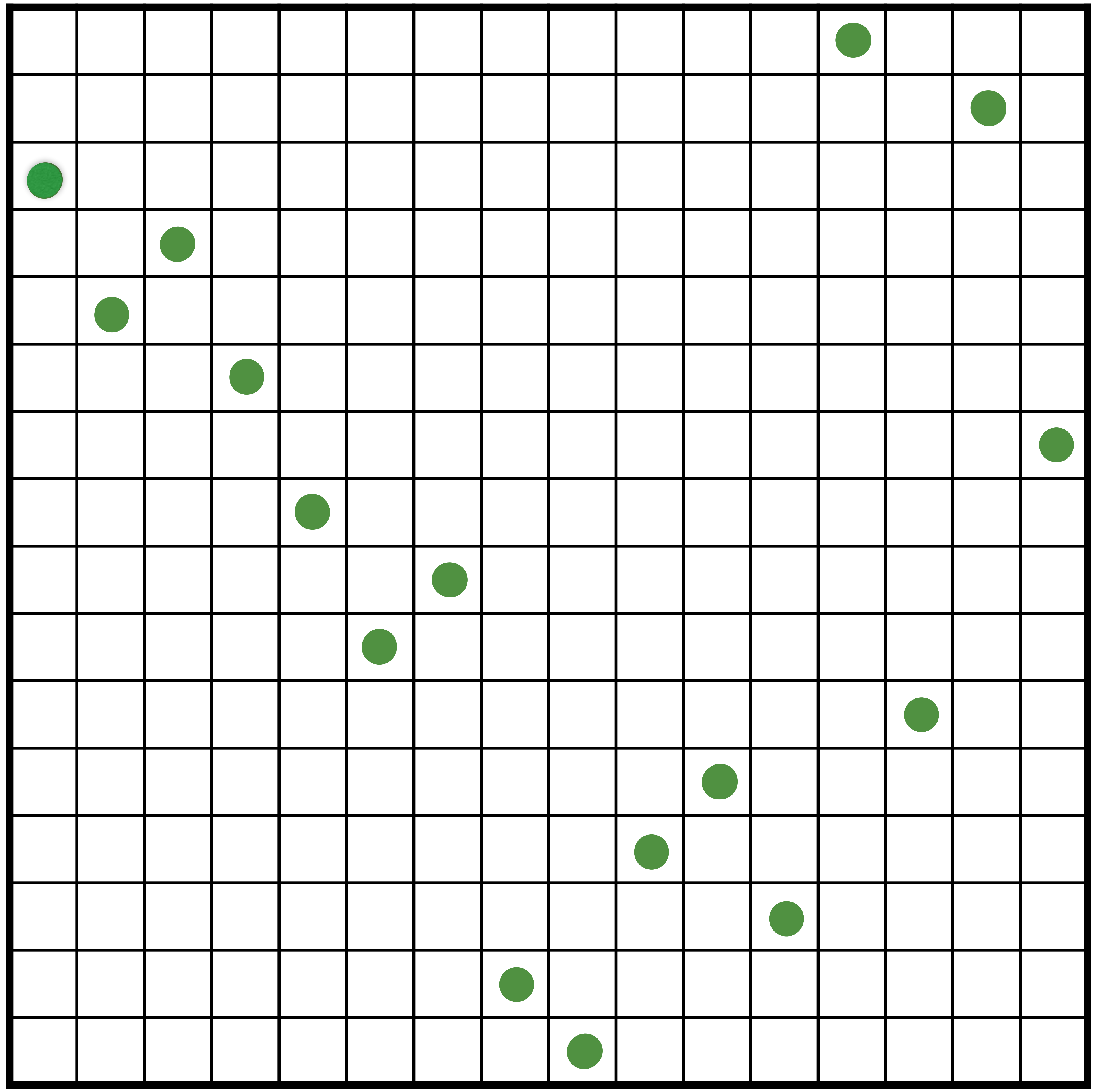


Latin Hypercube Sampler (N-rooks)

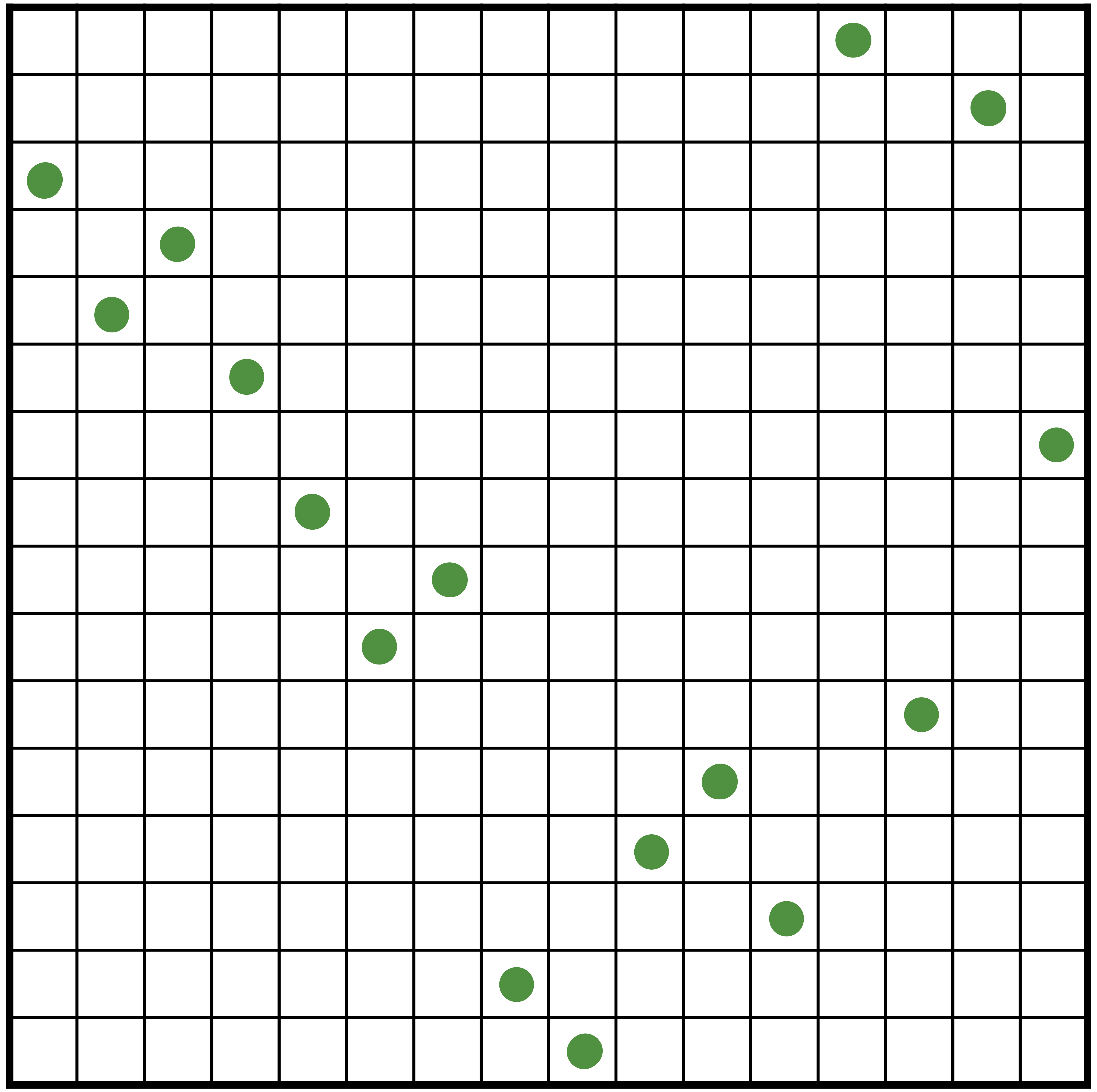
Shuffle columns



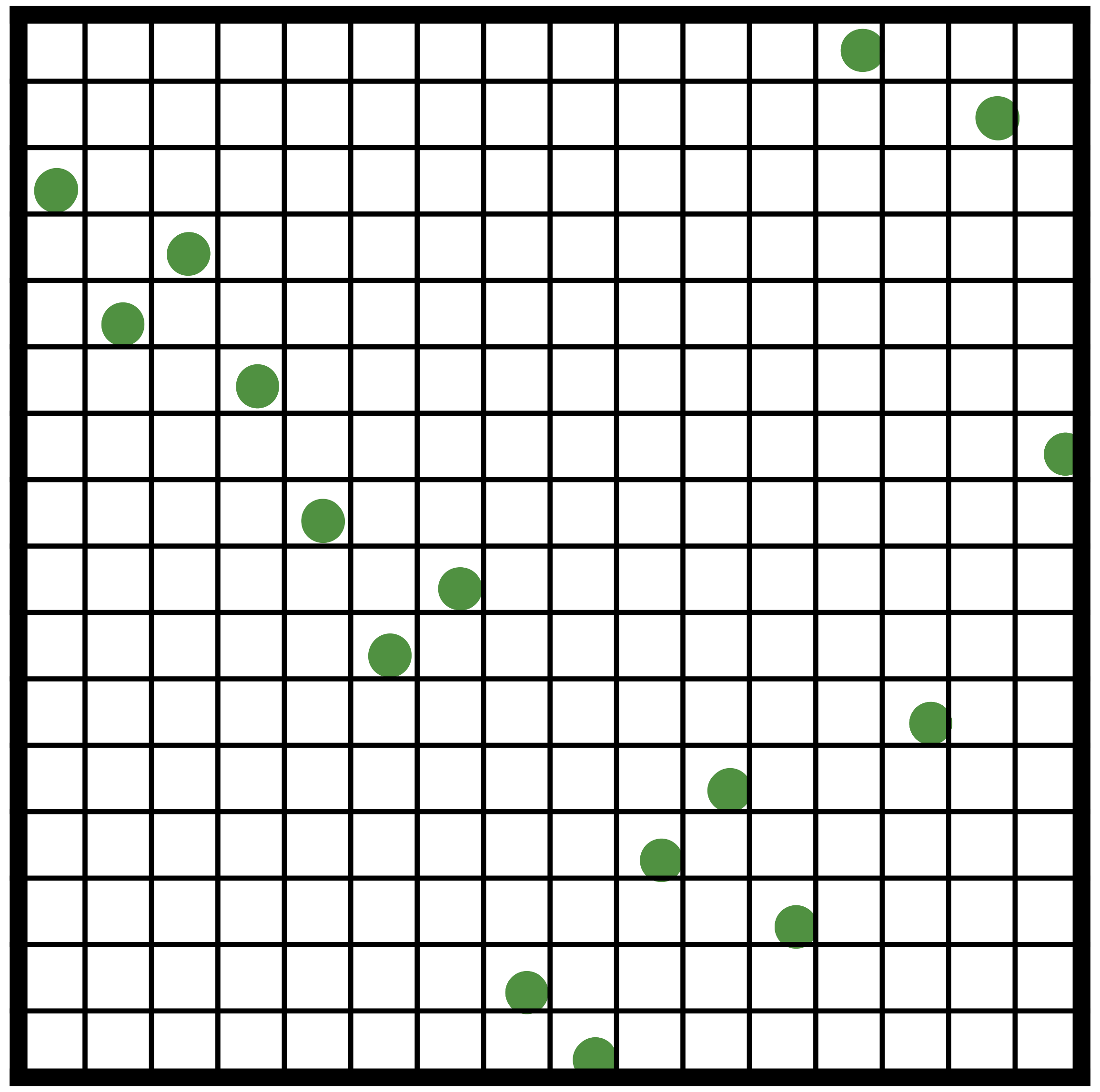
Latin Hypercube Sampler (N-rooks)



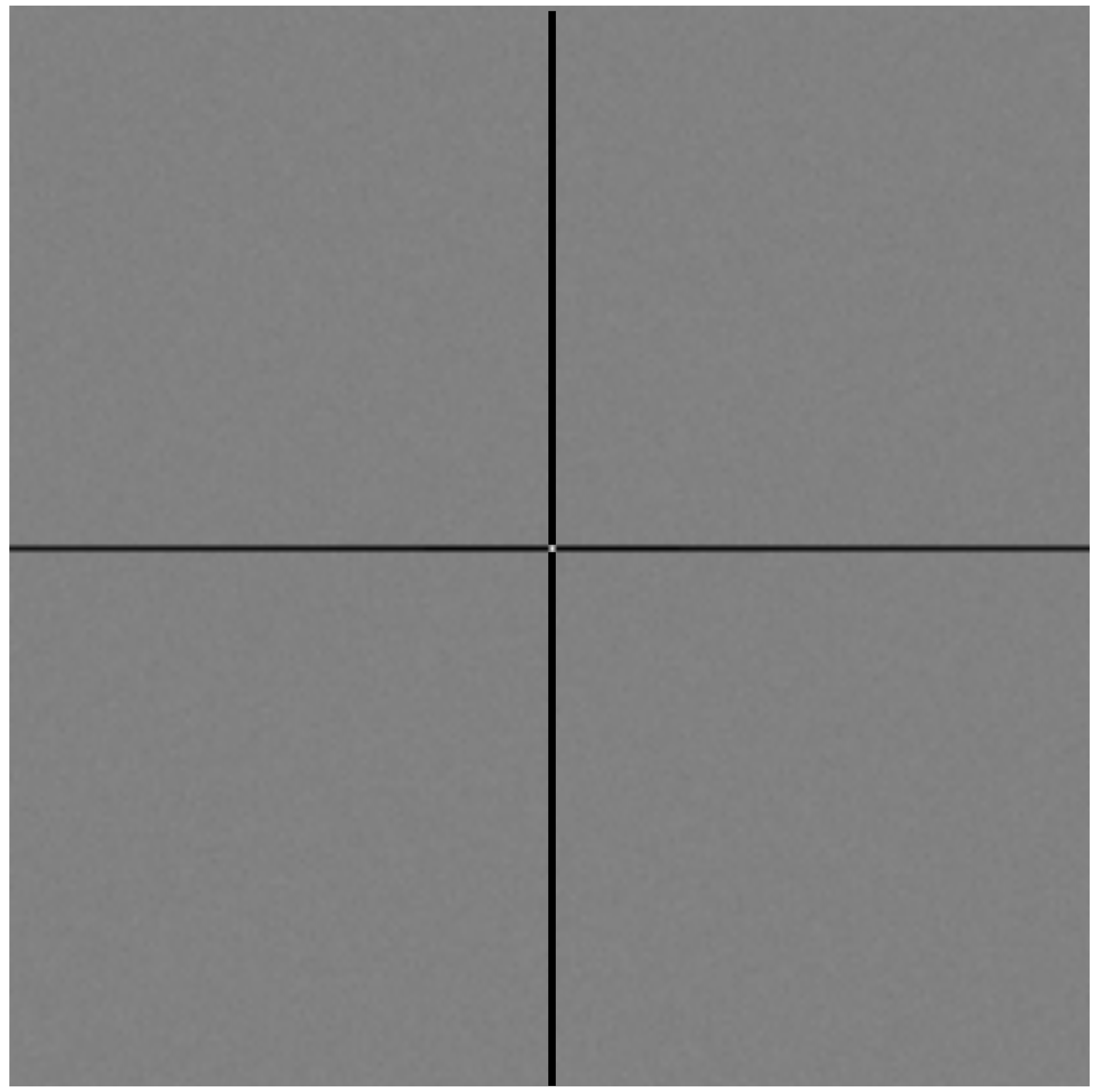
Latin Hypercube Sampler (N-rooks)



Anisotropic Sampling Power Spectra

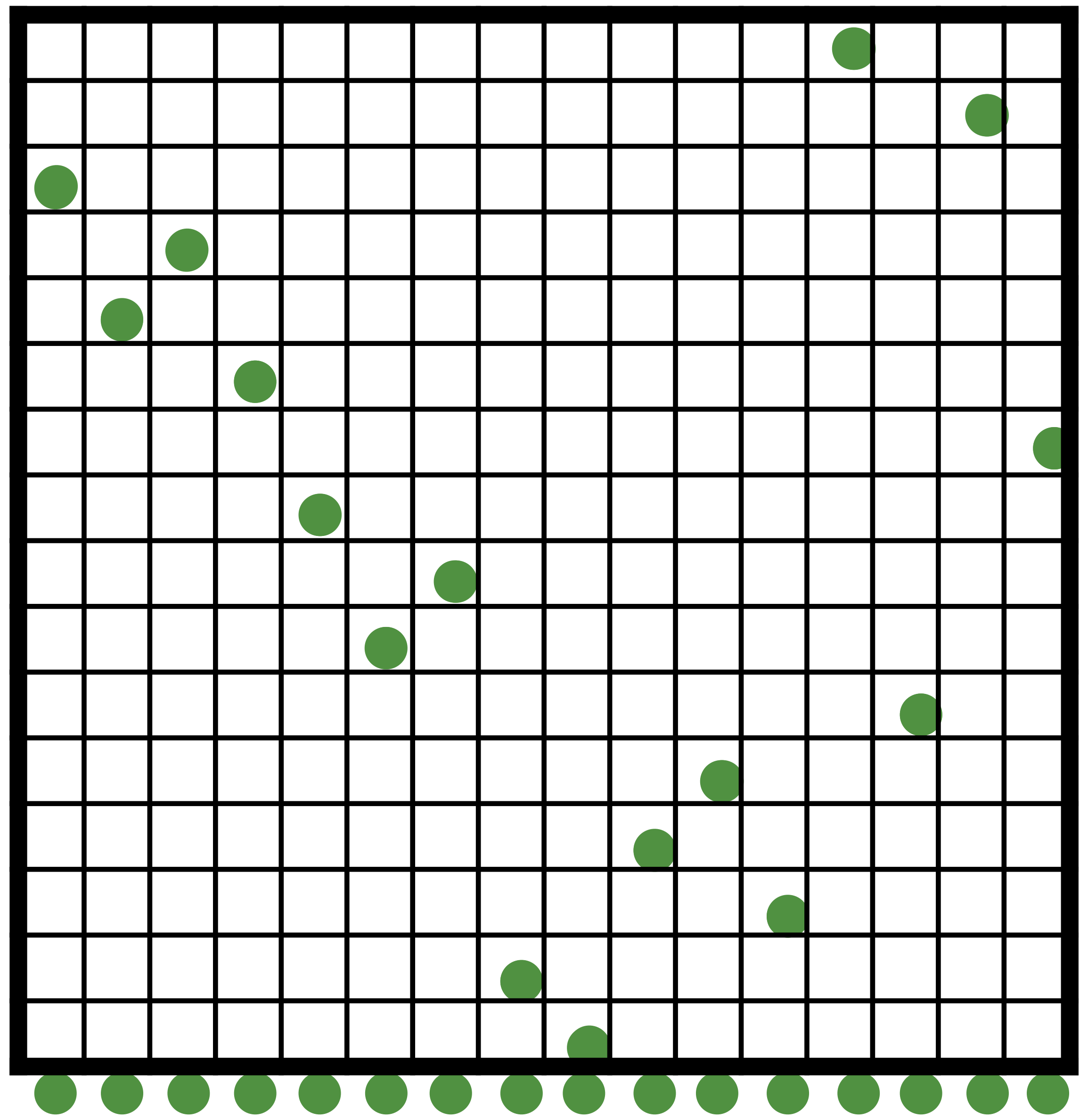


N-rooks /
Latin Hypercube

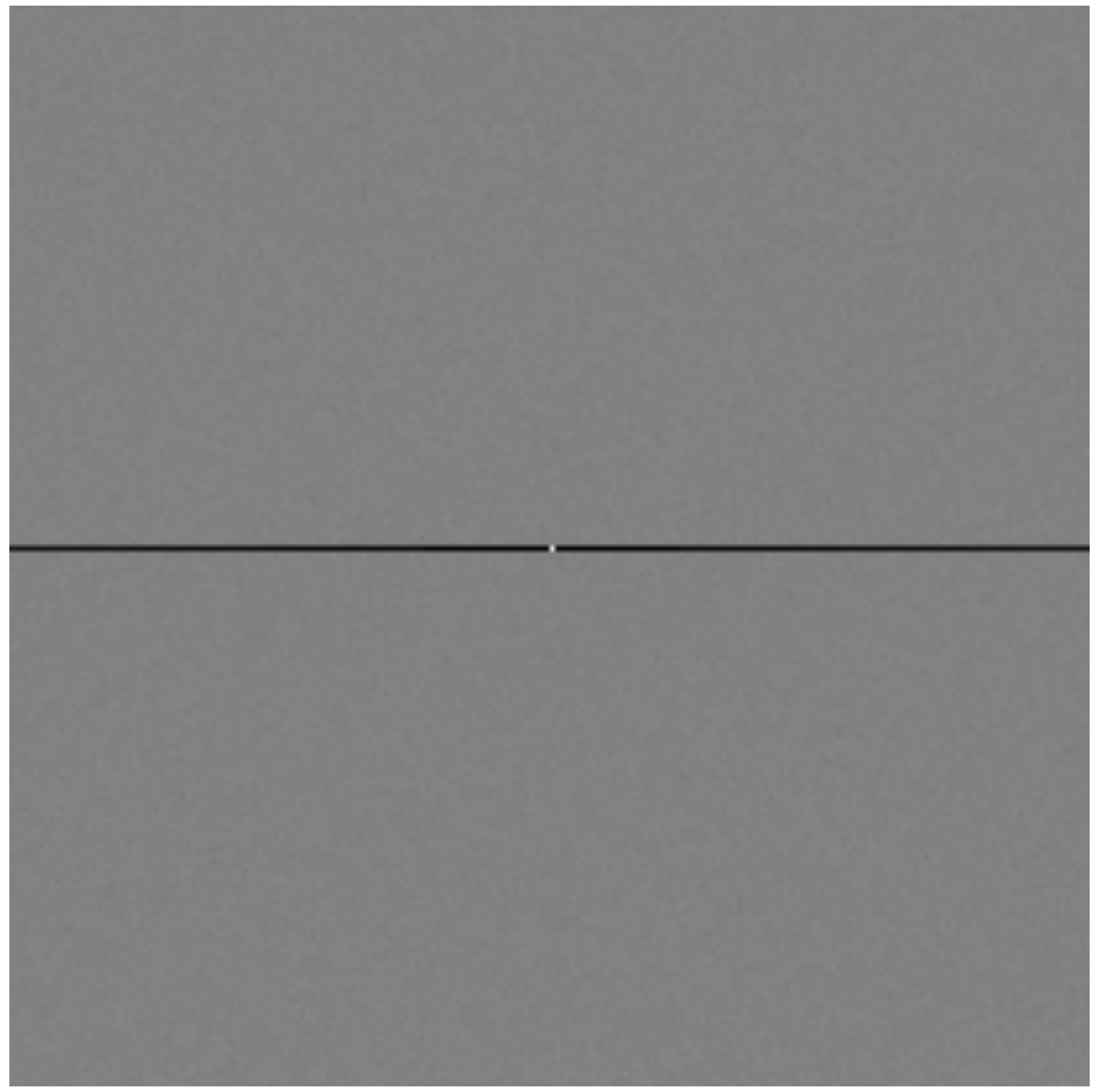


N-rooks
Spectrum

Anisotropic Sampling Power Spectra

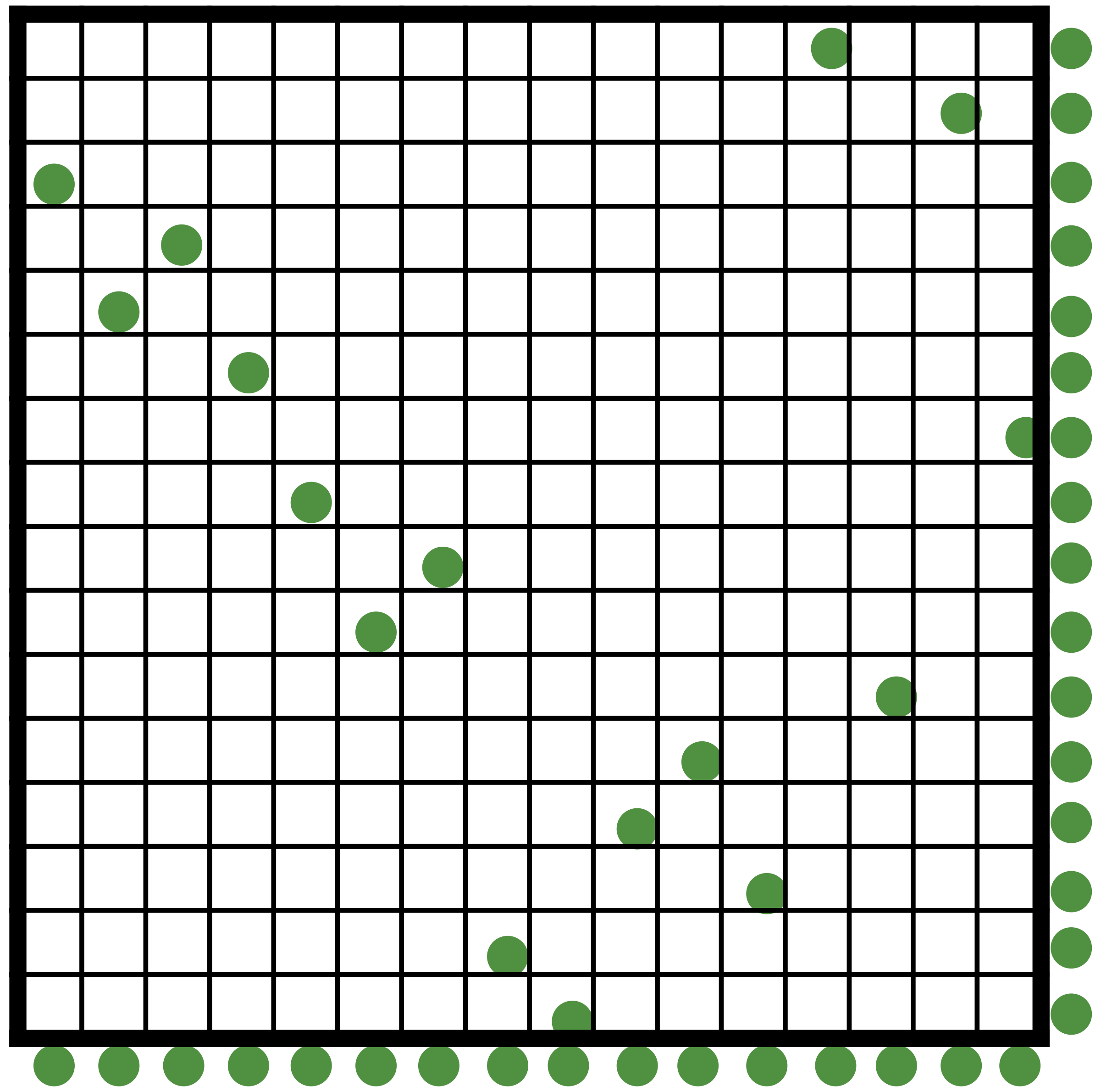


N-rooks /
Latin Hypercube

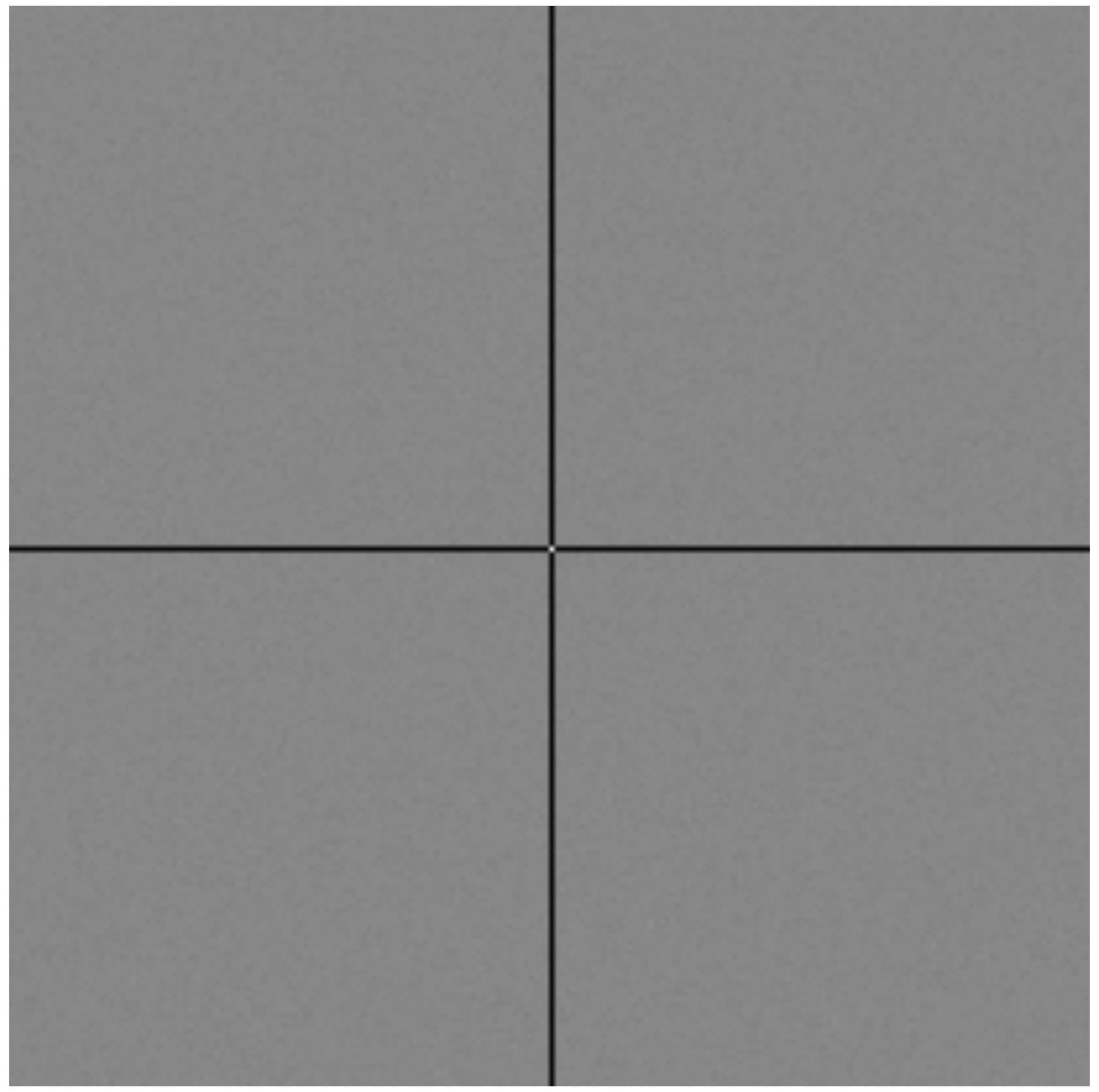


Spectrum

Anisotropic Sampling Power Spectra

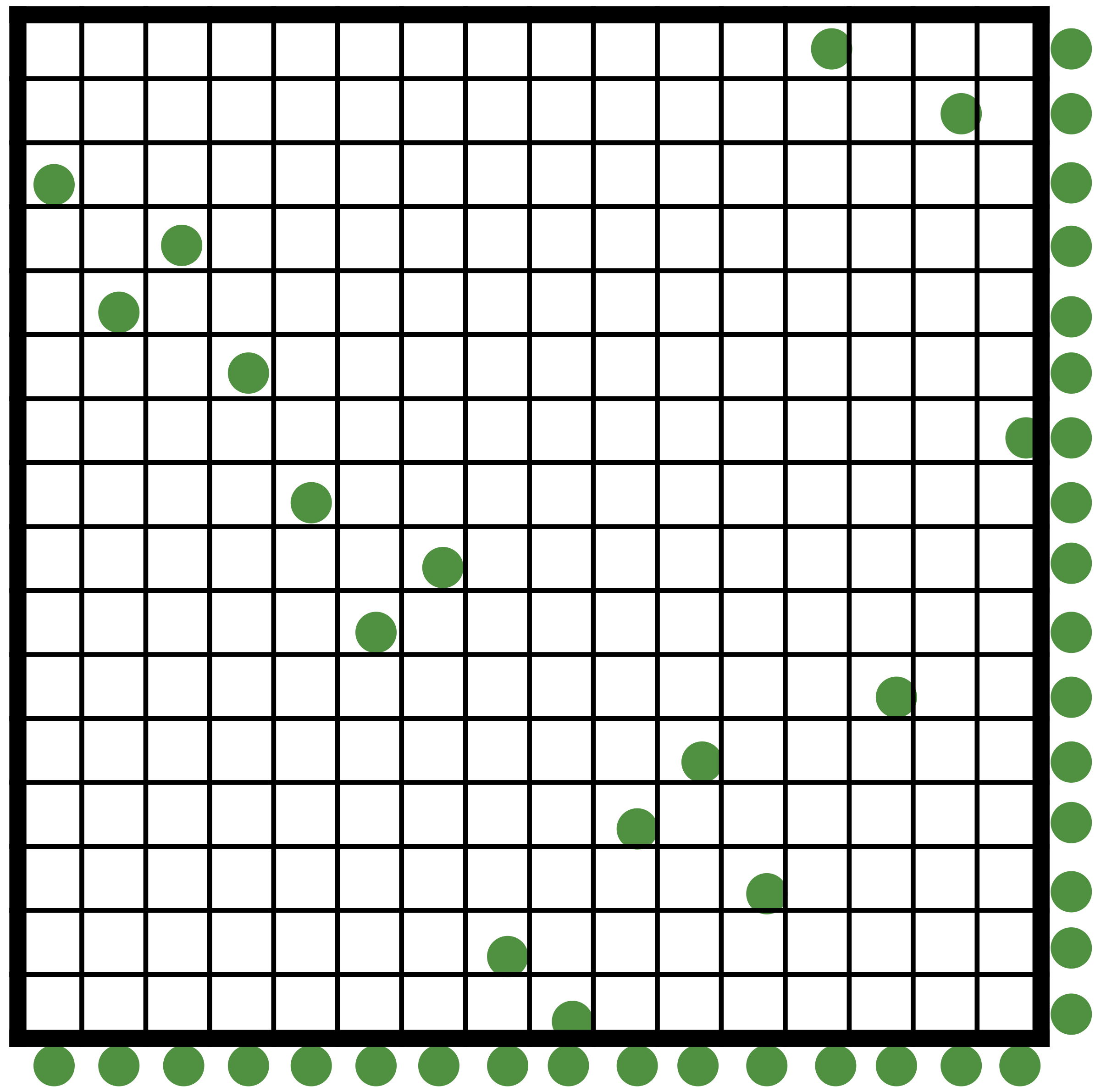


N-rooks /
Latin Hypercube

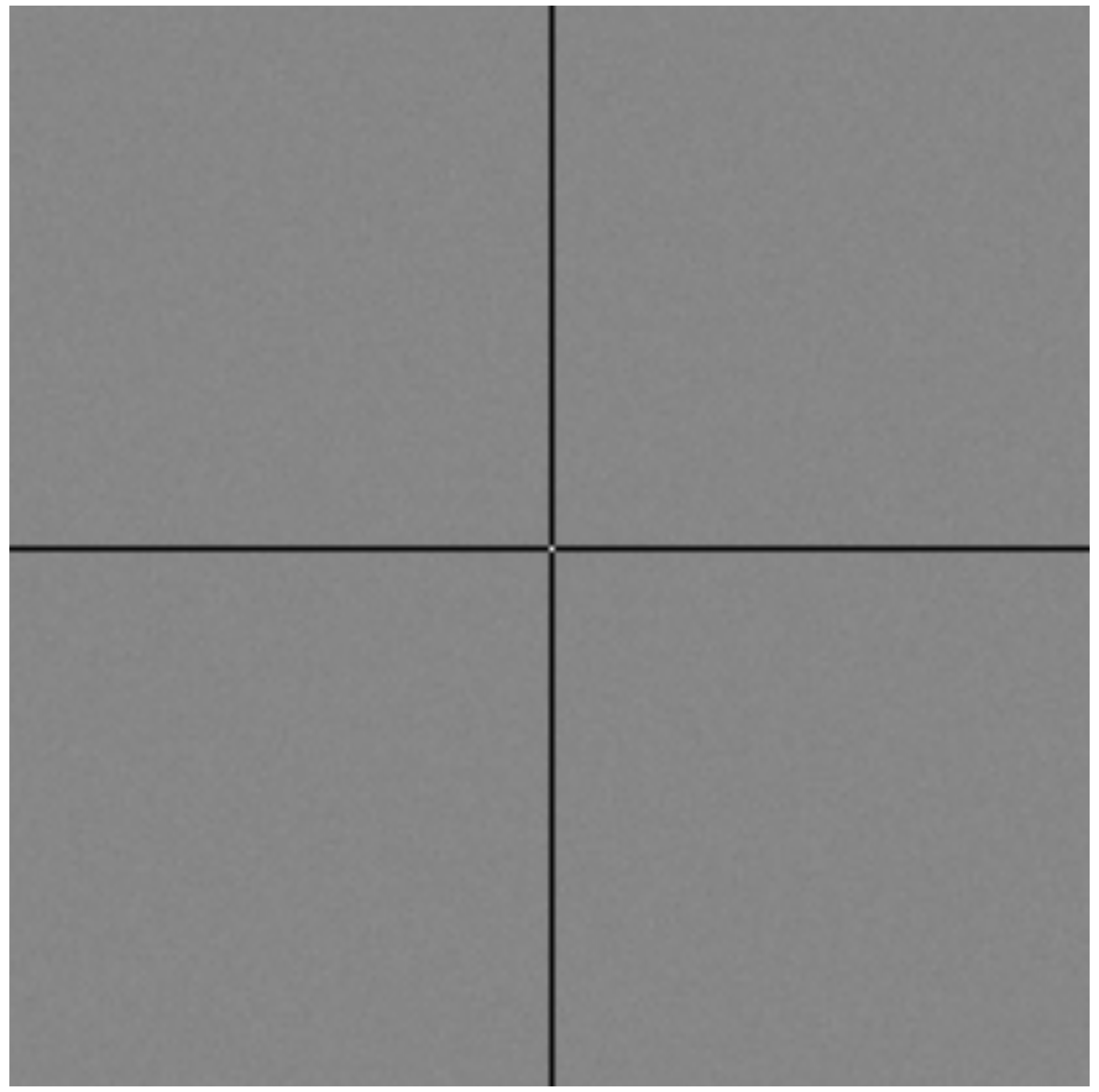


N-rooks
Spectrum

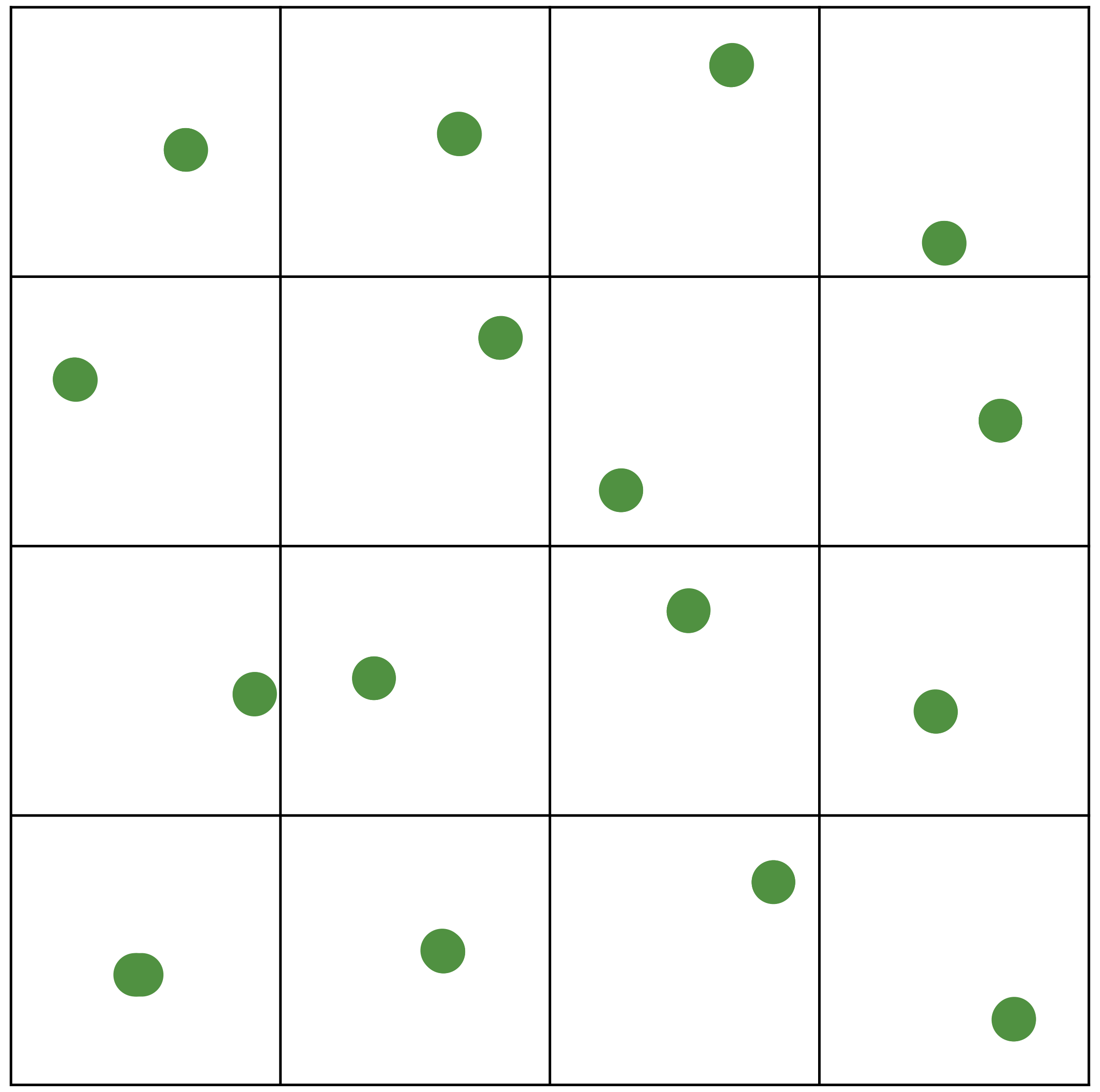
Anisotropic Sampling Power Spectra



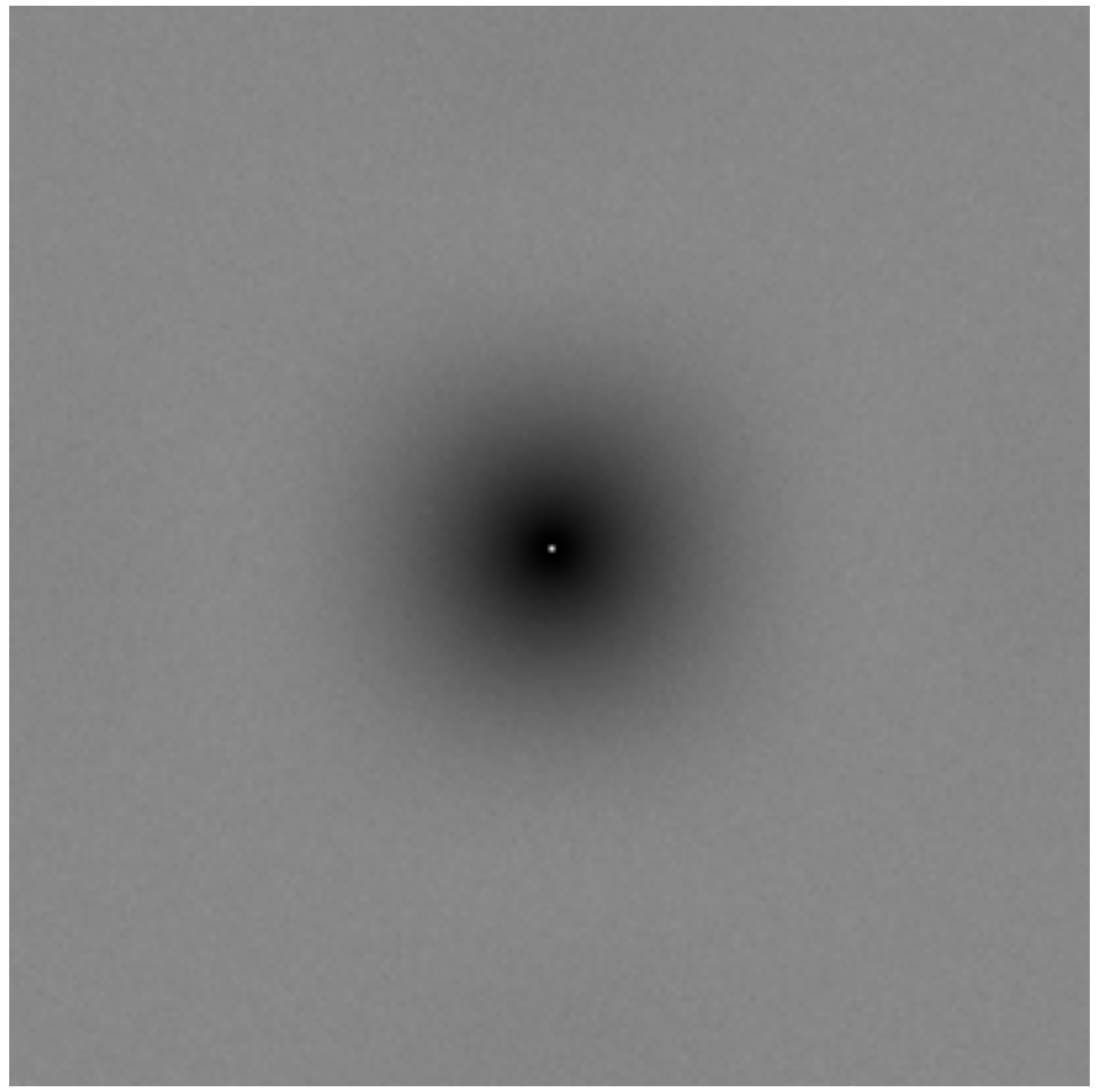
N-rooks /
Latin Hypercube



N-rooks
Spectrum

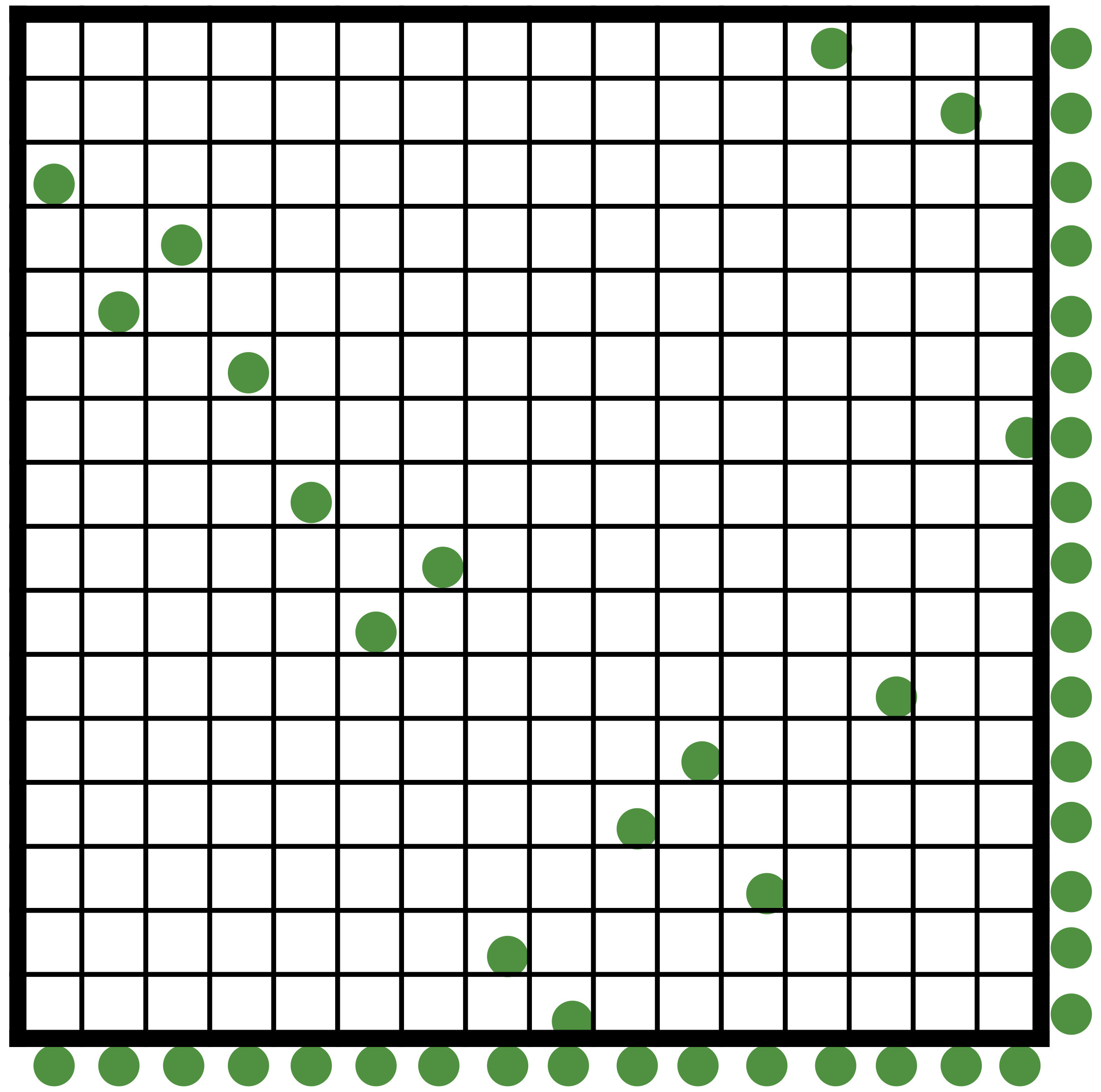


Jitter

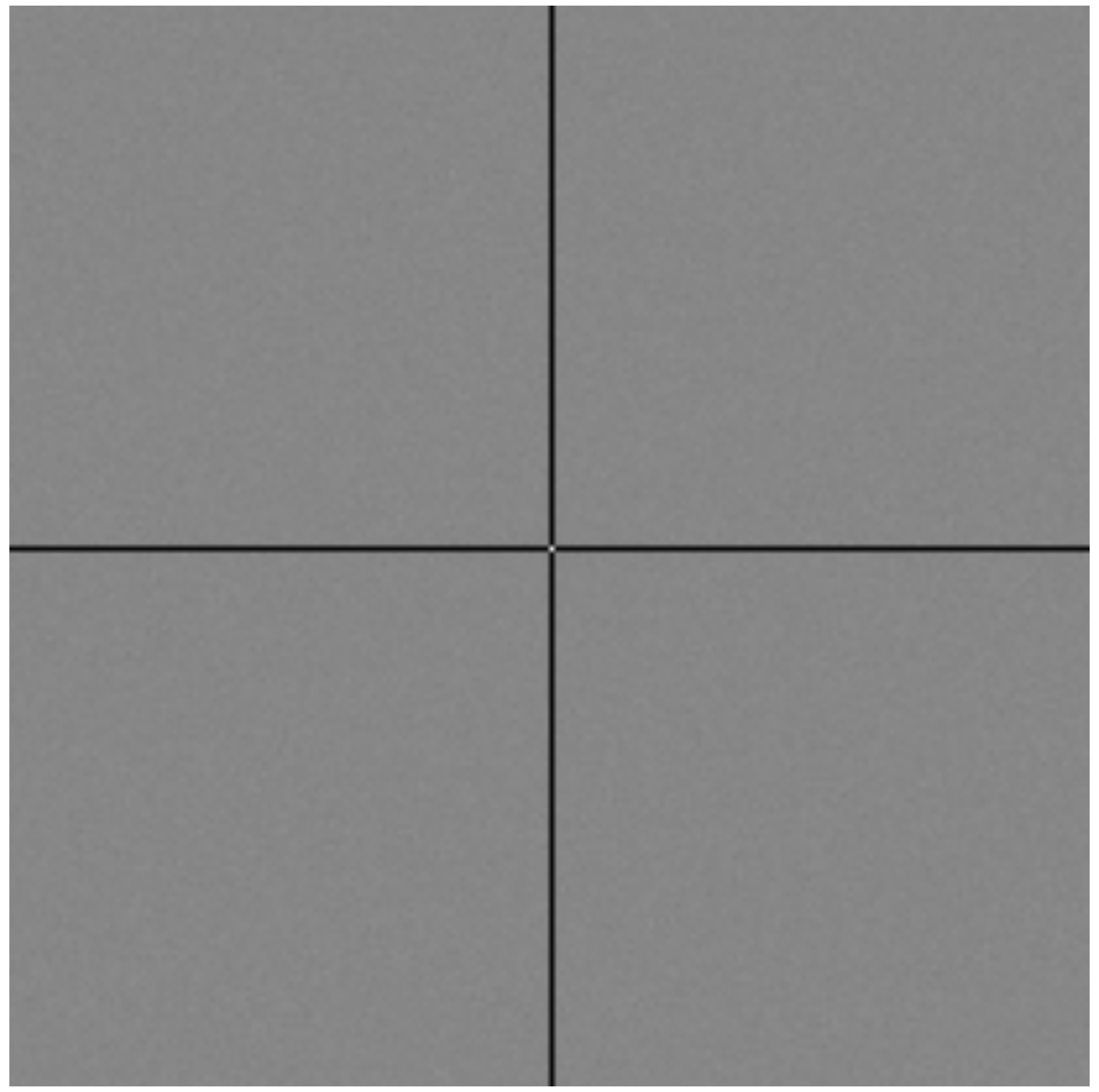


Jitter
Spectrum

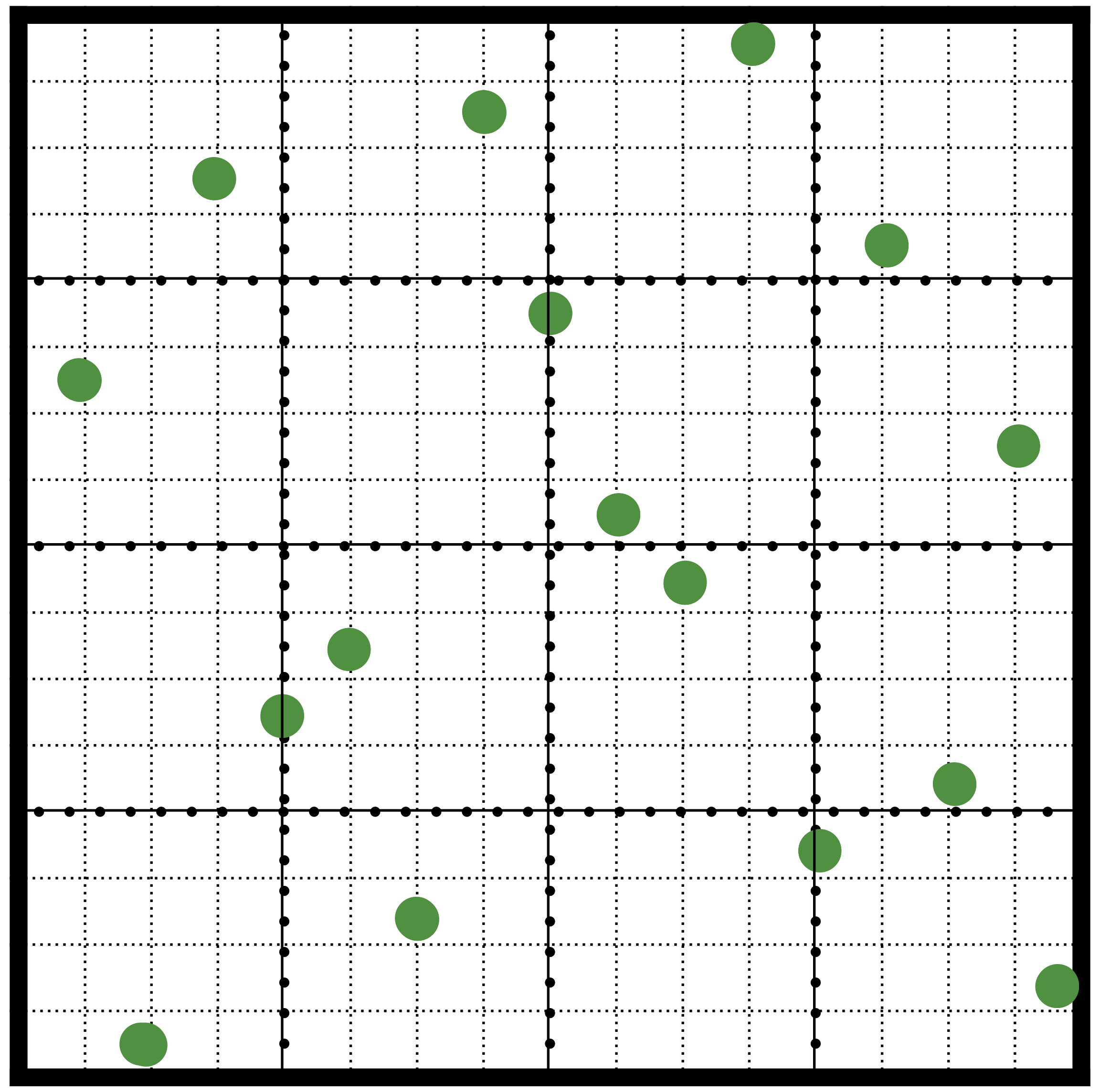
Anisotropic Sampling Power Spectra



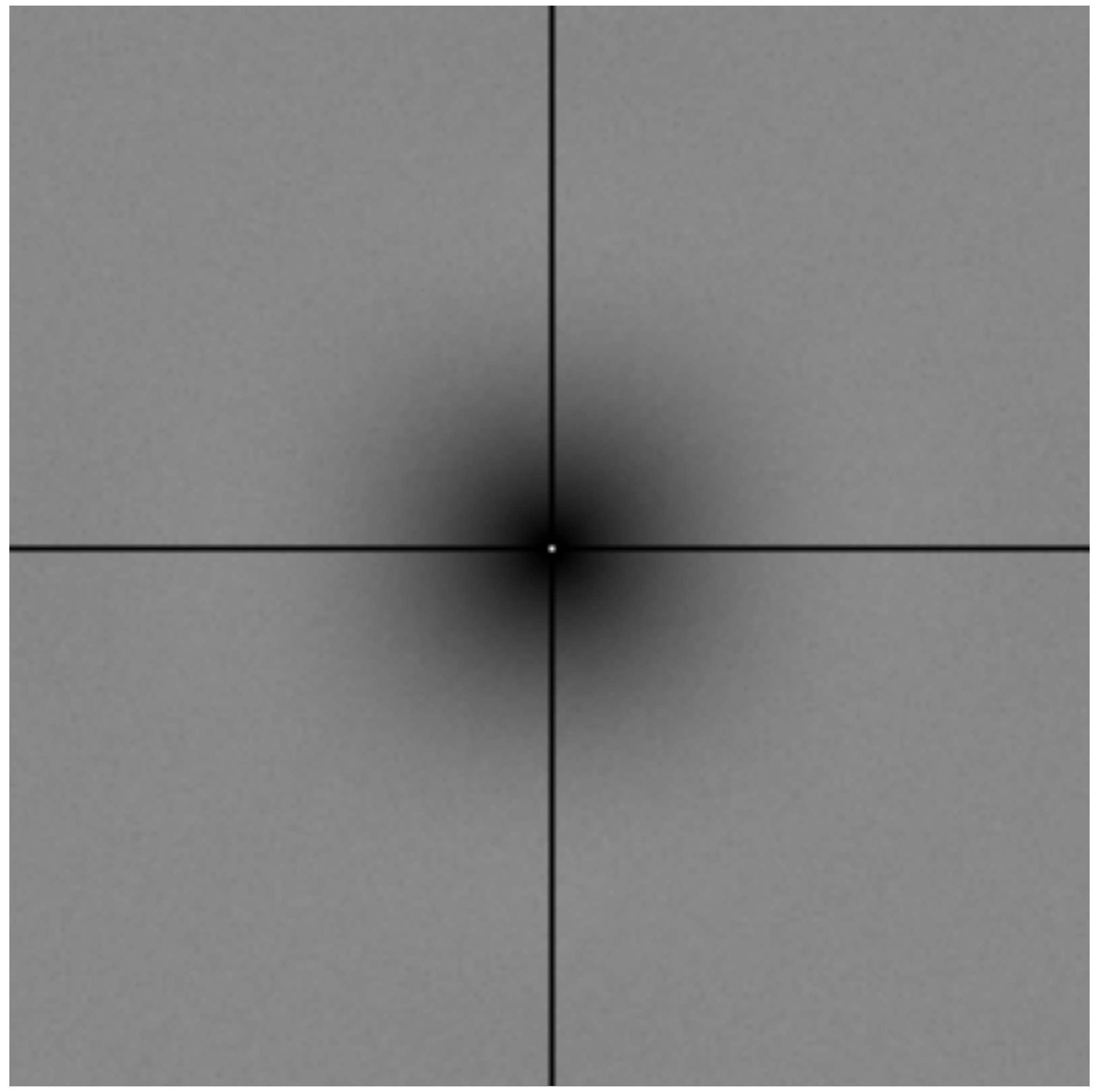
N-rooks /
Latin Hypercube



N-rooks
Spectrum



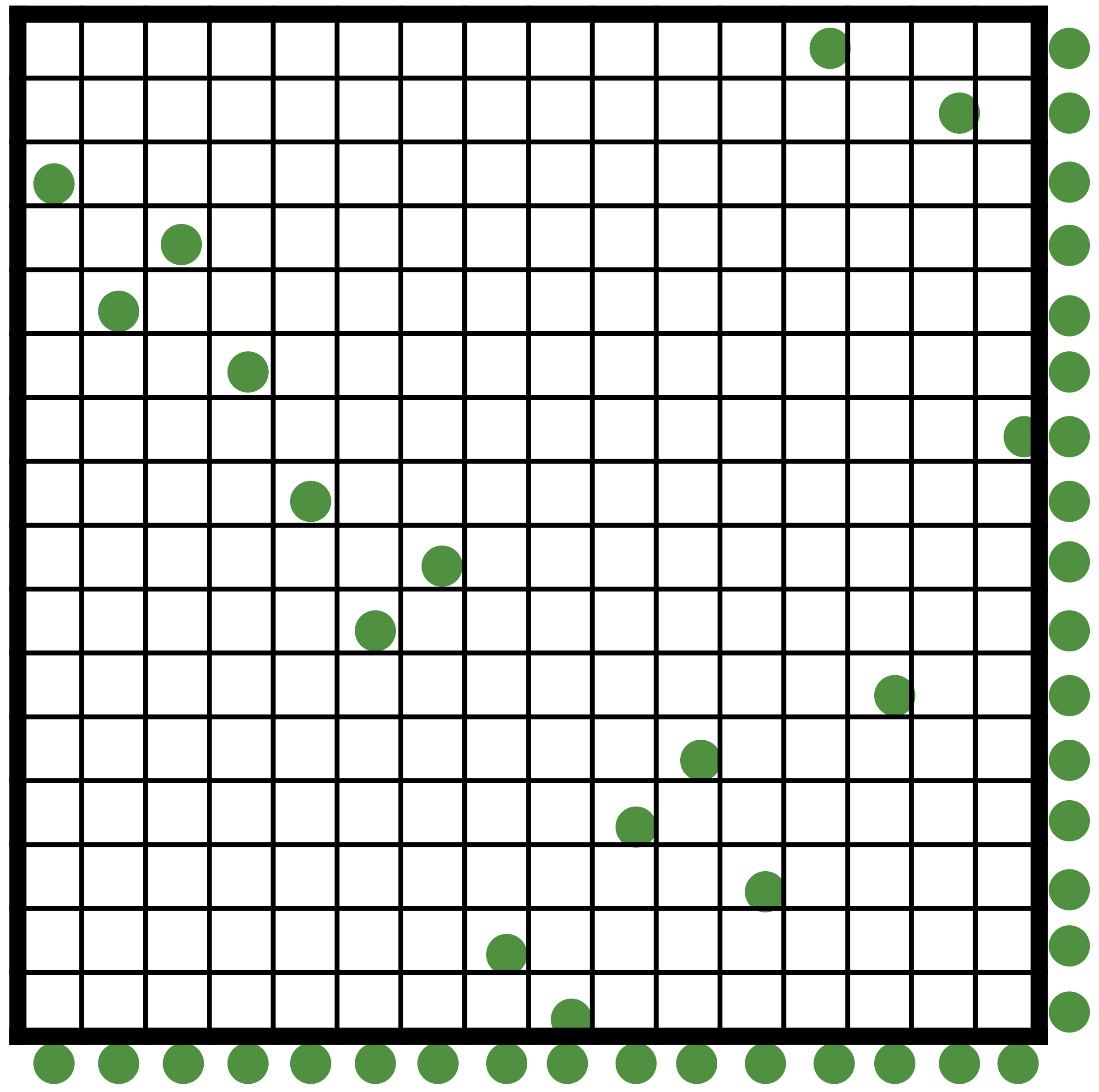
Multi-Jitter



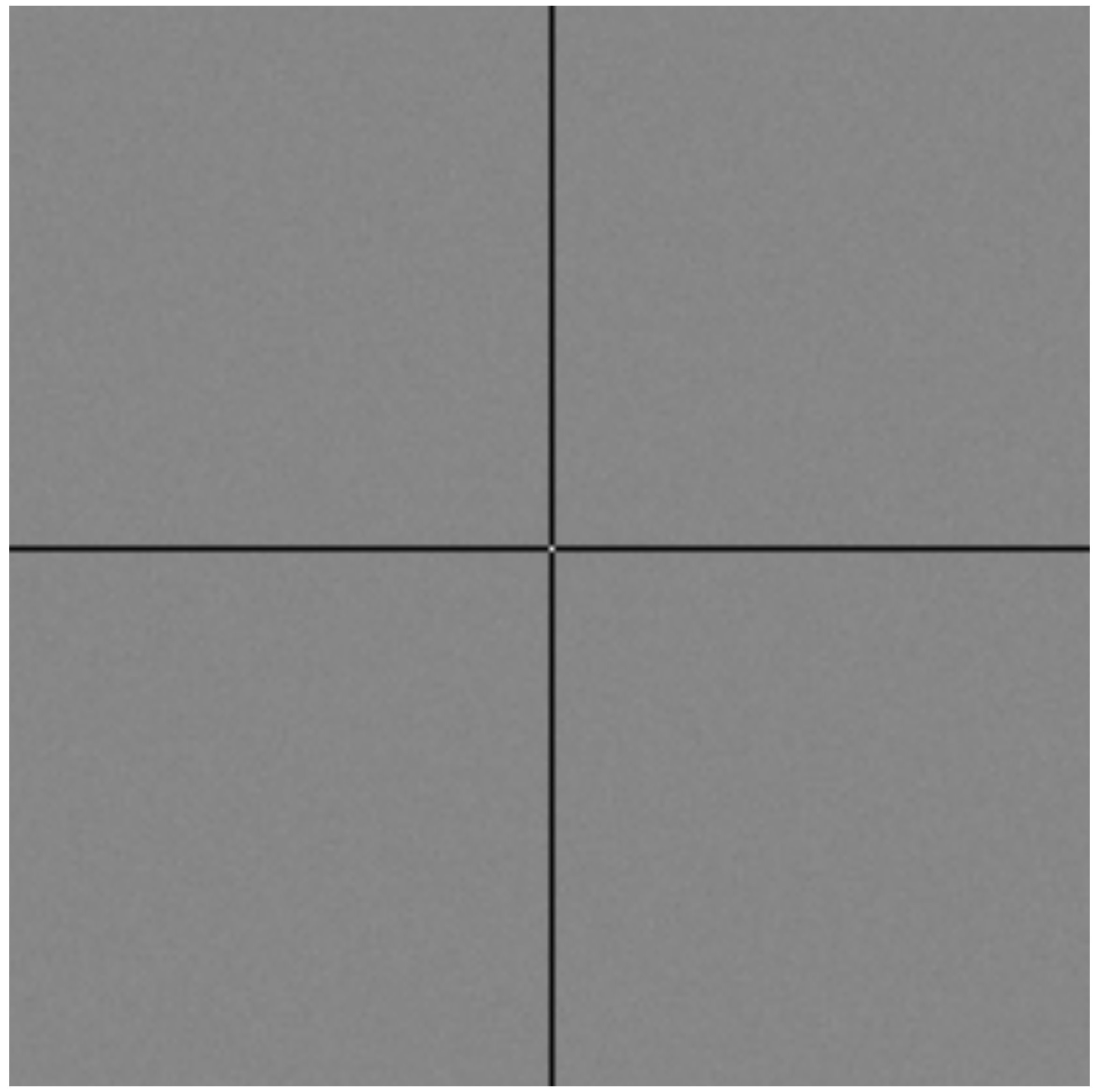
Multi-Jitter
Spectrum

Chiu et al. [1993]

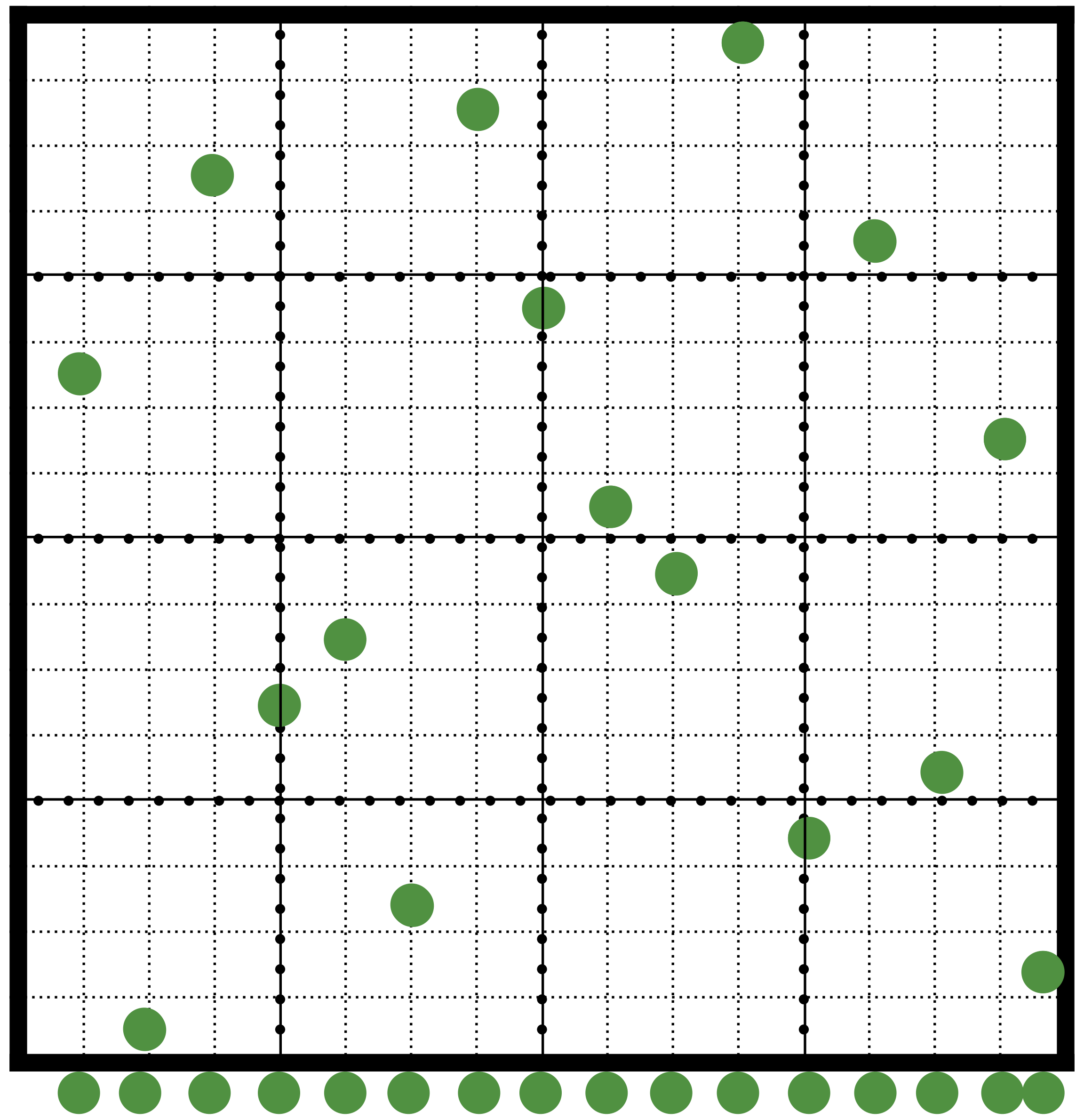
Anisotropic Sampling Power Spectra



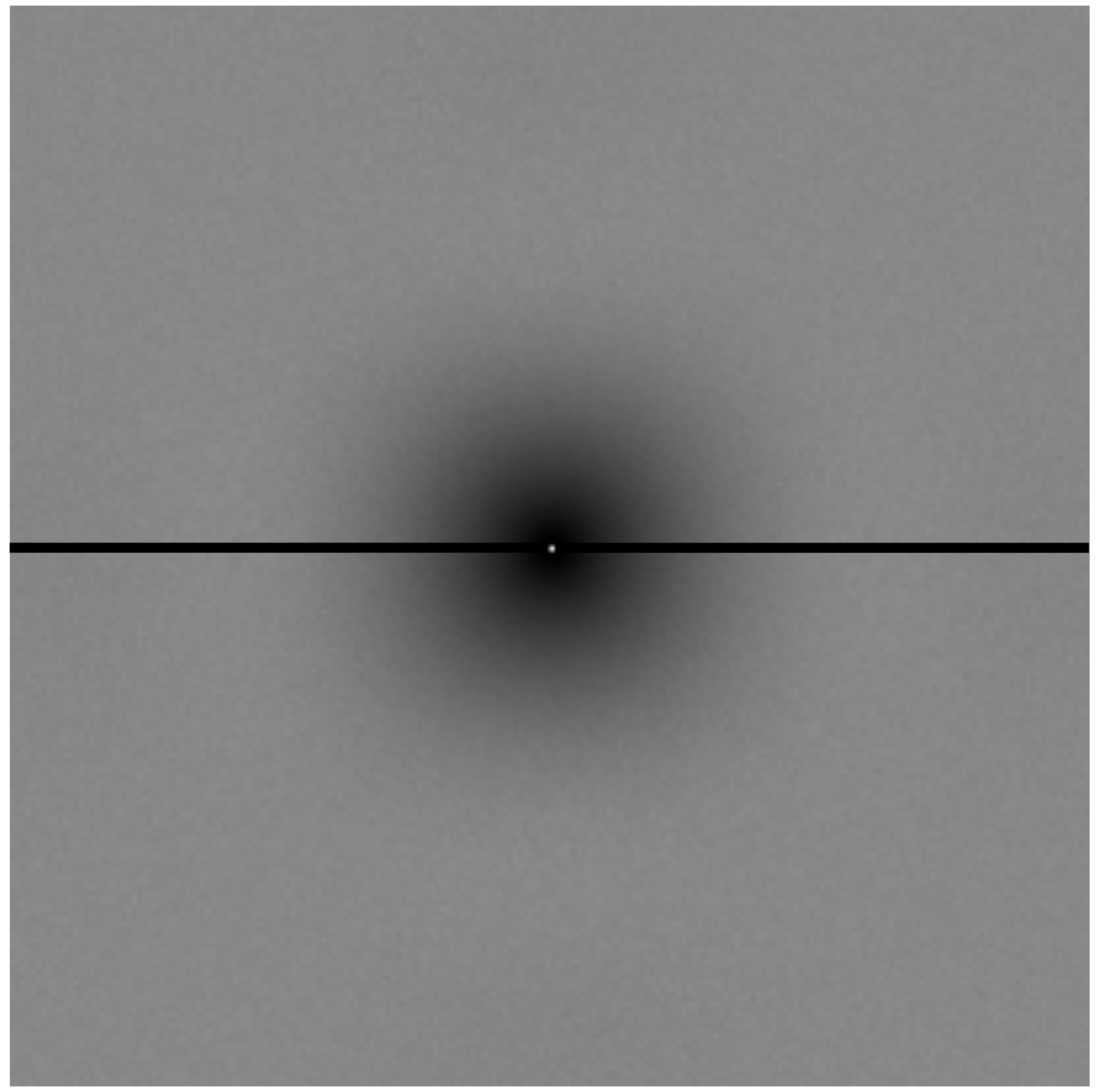
N-rooks /
Latin Hypercube



N-rooks
Spectrum



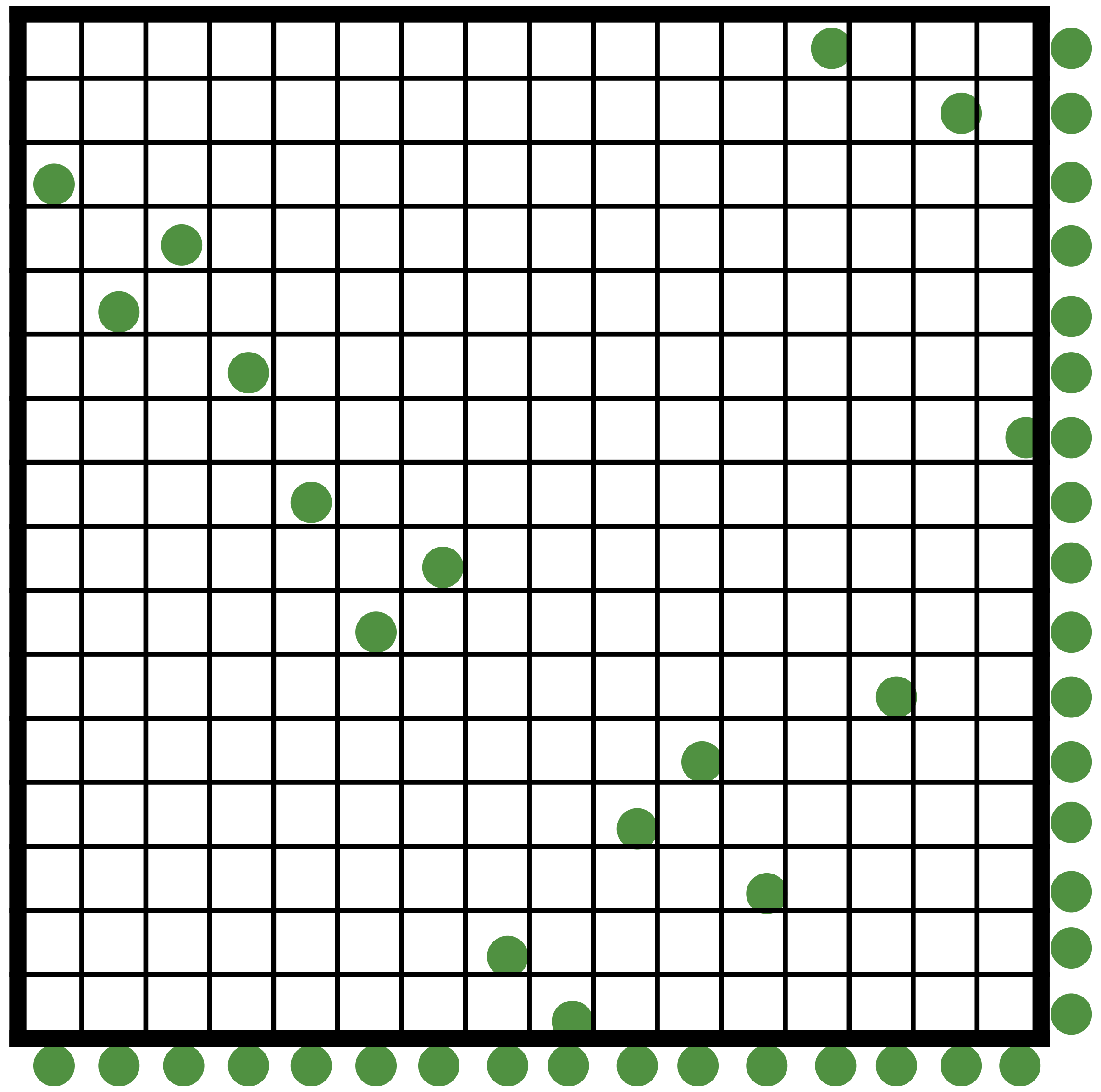
Multi-jitter



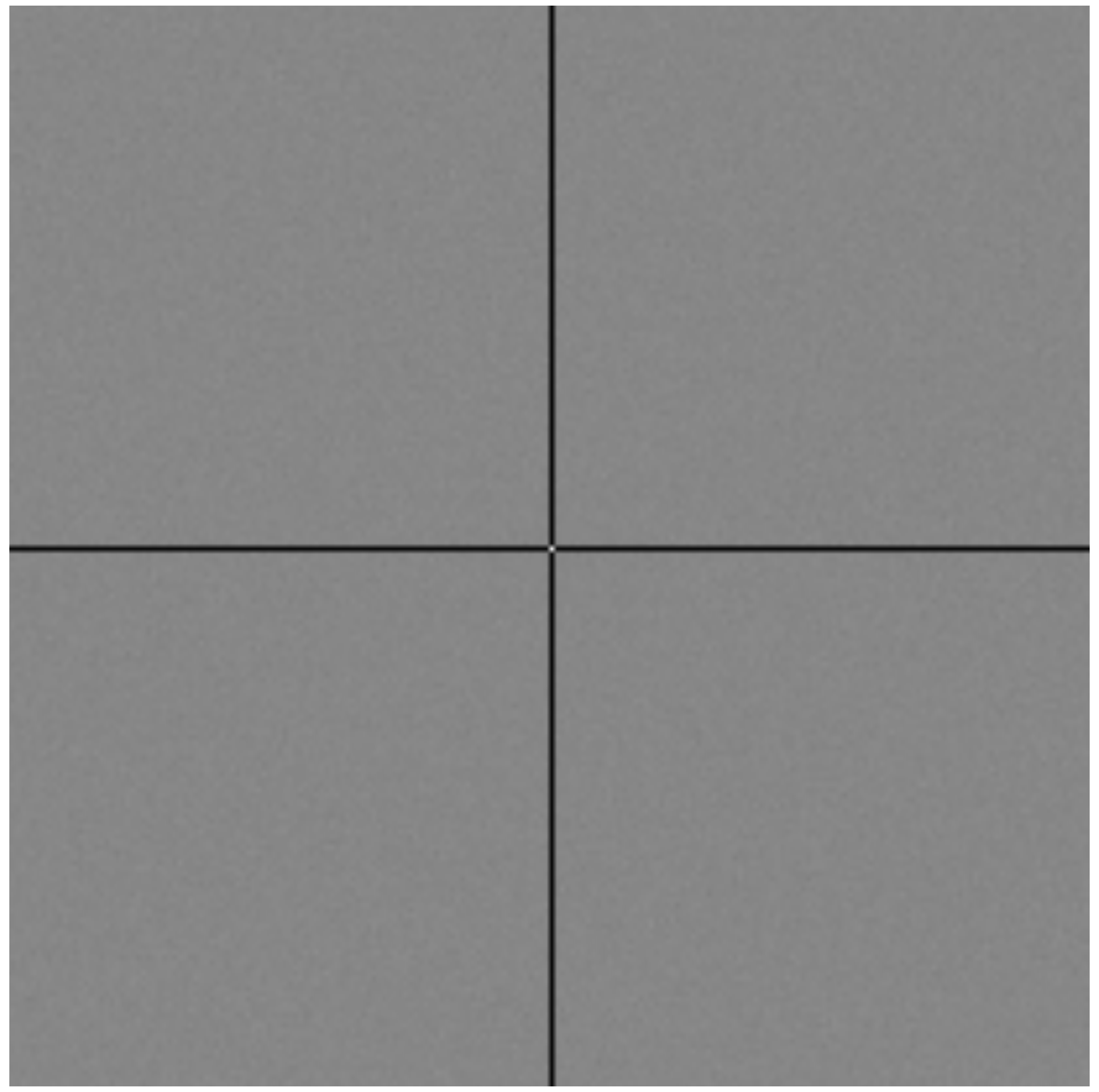
Multi-Jitter
Spectrum

Chiu et al. [1993]

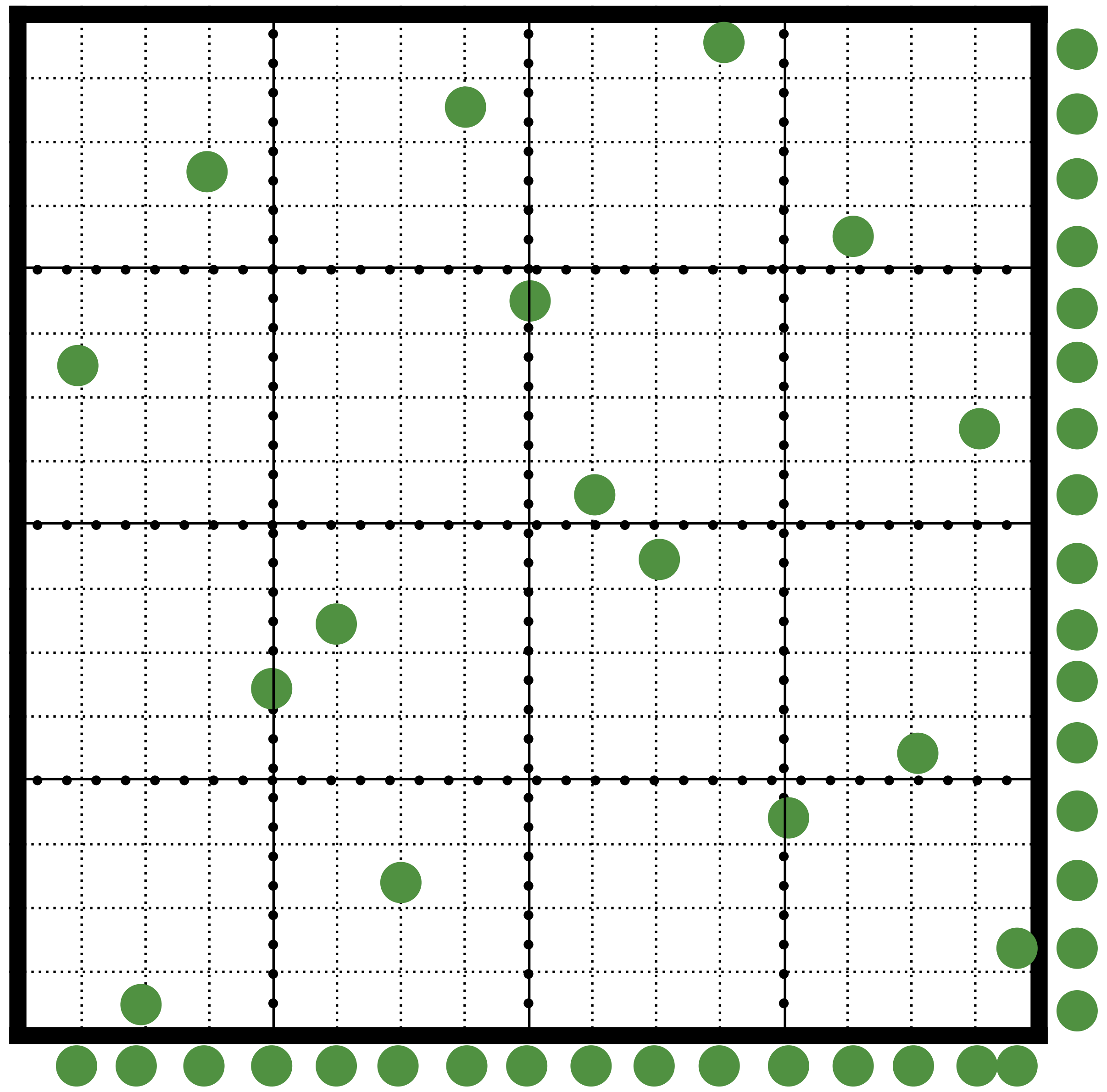
Anisotropic Sampling Power Spectra



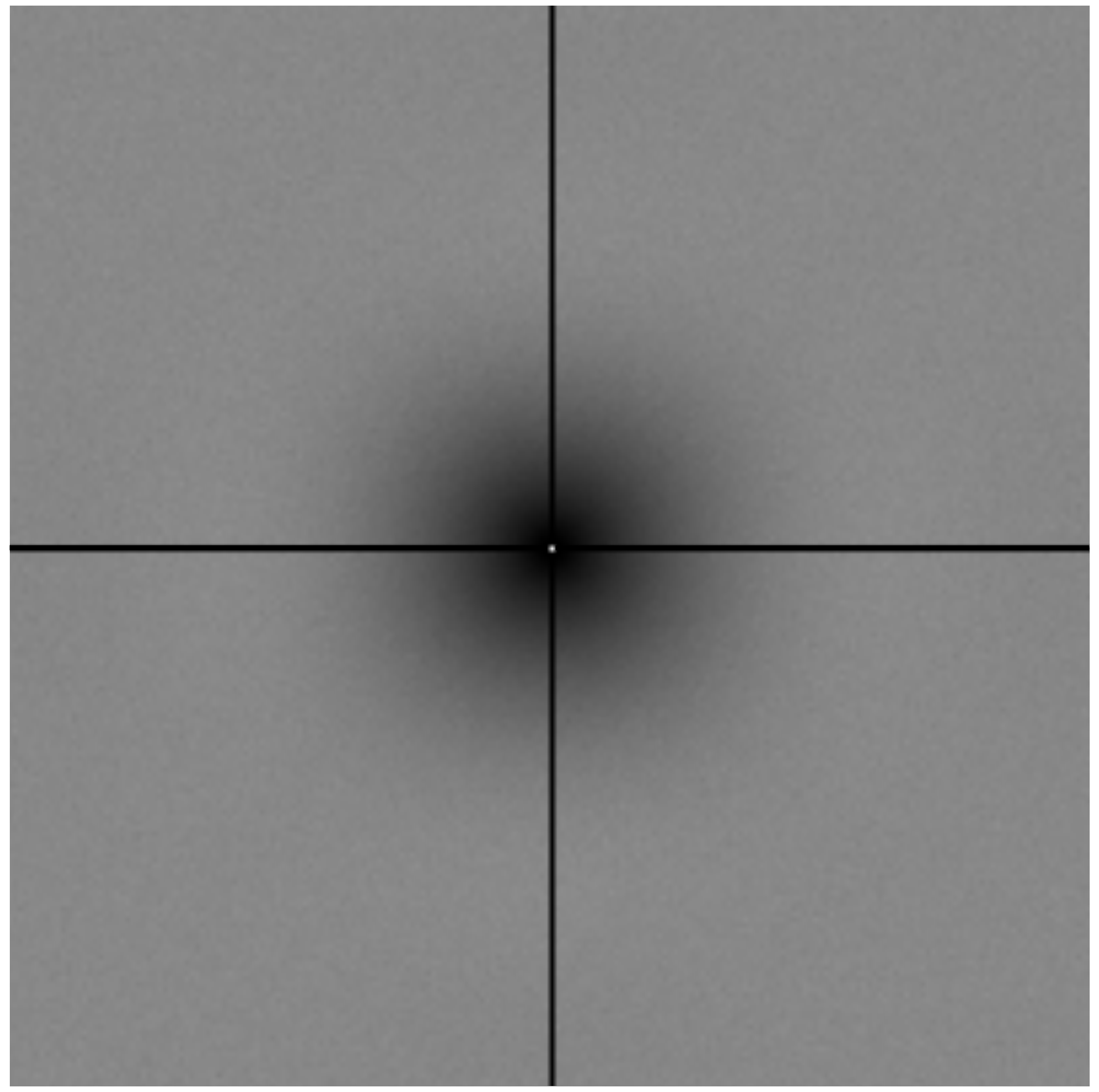
N-rooks /
Latin Hypercube



N-rooks
Spectrum



Multi-jitter

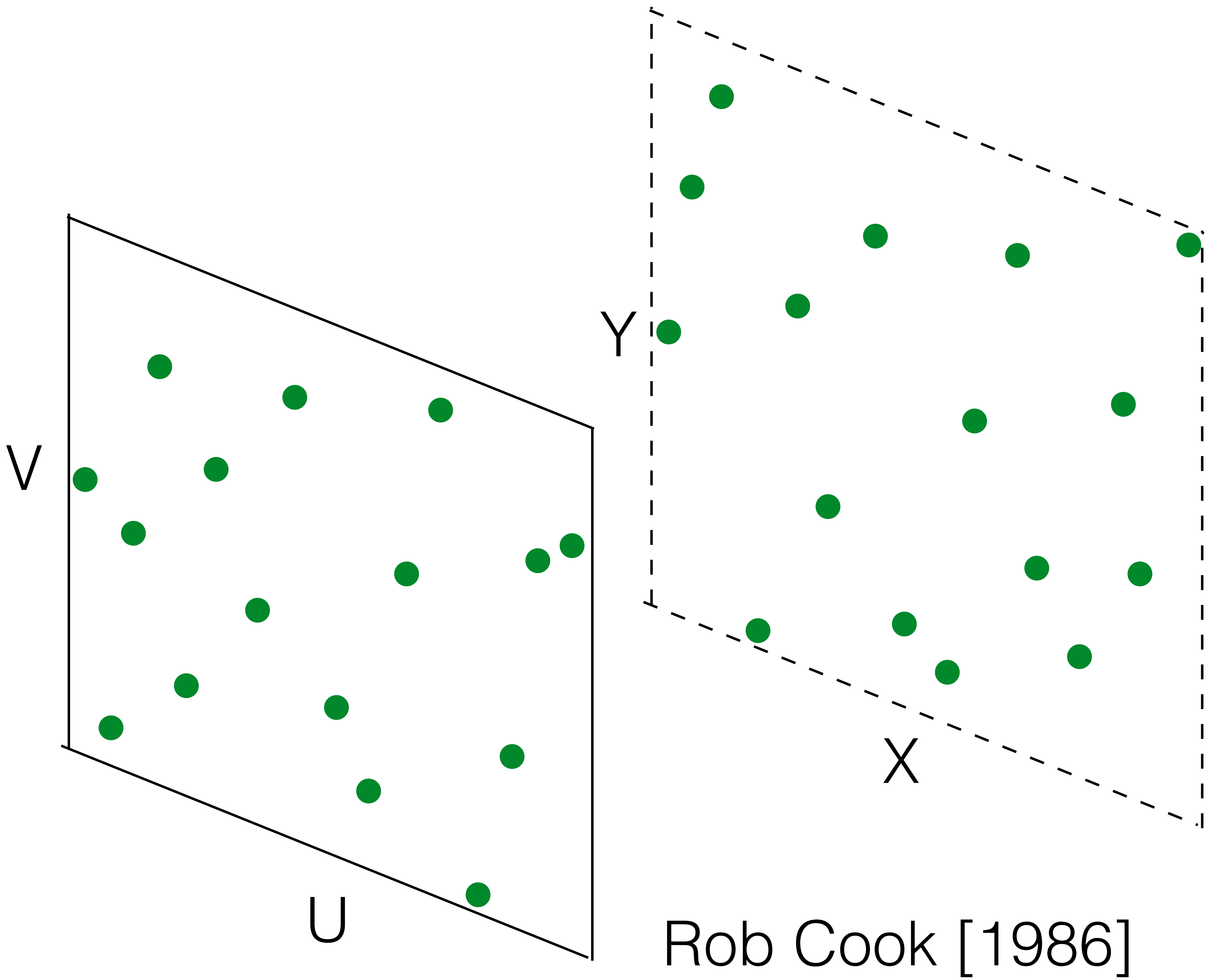


Multi-Jitter
Spectrum

Chiu et al. [1993]

Sampling in Higher Dimensions

4D Sampling



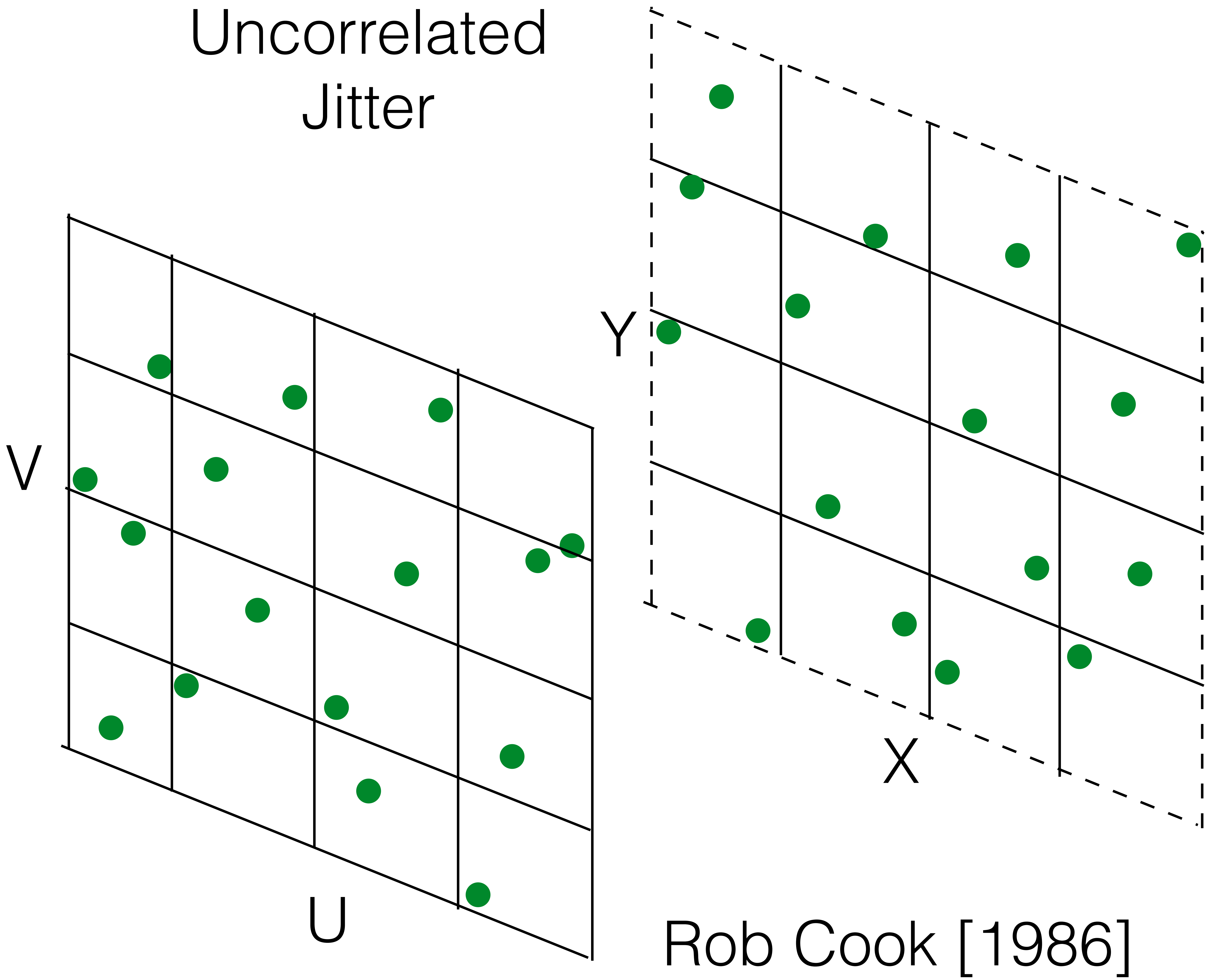
Rob Cook [1986]

2D	2D
(x_1, y_1)	(u_1, v_1)
(x_2, y_2)	(u_2, v_2)
(x_3, y_3)	(u_3, v_3)
(x_4, y_4)	(u_4, v_4)
$\vdots$	$\vdots$

4D

$$(x_1, y_1, u_3, v_3)$$
$$(x_2, y_2, u_1, v_1)$$
$$(x_3, y_3, u_4, v_4)$$
$$(x_4, y_4, u_2, v_2)$$
$$\vdots$$

4D Sampling



Rob Cook [1986]

2D | 2D

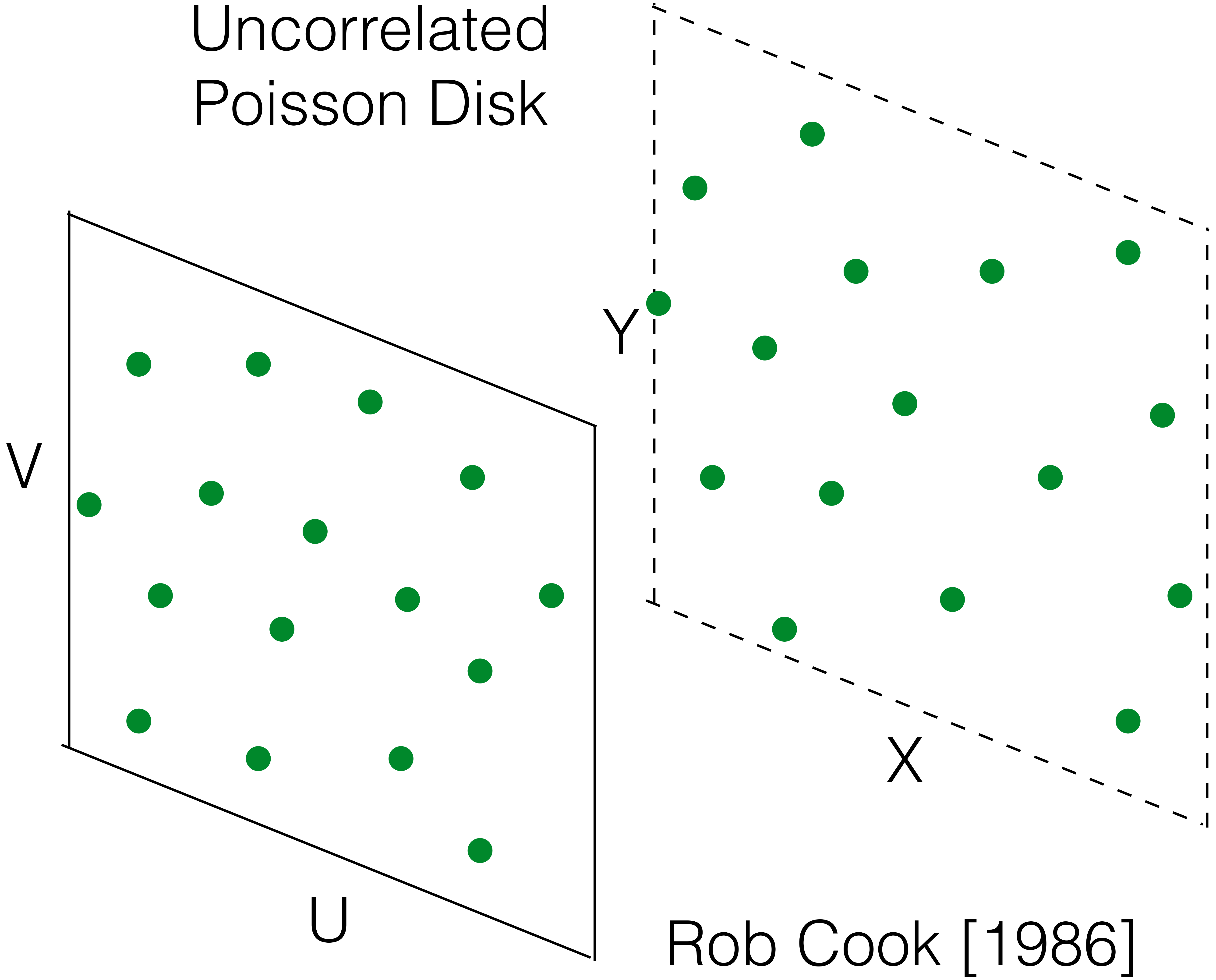
$$(x_1, y_1) \quad (u_1, v_1)$$
$$(x_2, y_2) \quad (u_2, v_2)$$
$$(x_3, y_3) \quad (u_3, v_3)$$
$$(x_4, y_4) \quad (u_4, v_4)$$
$$\vdots \qquad \vdots$$

4D

$$(x_1, y_1, u_3, v_3)$$
$$(x_2, y_2, u_1, v_1)$$
$$(x_3, y_3, u_4, v_4)$$
$$(x_4, y_4, u_2, v_2)$$
$$\vdots$$

4D Sampling

Uncorrelated Poisson Disk



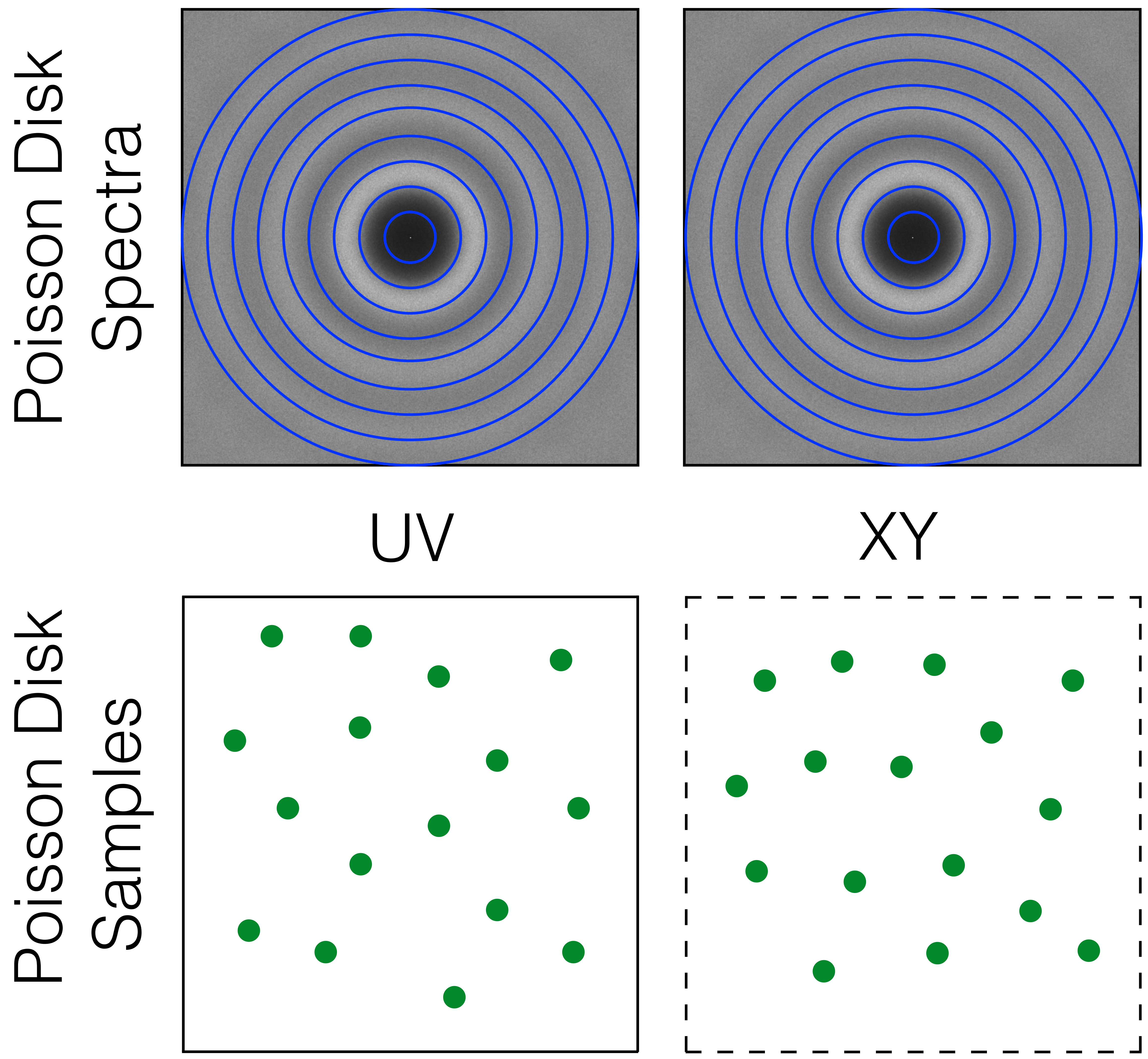
Rob Cook [1986]

2D	2D
(x_1, y_1)	(u_1, v_1)
(x_2, y_2)	(u_2, v_2)
(x_3, y_3)	(u_3, v_3)
(x_4, y_4)	(u_4, v_4)
$\vdots$	$\vdots$

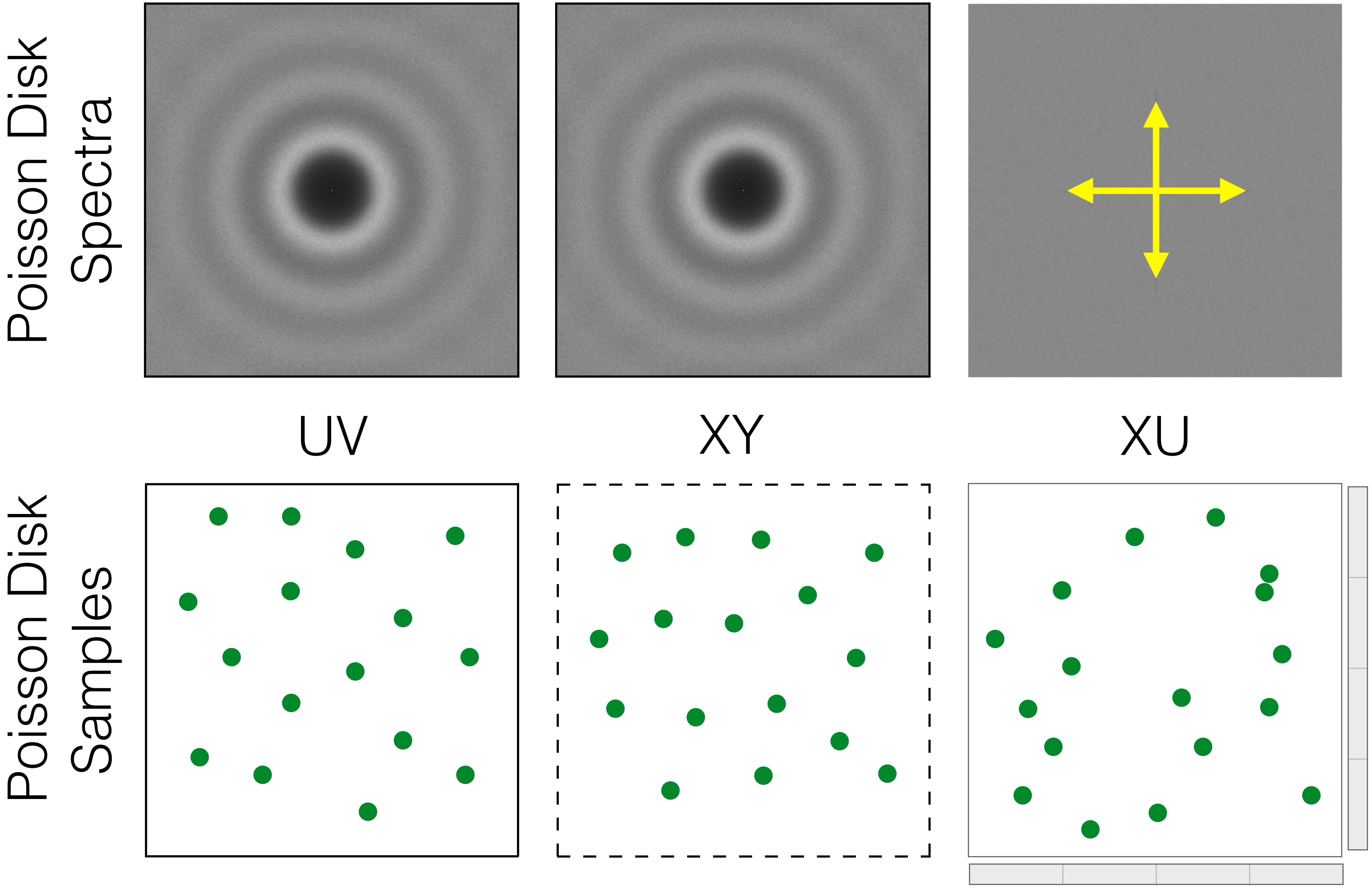
4D

$$(x_1, y_1, u_3, v_3)$$
$$(x_2, y_2, u_1, v_1)$$
$$(x_3, y_3, u_4, v_4)$$
$$(x_4, y_4, u_2, v_2)$$
$$\vdots$$

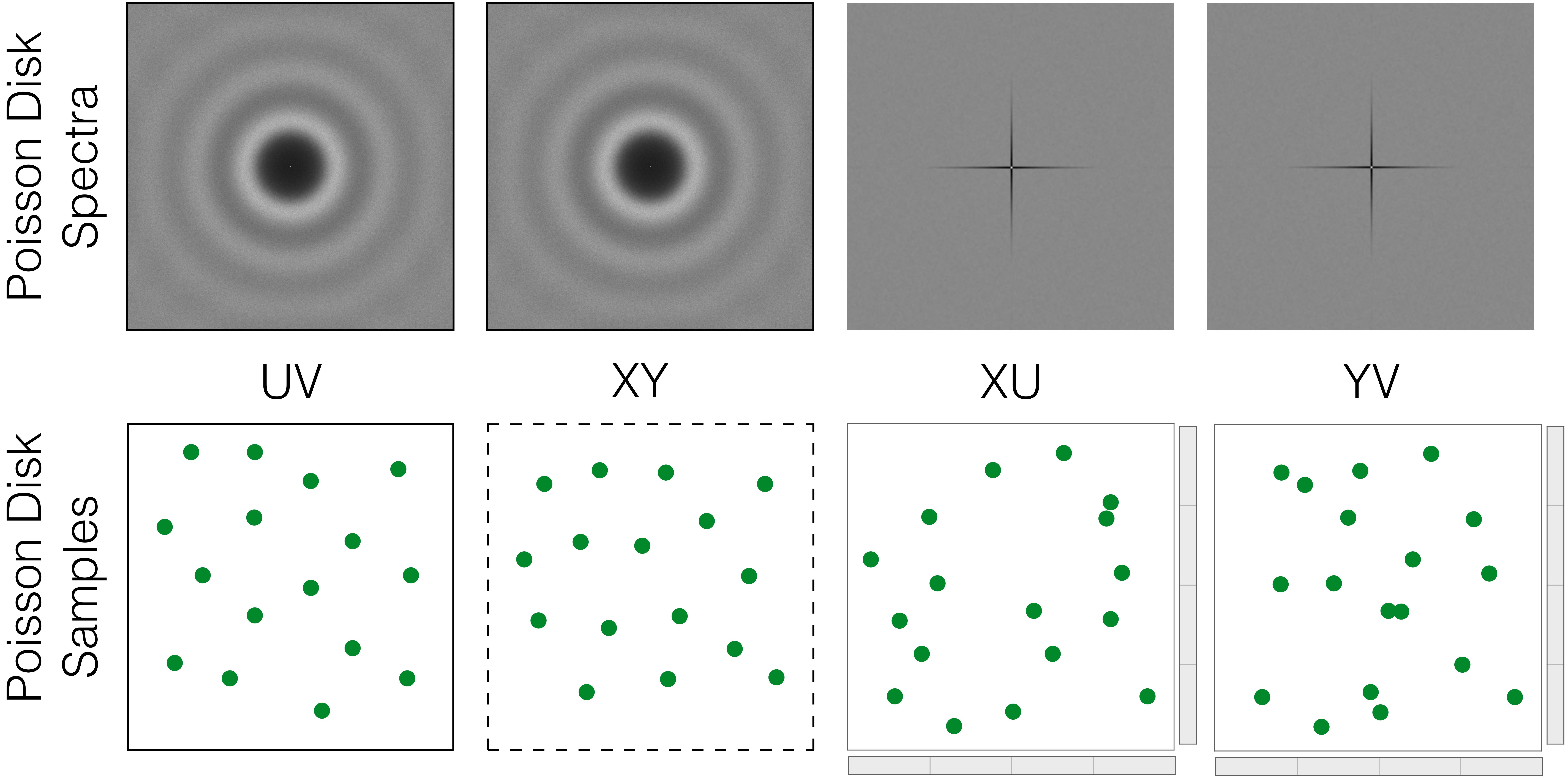
4D Sampling Spectra along Projections



4D Sampling Spectra along Projections



4D Sampling Spectra along Projections

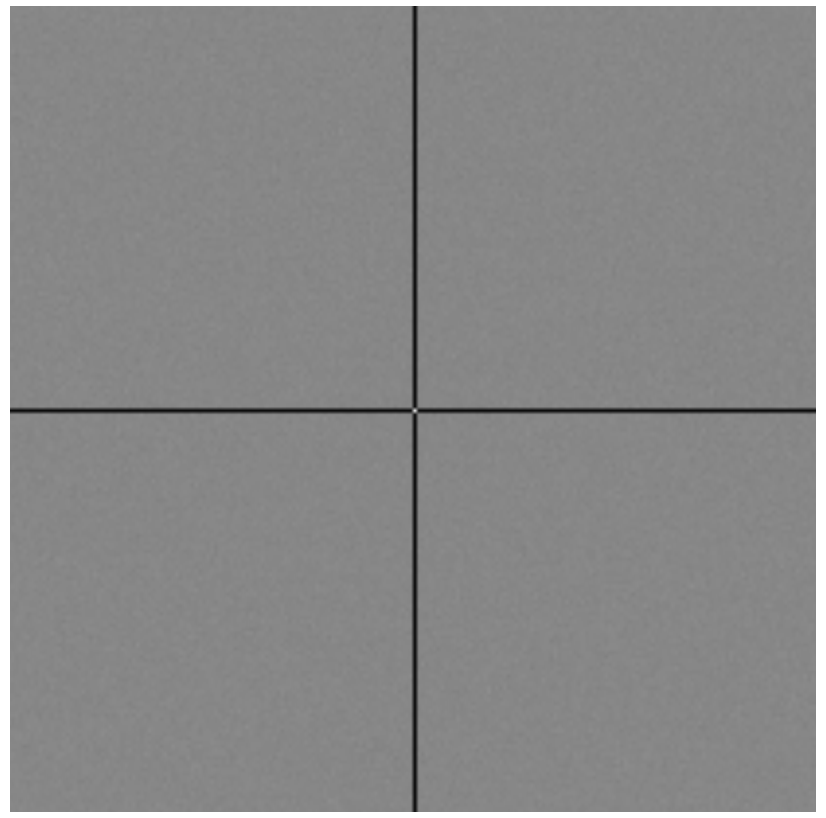


How can we perform Convergence Analysis for Anisotropic Sampling Spectra ?

Variance Formulation for Anisotropic Sampling Spectra

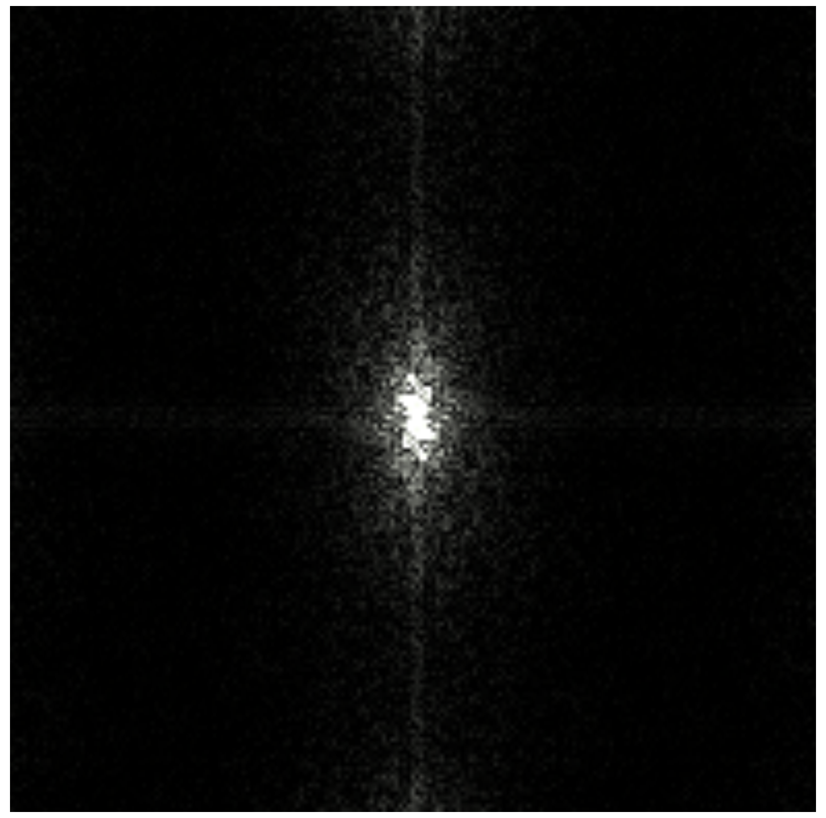
$$\mathrm{Var}(I_N) = \int_\Omega \langle \mathcal{P}_{S_N}(\nu) \rangle \times \mathcal{P}_f(\nu) \, d\nu$$

$\langle \mathcal{P}_{S_N}(\nu) \rangle$



N-rooks spectrum

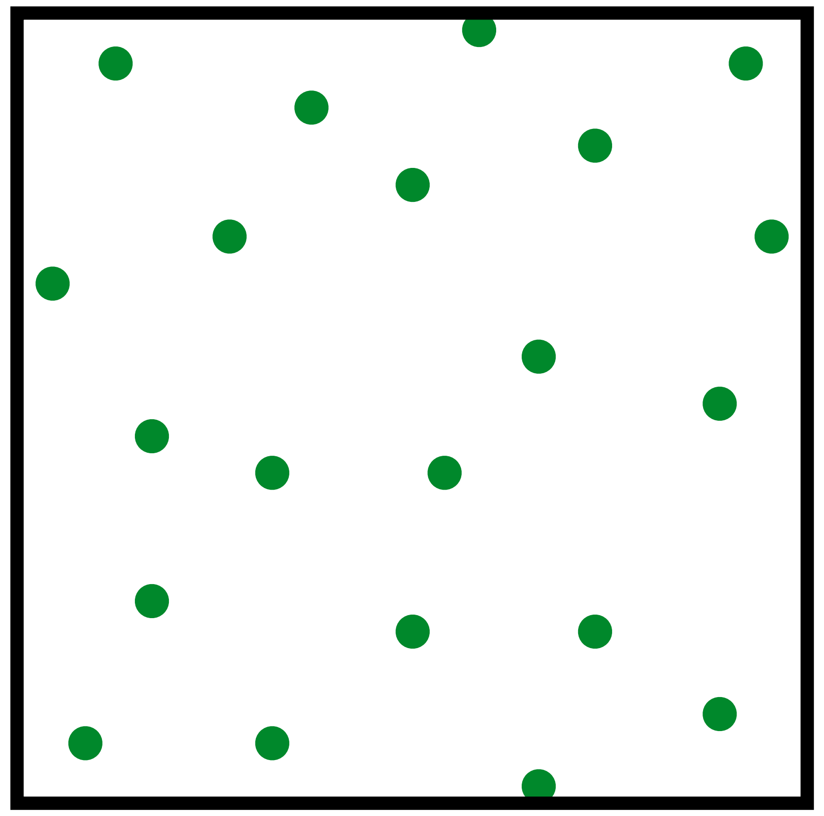
$\mathcal{P}_f(\nu)$



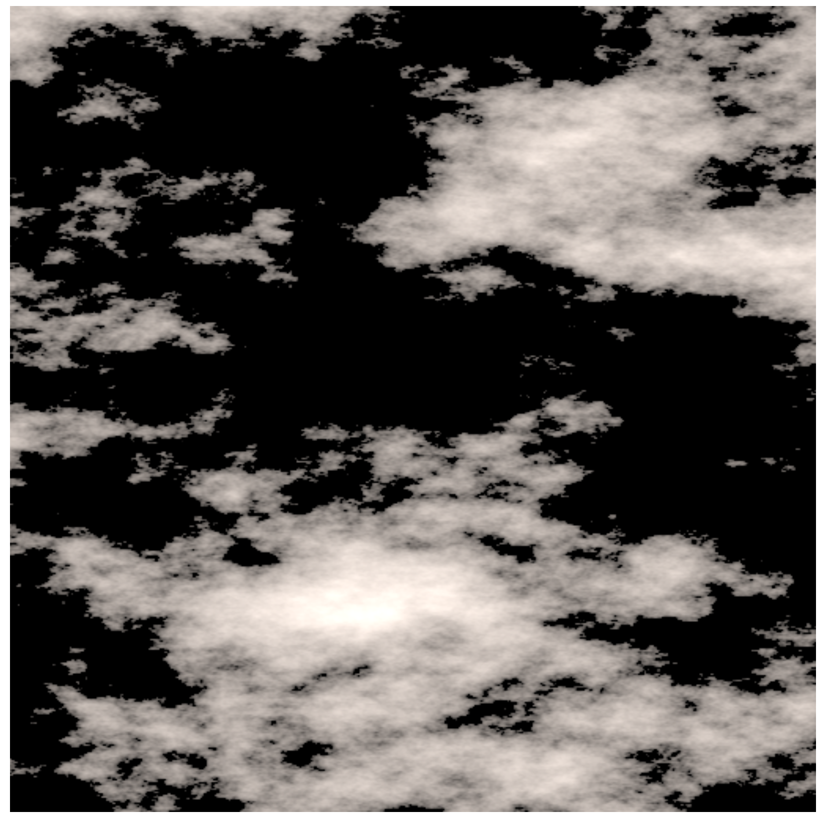
Integrand spectrum

$S_N(\vec{x})$

N-rooks



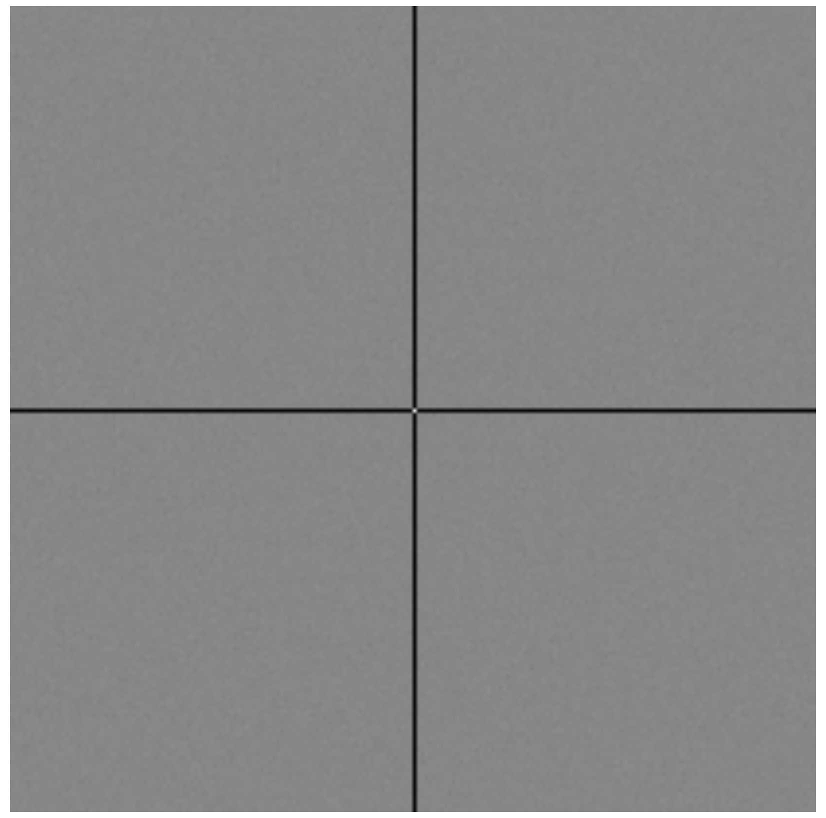
$f(\vec{x})$



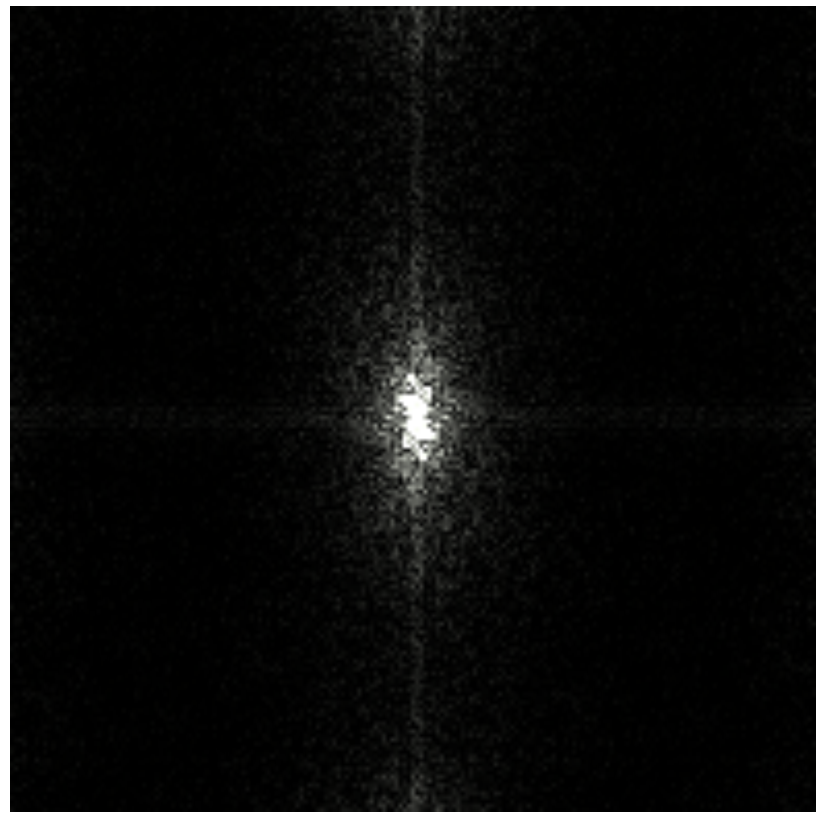
Variance Formulation for Anisotropic Sampling Spectra

$\left\langle \mathcal{P}_{S_N}(\rho\mathbf{n}) \right\rangle$ $\qquad$ $\mathcal{P}_f(\rho\mathbf{n})$

$$\mathrm{Var}(I_N) = \int_0^{\infty} \rho^{d-1} \int_{\mathcal{S}^{d-1}}$$



$$\times$$

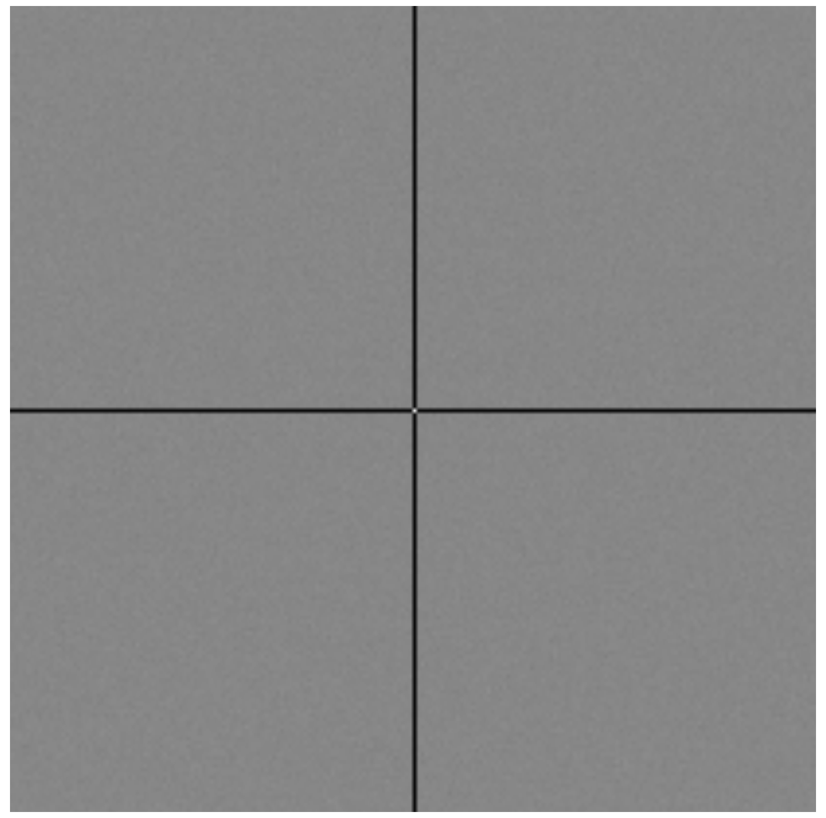


$$d\mathbf{n}\, d\rho$$

Variance Formulation for Anisotropic Sampling Spectra

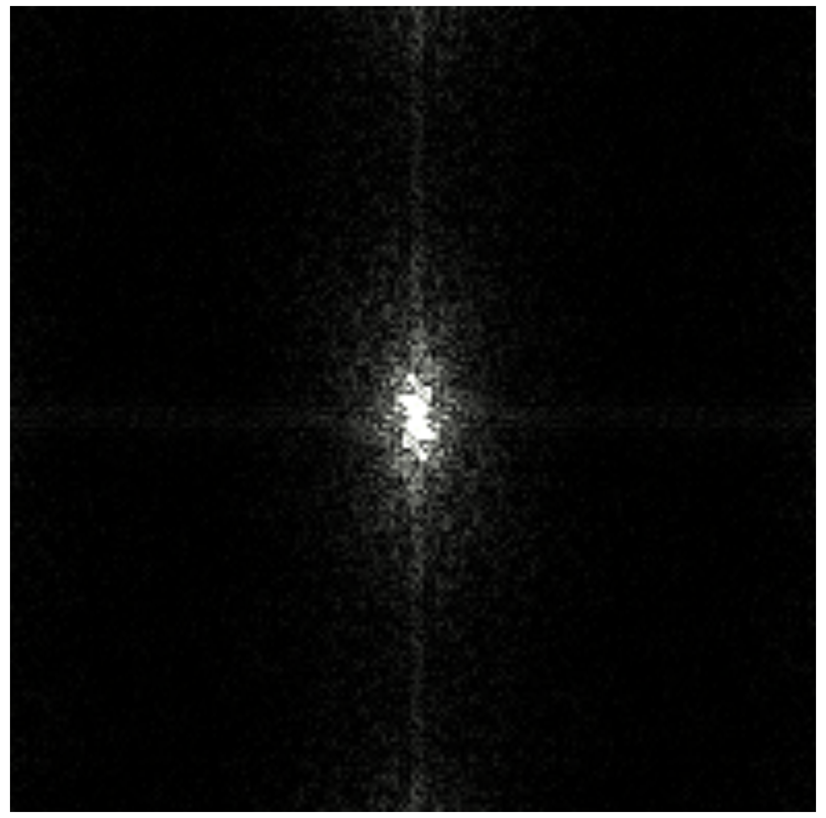
$$\mathrm{Var}(I_N) = \int_{\mathcal{S}^{d-1}} \int_0^\infty \rho^{d-1}$$

$\left\langle \mathcal{P}_{S_N}(\rho\mathbf{n}) \right\rangle$



$\times$

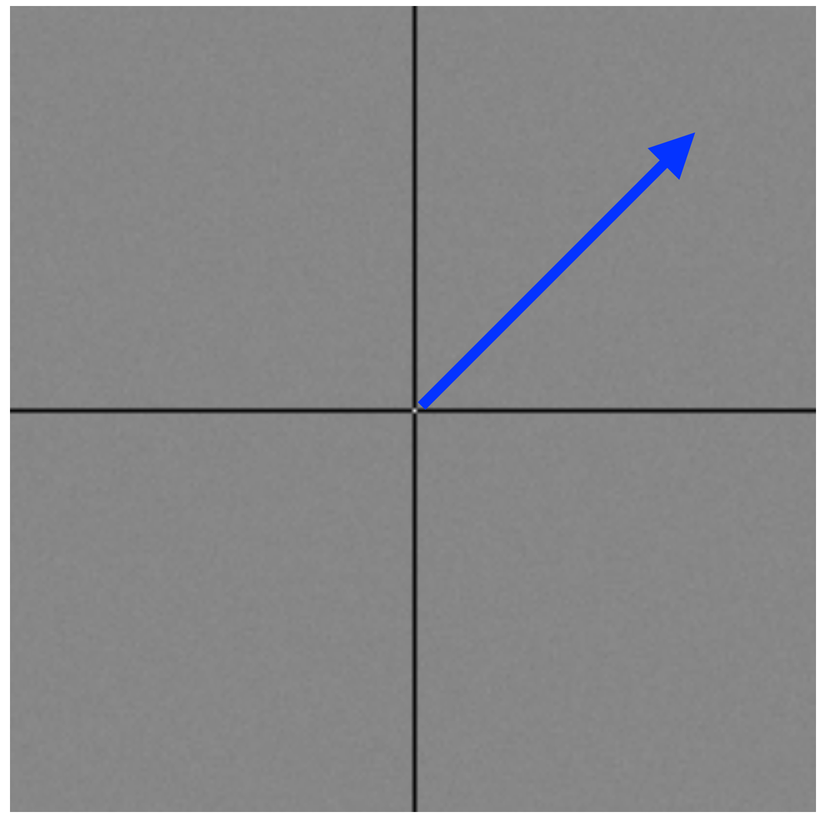
$\mathcal{P}_f(\rho\mathbf{n})$

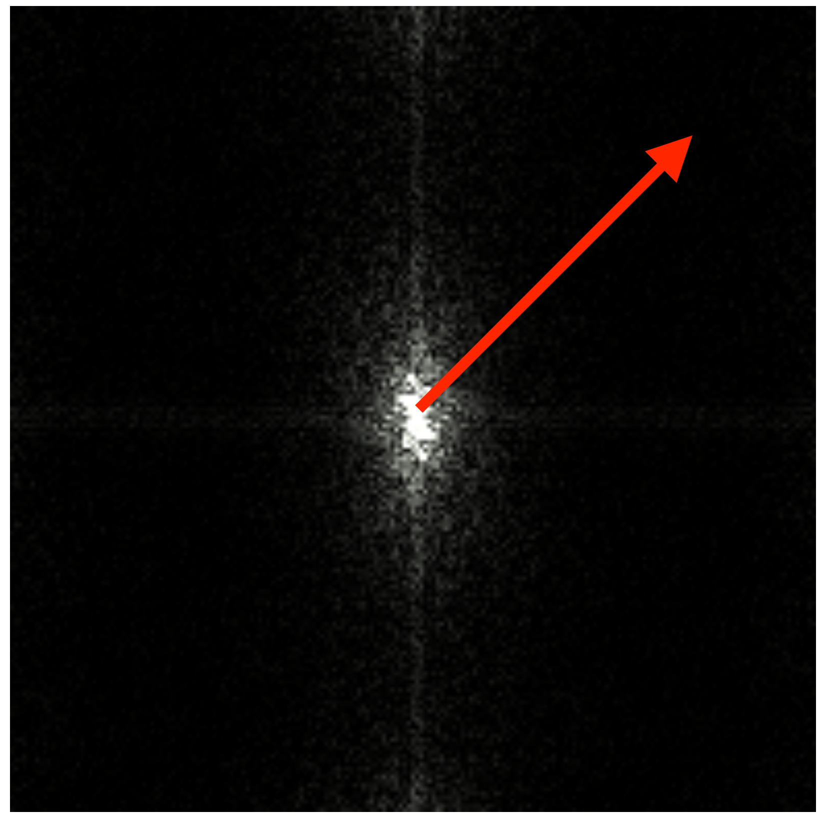


$d\rho\, d\mathbf{n}$

Variance Formulation for Anisotropic Sampling Spectra

$$\mathrm{Var}(I_N) = \int_{\mathcal{S}^{d-1}} \int_0^{\infty} \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n}_{\mathbf{k}}) \right\rangle \times \mathcal{P}_f(\rho_k \mathbf{n}_k)\, d\rho\, d\mathbf{n}$$

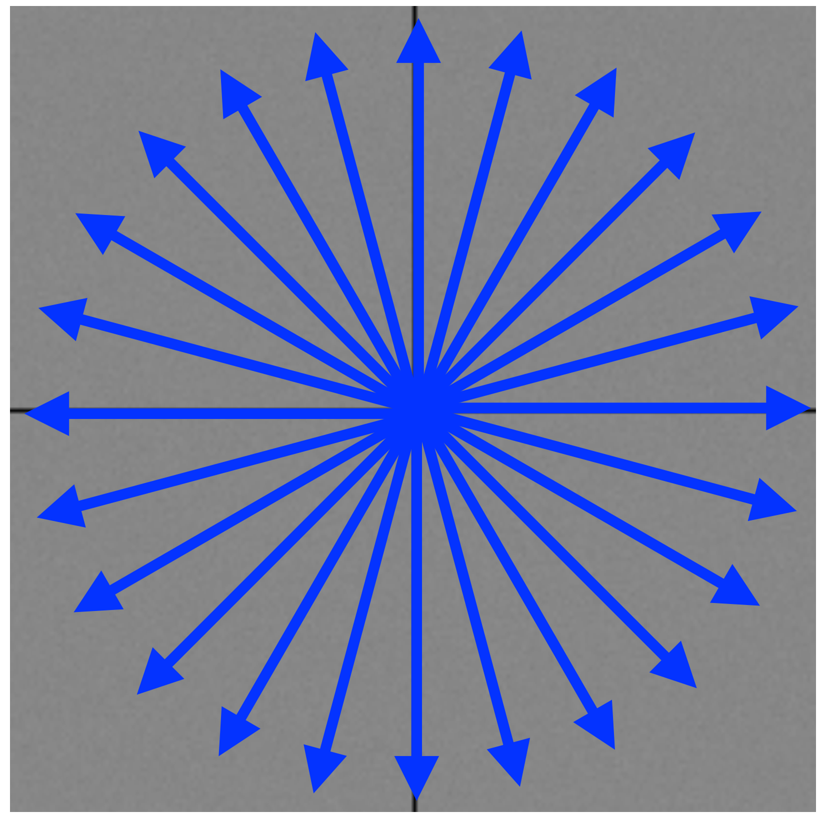




Variance Formulation for Anisotropic Sampling Spectra

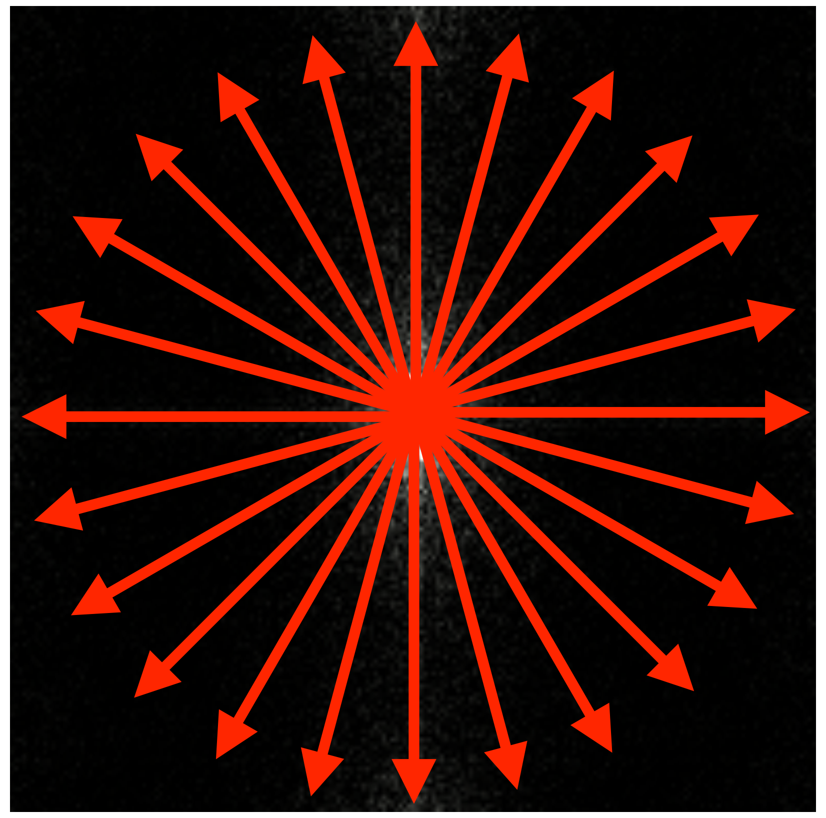
$$\mathrm{Var}(I_N) = \int_{\mathcal{S}^{d-1}} \int_0^\infty \rho^{d-1}$$

$\left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle$



$\times$

$\mathcal{P}_f(\rho_k \mathbf{n}_k)$

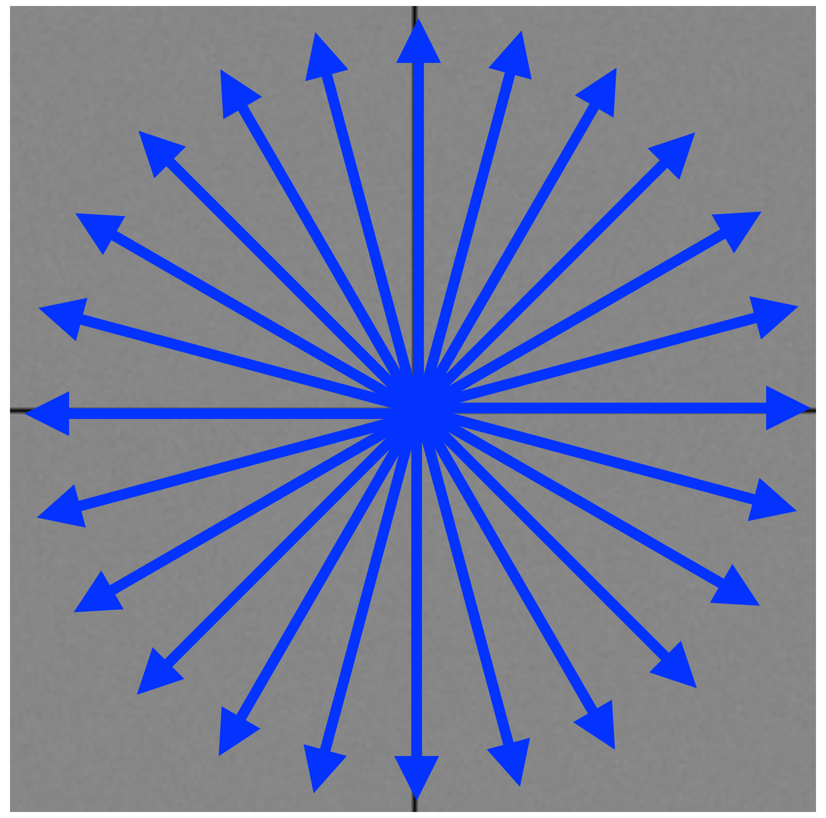


$$d\rho \, d\mathbf{n}$$

Variance Formulation for Anisotropic Sampling Spectra

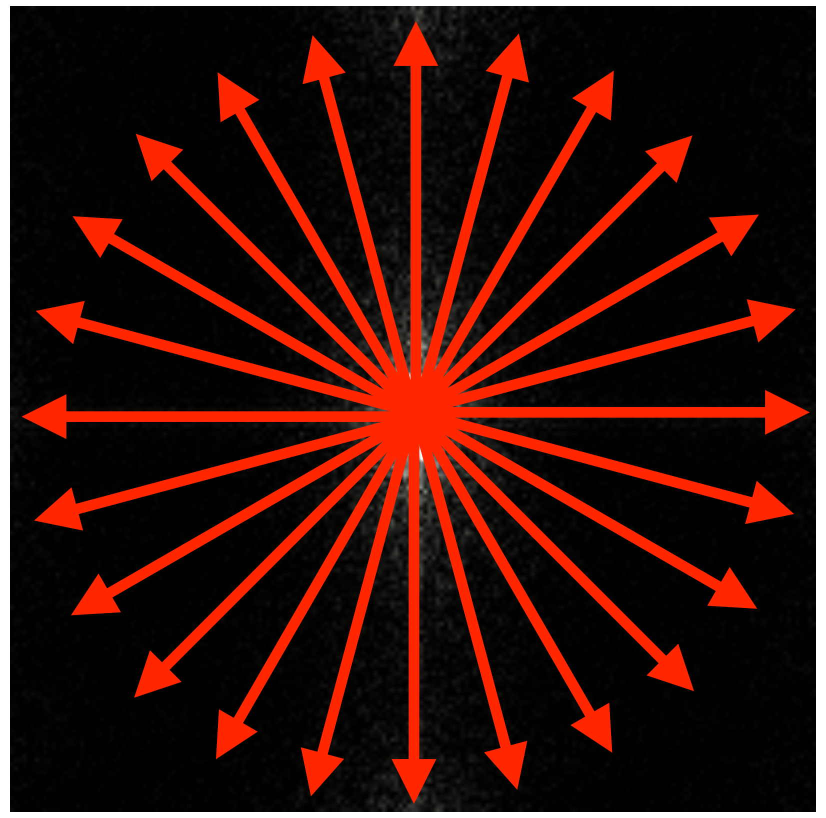
$$\mathrm{Var}(I_N) = \lim_{m\to\infty} \sum_{k=1}^{m} \int_0^{\infty} \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle \times \mathcal{P}_f(\rho_k \mathbf{n}_k) \, d\rho \; \Delta\mathbf{n}_k$$

$\left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle$



$\times$

$\mathcal{P}_f(\rho_k \mathbf{n}_k)$



Variance Formulation for Anisotropic Sampling Spectra

$$\mathrm{Var}(I_N) = \lim_{m\to\infty} \sum_{k=1}^{m} \int_0^{\infty} \rho^{d-1} \quad \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle \quad \times \quad \mathcal{P}_f(\rho_k \mathbf{n}_k) \quad d\rho\, \Delta\mathbf{n}_k$$

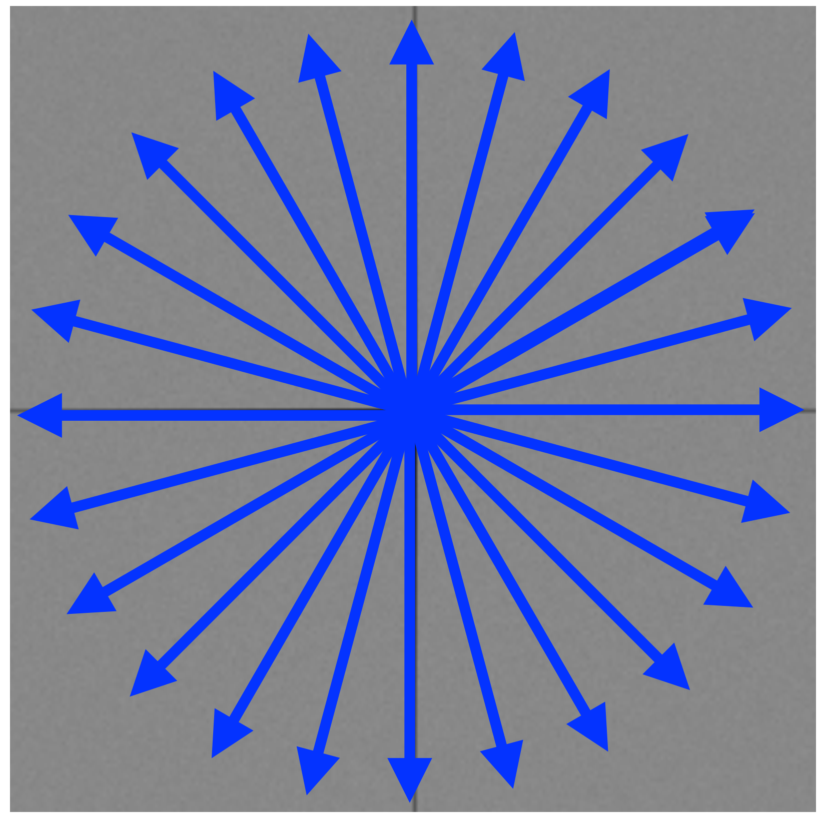
Variance Formulation for Anisotropic Sampling Spectra

$$\mathrm{Var}(I_N) = \lim_{m\to\infty} \sum_{k=1}^{m} \int_0^{\infty} \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle \mathcal{P}_f(\rho_k \mathbf{n}_k)\, d\rho\, \Delta \mathbf{n}_k$$

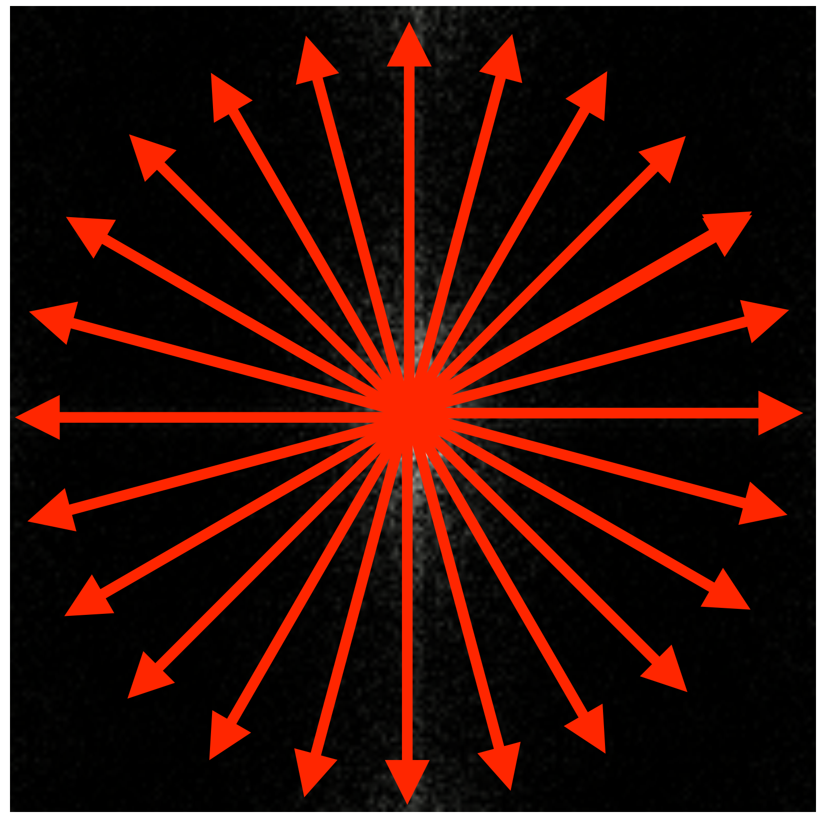
Variance Formulation for Anisotropic Sampling Spectra

$$\mathrm{Var}(I_N) = \lim_{m\to\infty} \sum_{k=1}^{m} \boxed{\int_0^\infty \rho^{d-1} \left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle \mathcal{P}_f(\rho_k \mathbf{n}_k)\, d\rho} \, \Delta\mathbf{n}_k$$

$\left\langle \mathcal{P}_{S_N}(\rho_k \mathbf{n_k}) \right\rangle$

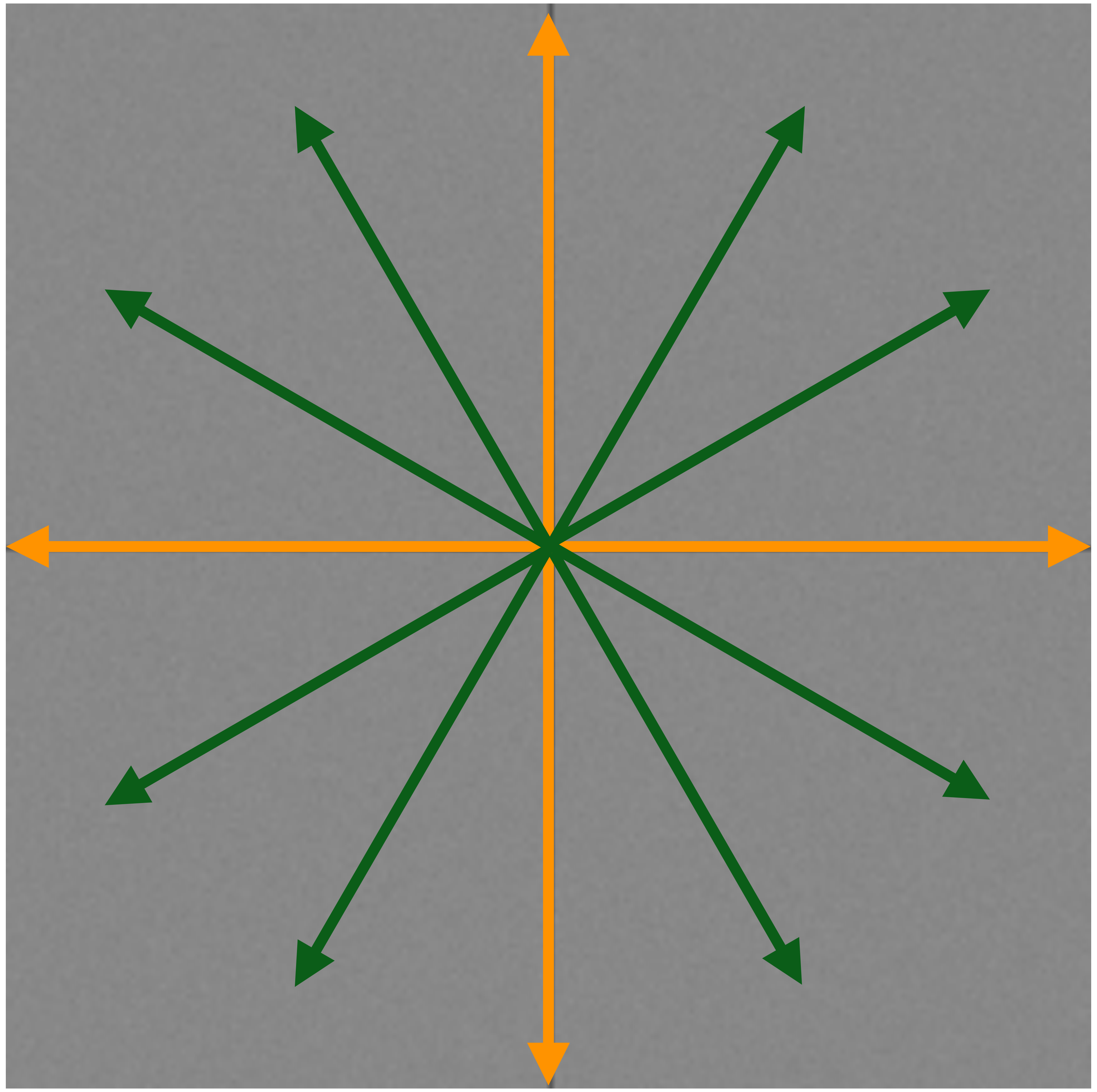


$\mathcal{P}_f(\rho_k \mathbf{n}_k)$

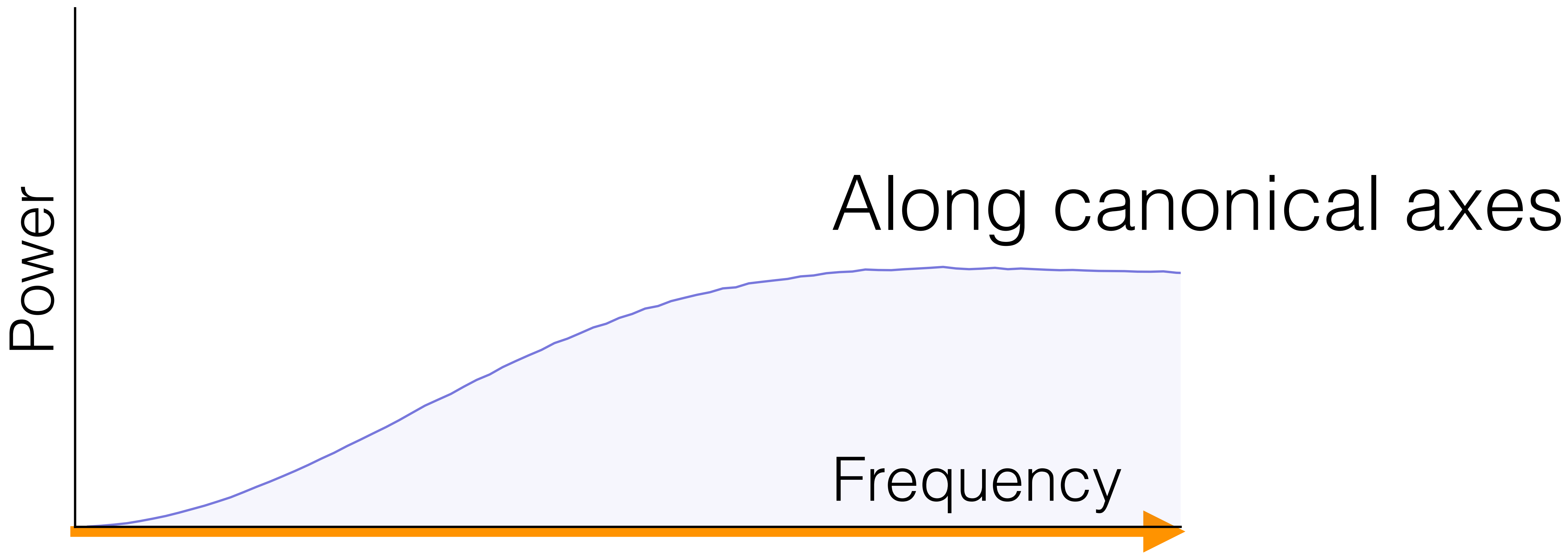


Convergence Analysis for Anisotropic Sampling Spectra

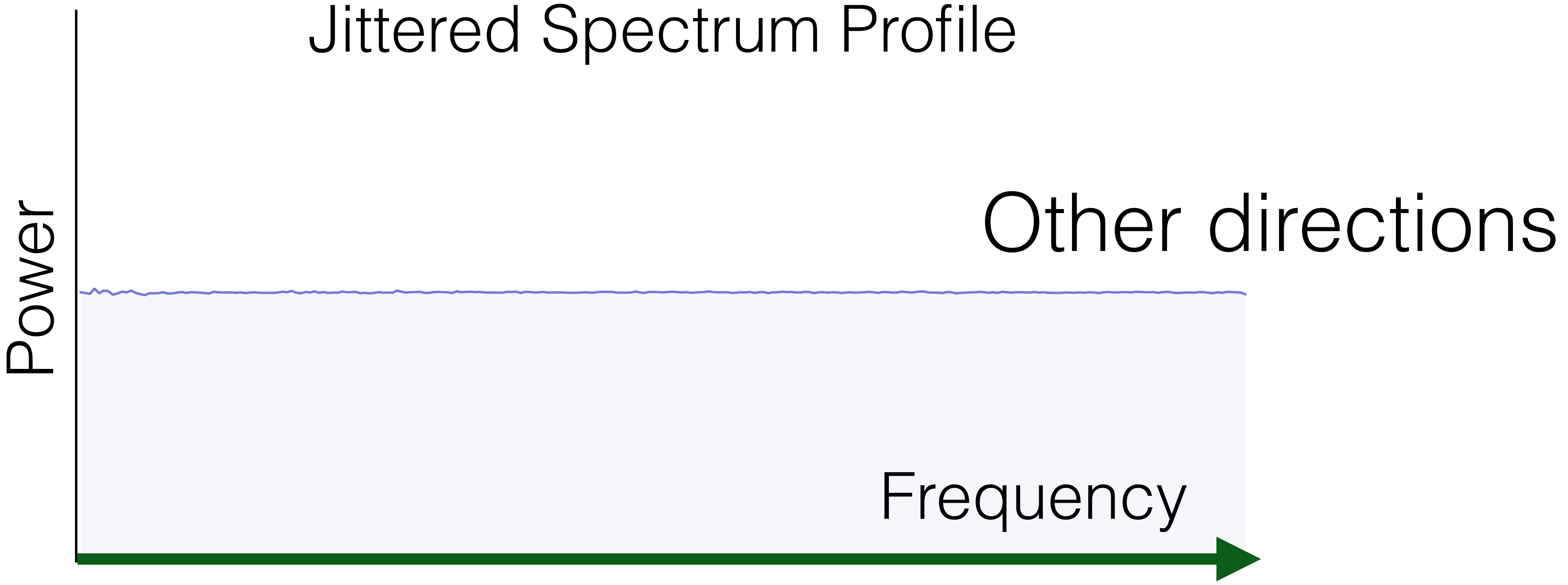
Power Spectrum



Radial Power Spectrum



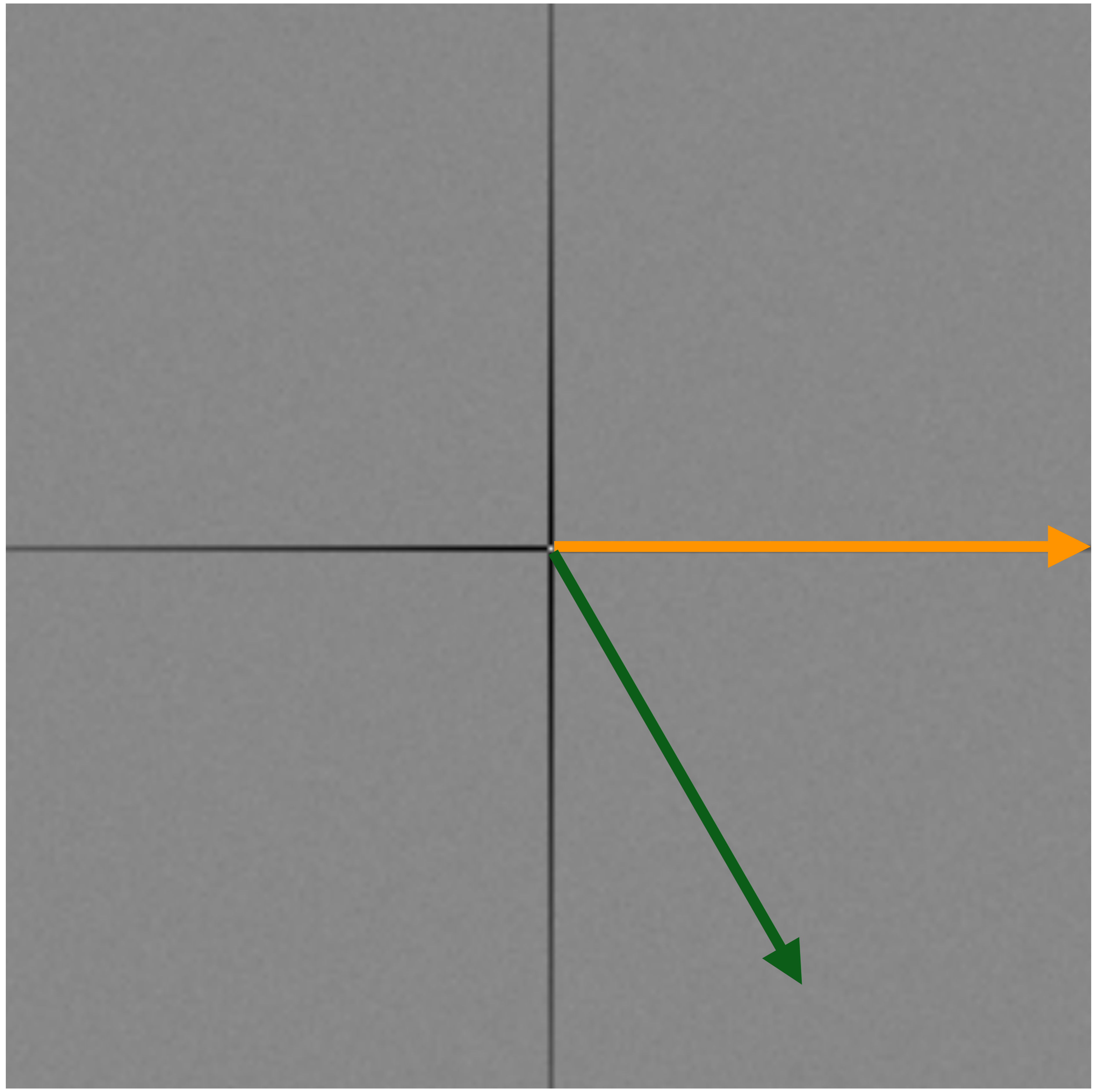
Jittered Spectrum Profile



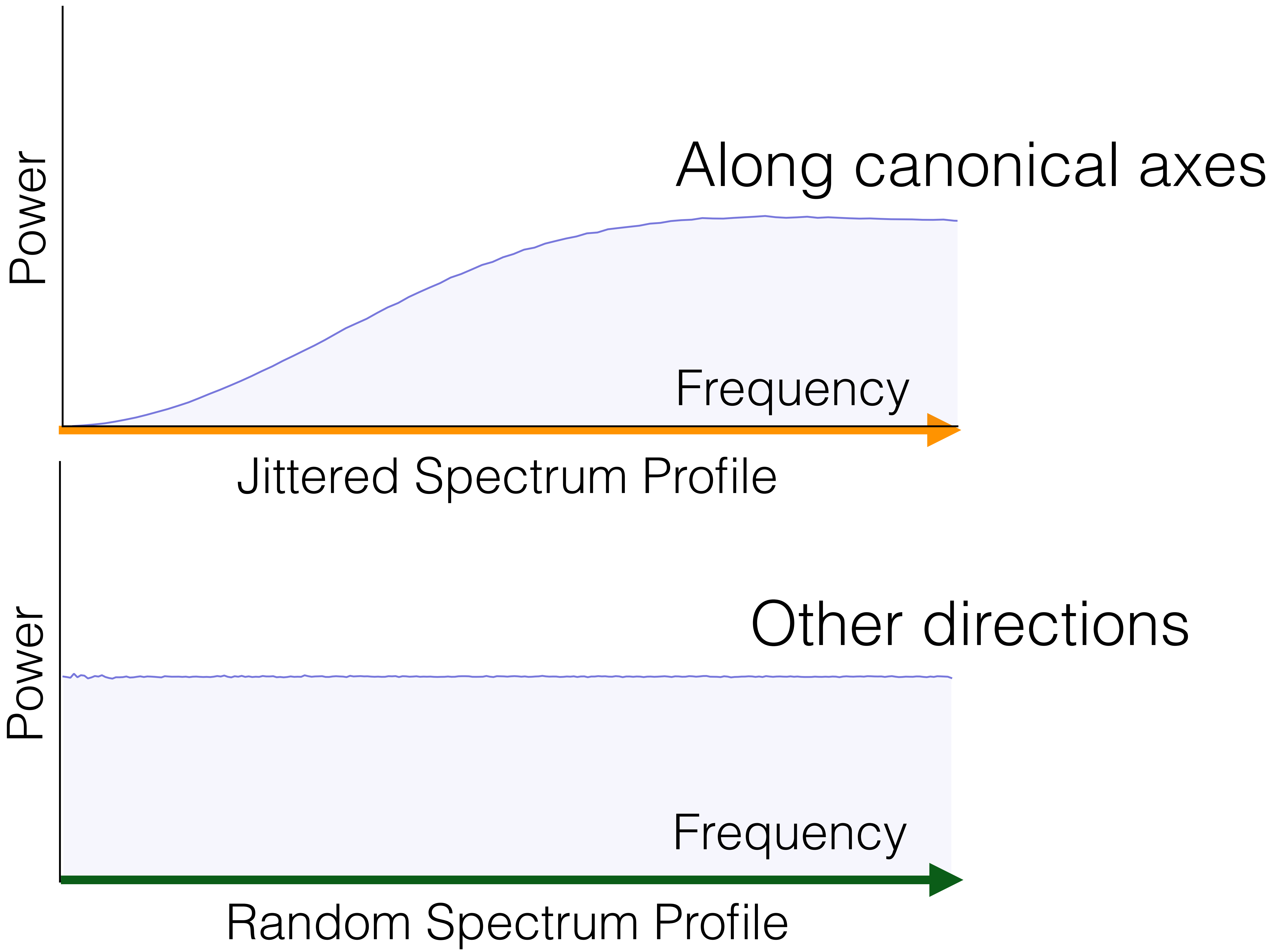
Random Spectrum Profile

Convergence Analysis for Anisotropic Sampling Spectra

Power Spectrum



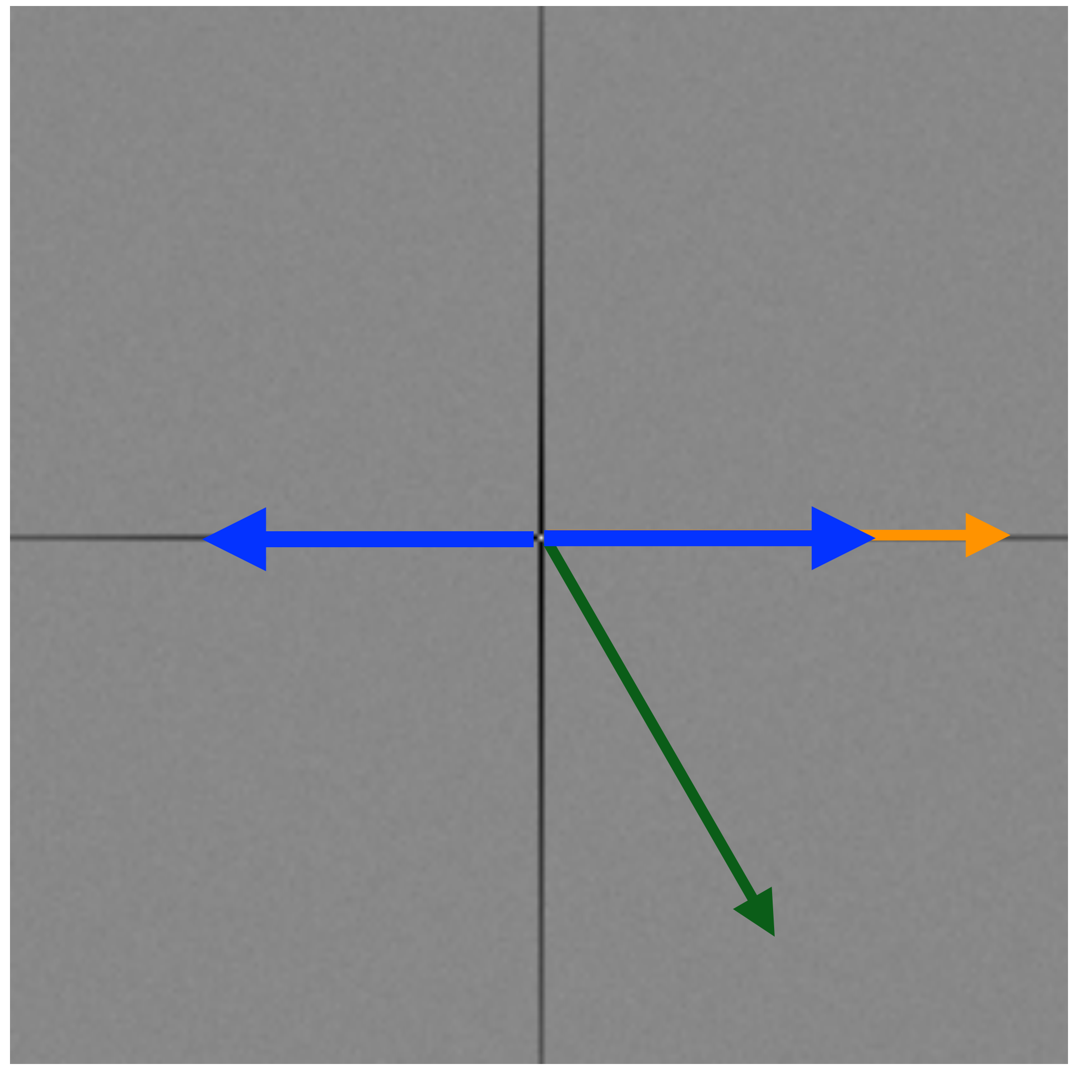
Radial Power Spectrum

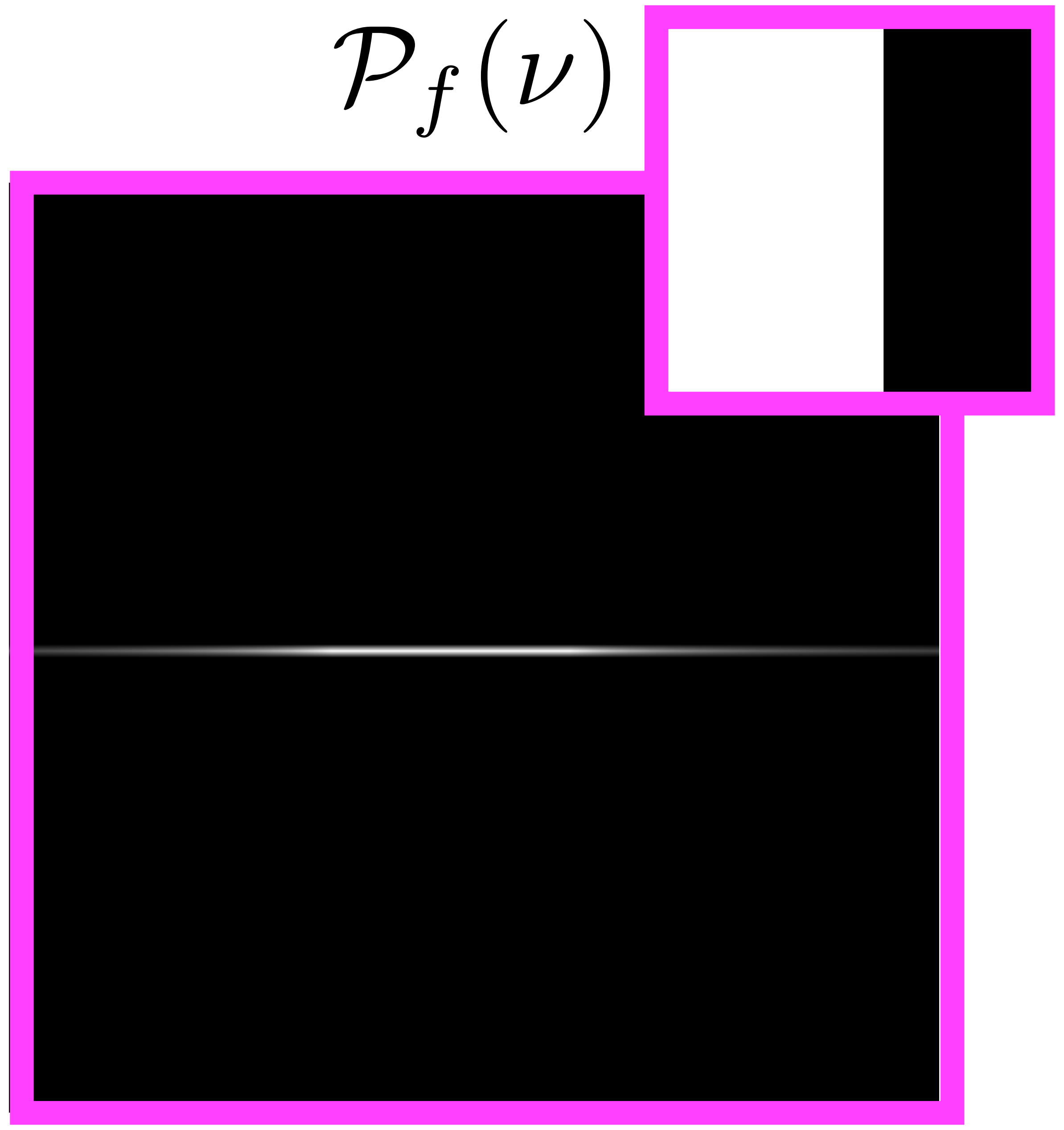


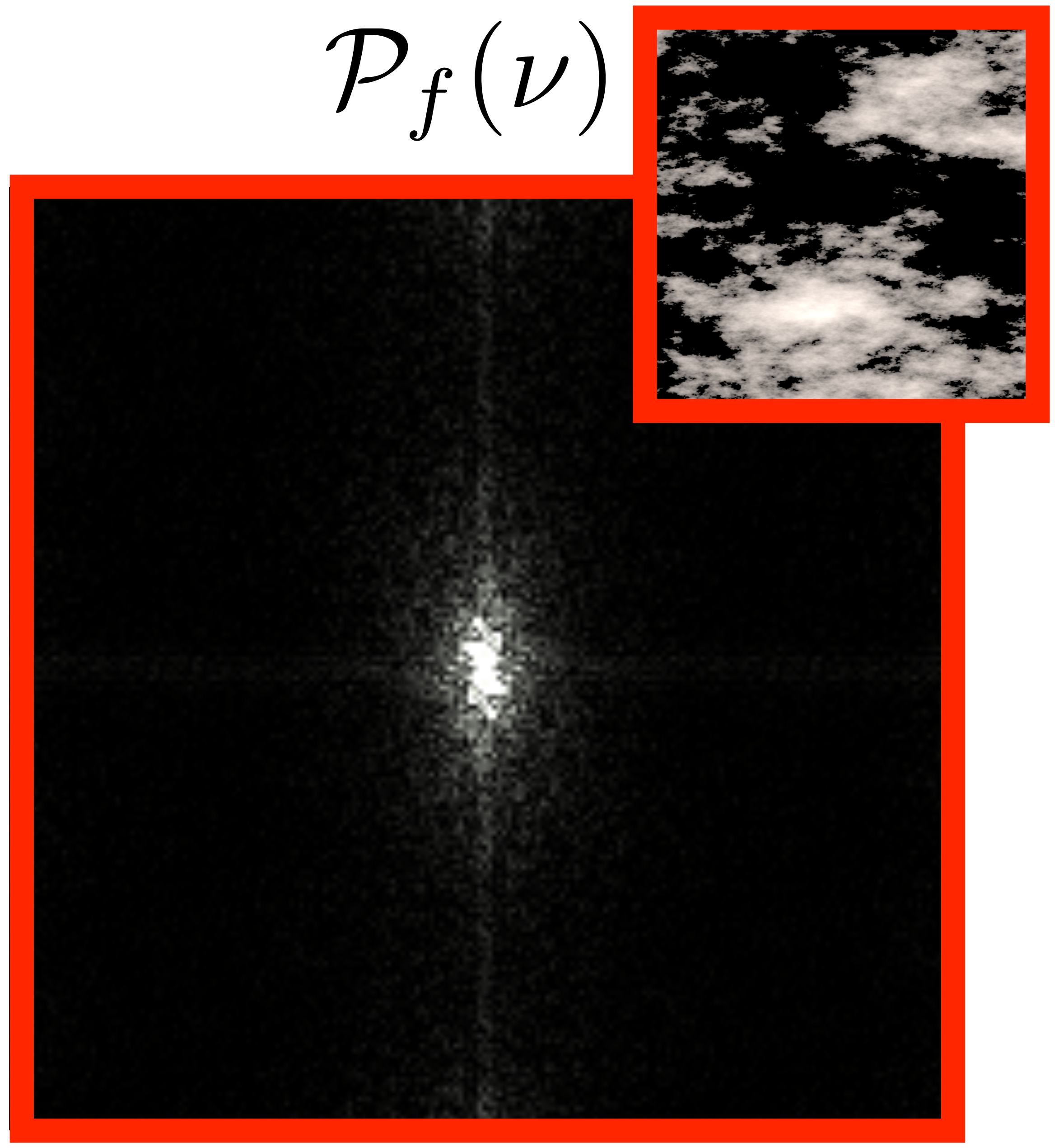
Convergence Analysis for Anisotropic Sampling Spectra

$\langle \mathcal{P}_{S_N}(\nu) \rangle$ $\quad$ $\mathcal{P}_f(\nu)$ $\quad$ $\mathcal{P}_f(\nu)$

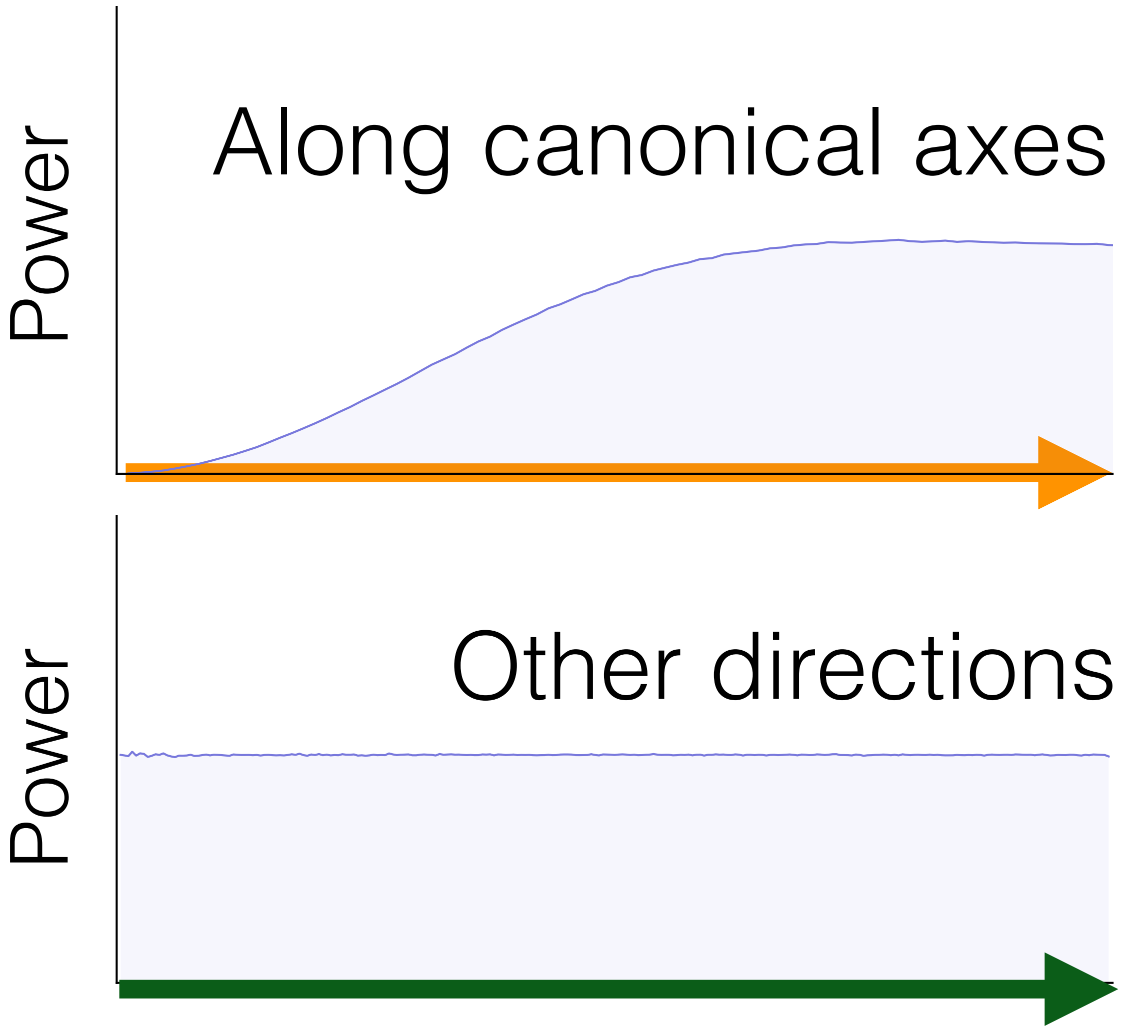
Spectrum







Radial Spectrum



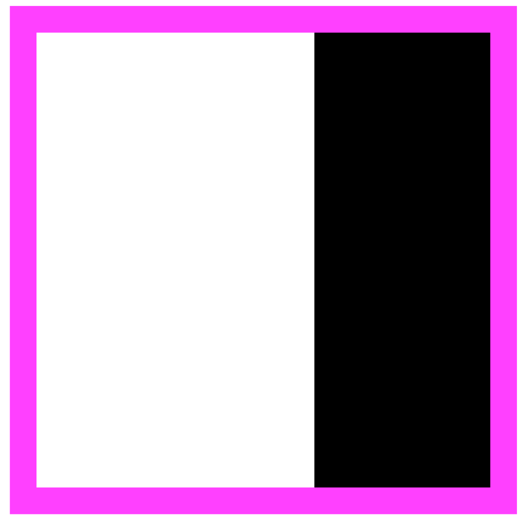
$O(N^{-2})$ $\quad$ $O(N^{-2})$

No Impact $\quad$ $O(N^{-1})$

$\left.\begin{matrix} O(N^{-2}) \\ O(N^{-1}) \end{matrix}\right\} O(N^{-1})$

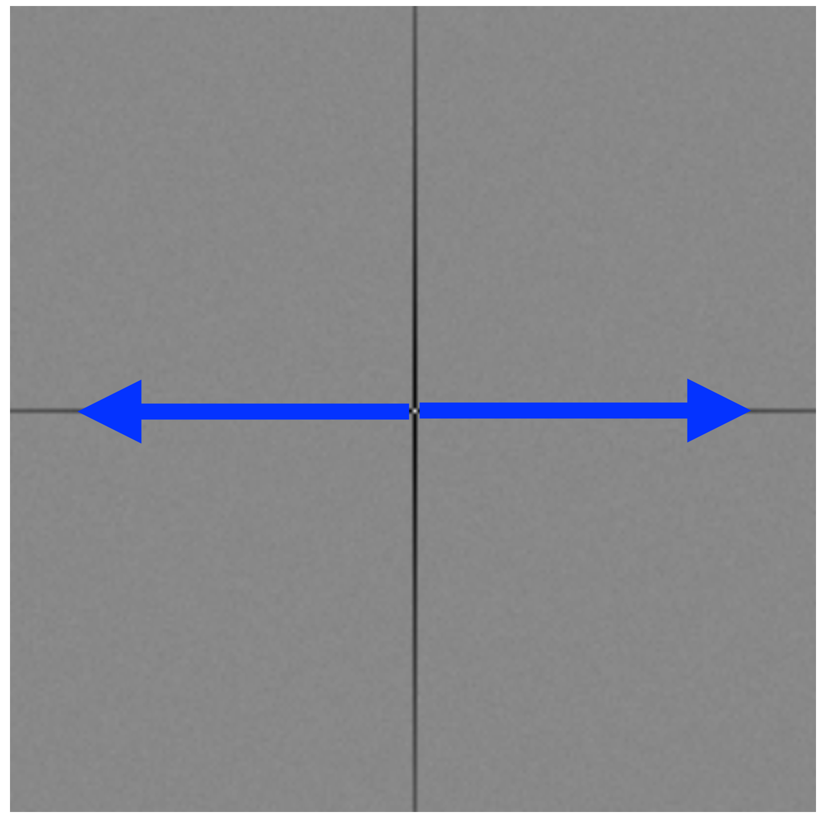
Variance due to N-rooks Sampler

$f(\vec{x})$



$\mathrm{Var}(I_N) = \int_\Omega$

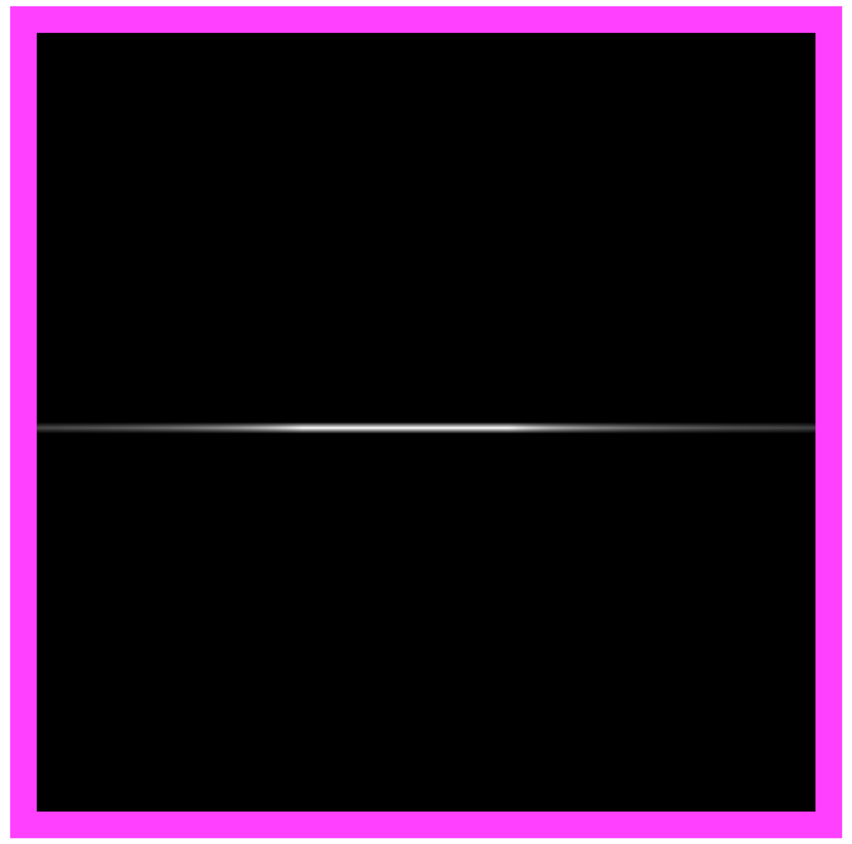
$\langle \mathcal{P}_{S_N}(\nu) \rangle$



N-rooks spectrum

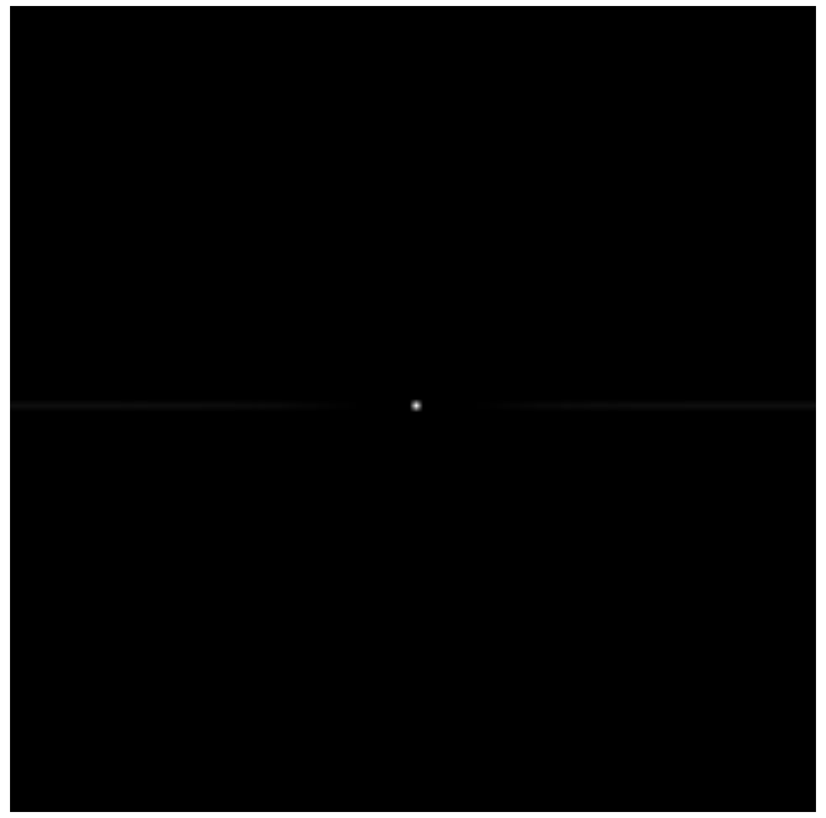
X

$\mathcal{P}_f(\nu)$



Integrand spectrum

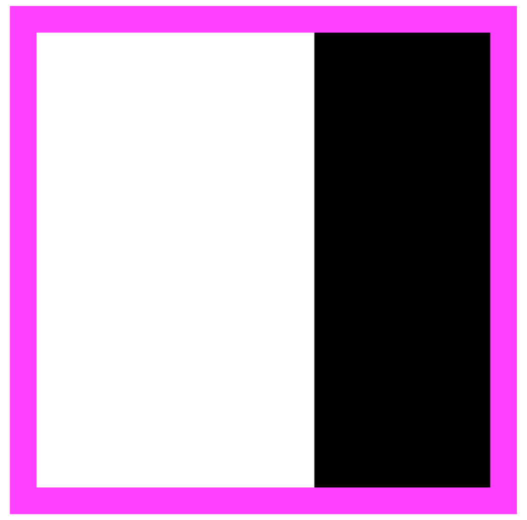
$d\nu = \int_\Omega$



$d\nu$

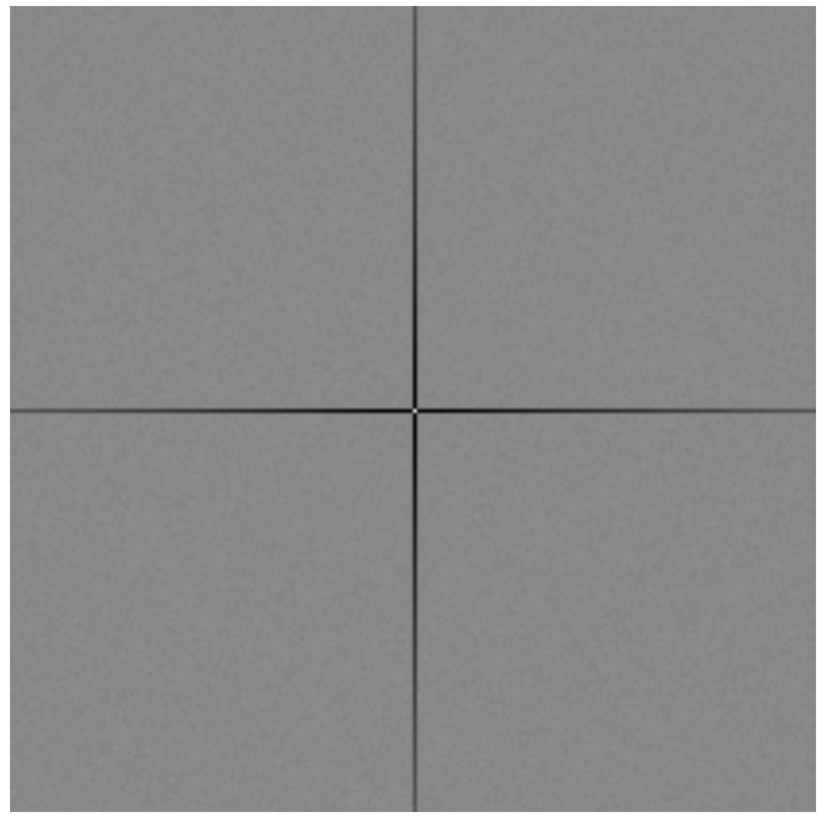
Variance due to N-rooks Sampler

$f(\vec{x})$



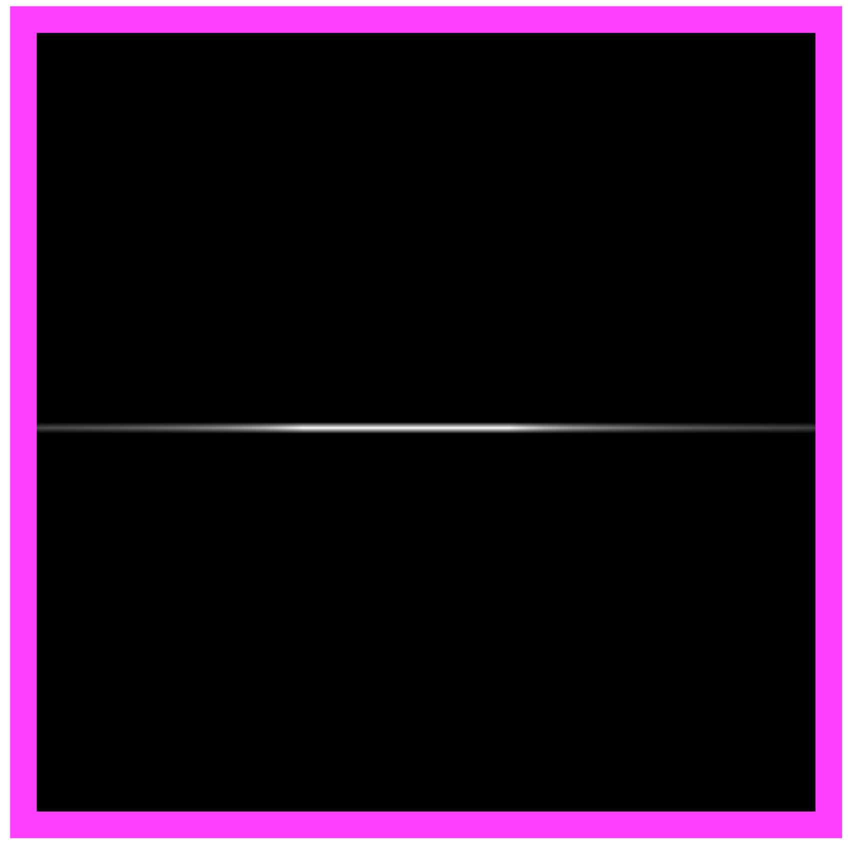
$\langle \mathcal{P}_{S_N}(\nu) \rangle$ $\quad$ $\mathcal{P}_f(\nu)$

$$\mathrm{Var}(I_N) = \int_\Omega$$



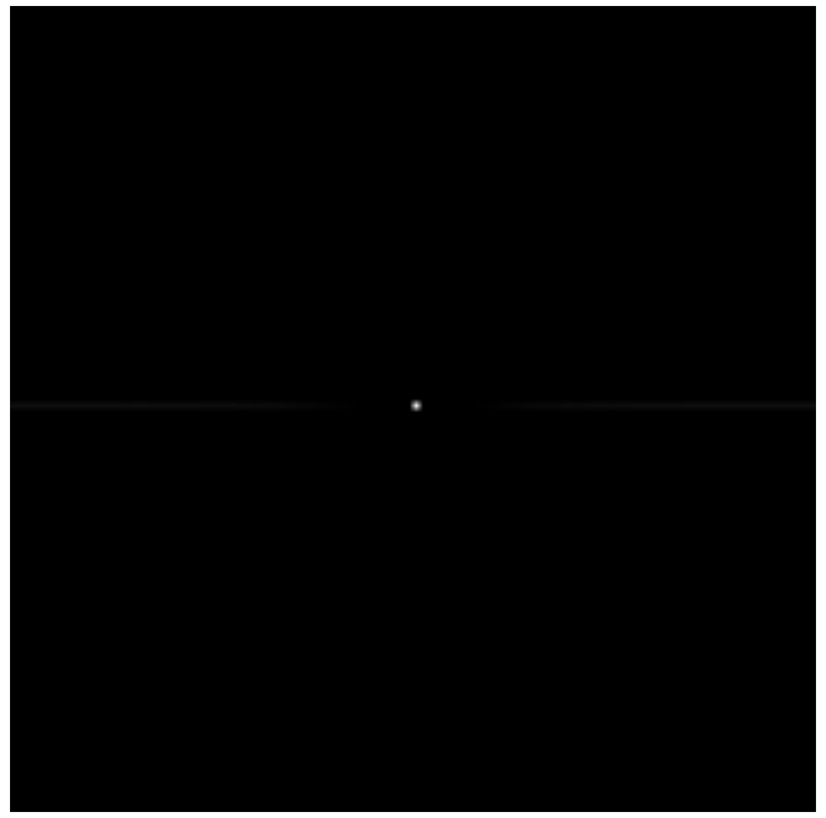
N-rooks spectrum

X



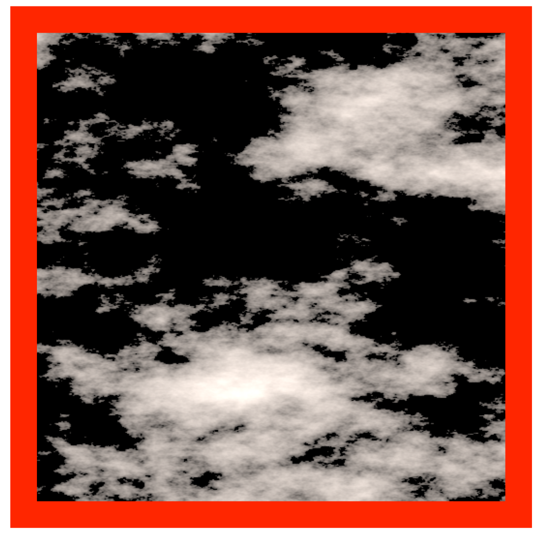
Integrand spectrum

$$d\nu = \int_\Omega$$

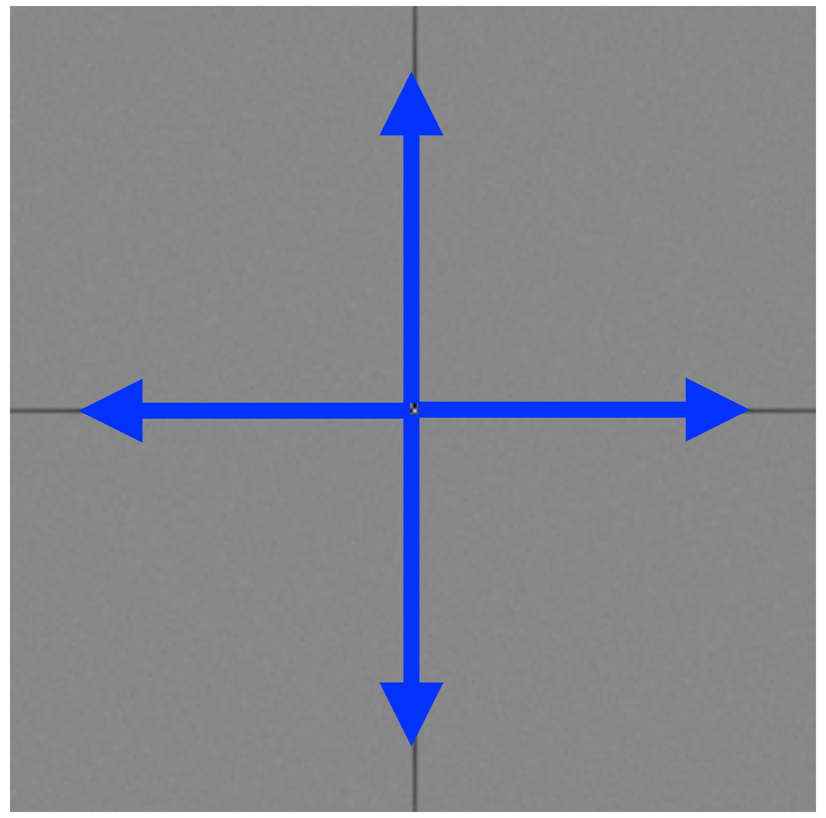


$$d\nu$$

$f(\vec{x})$

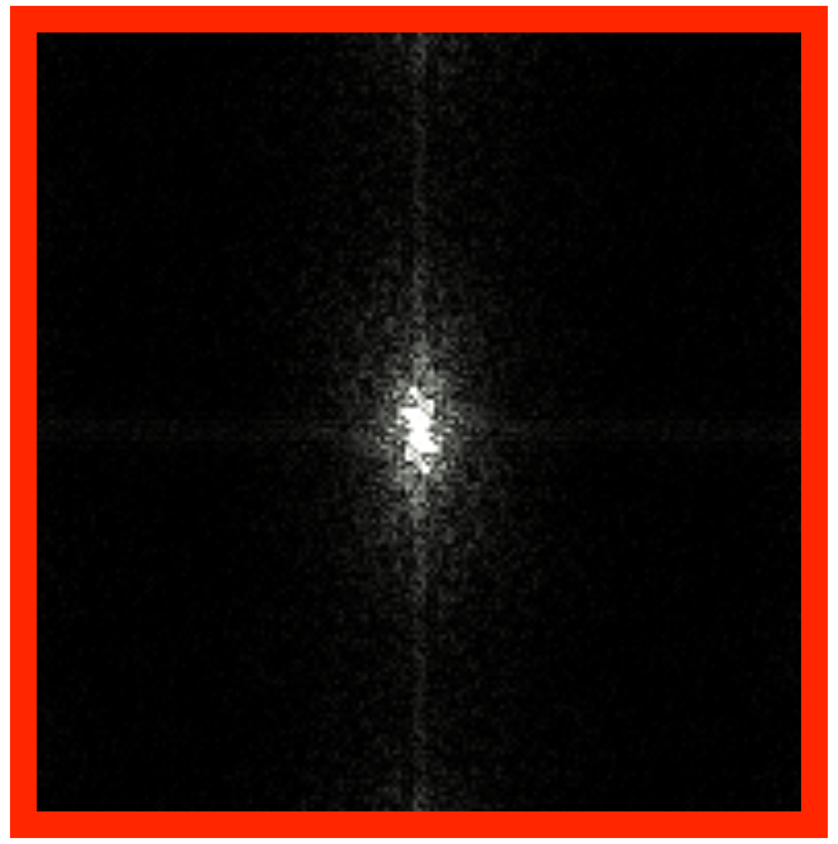


$$\mathrm{Var}(I_N) = \int_\Omega$$



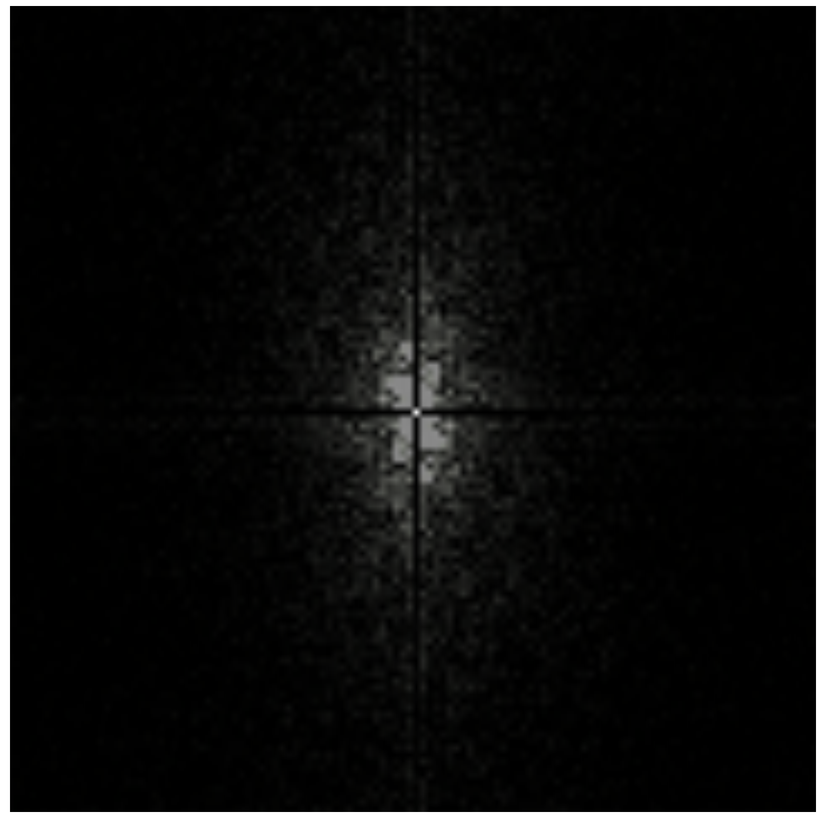
N-rooks spectrum

X



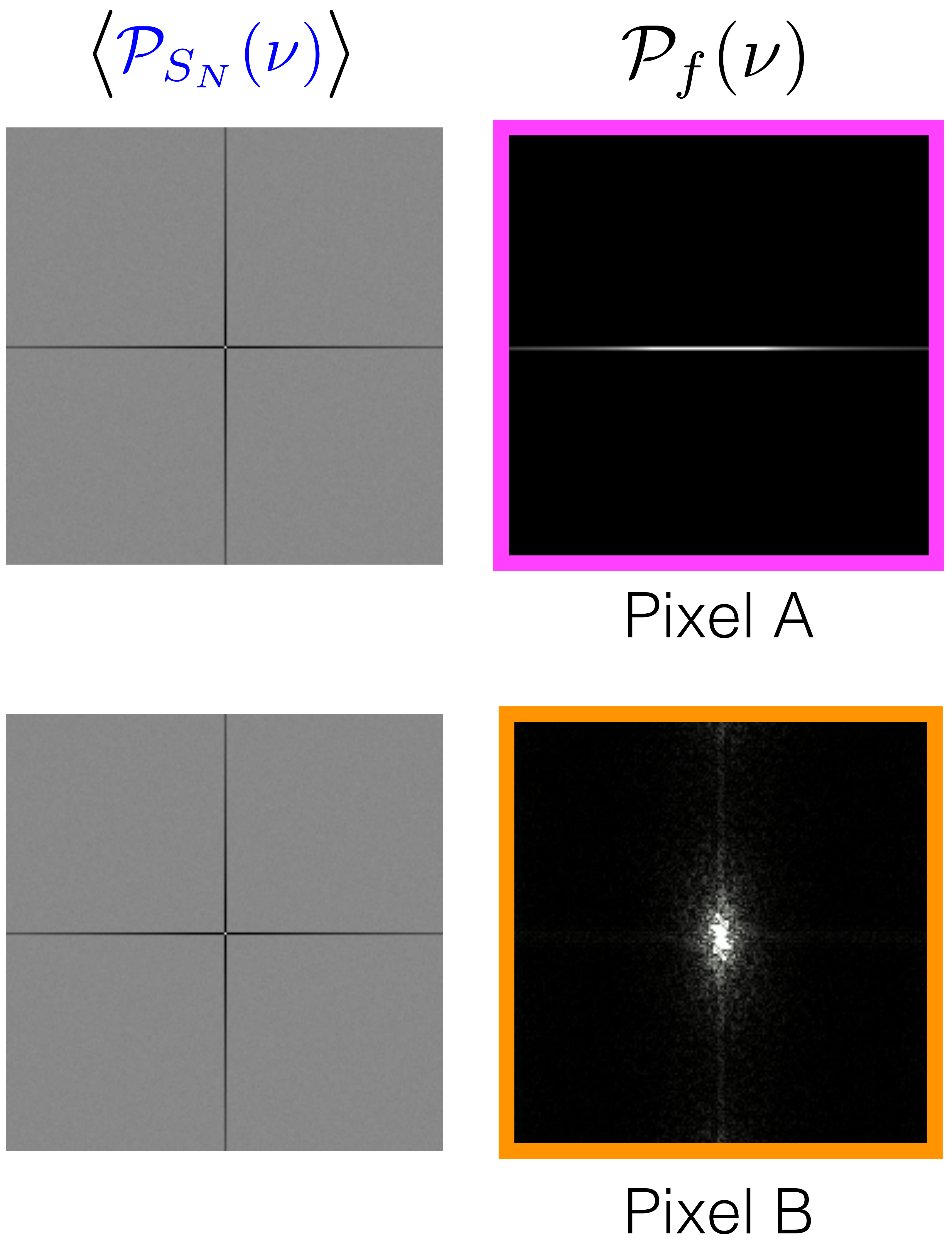
Integrand spectrum

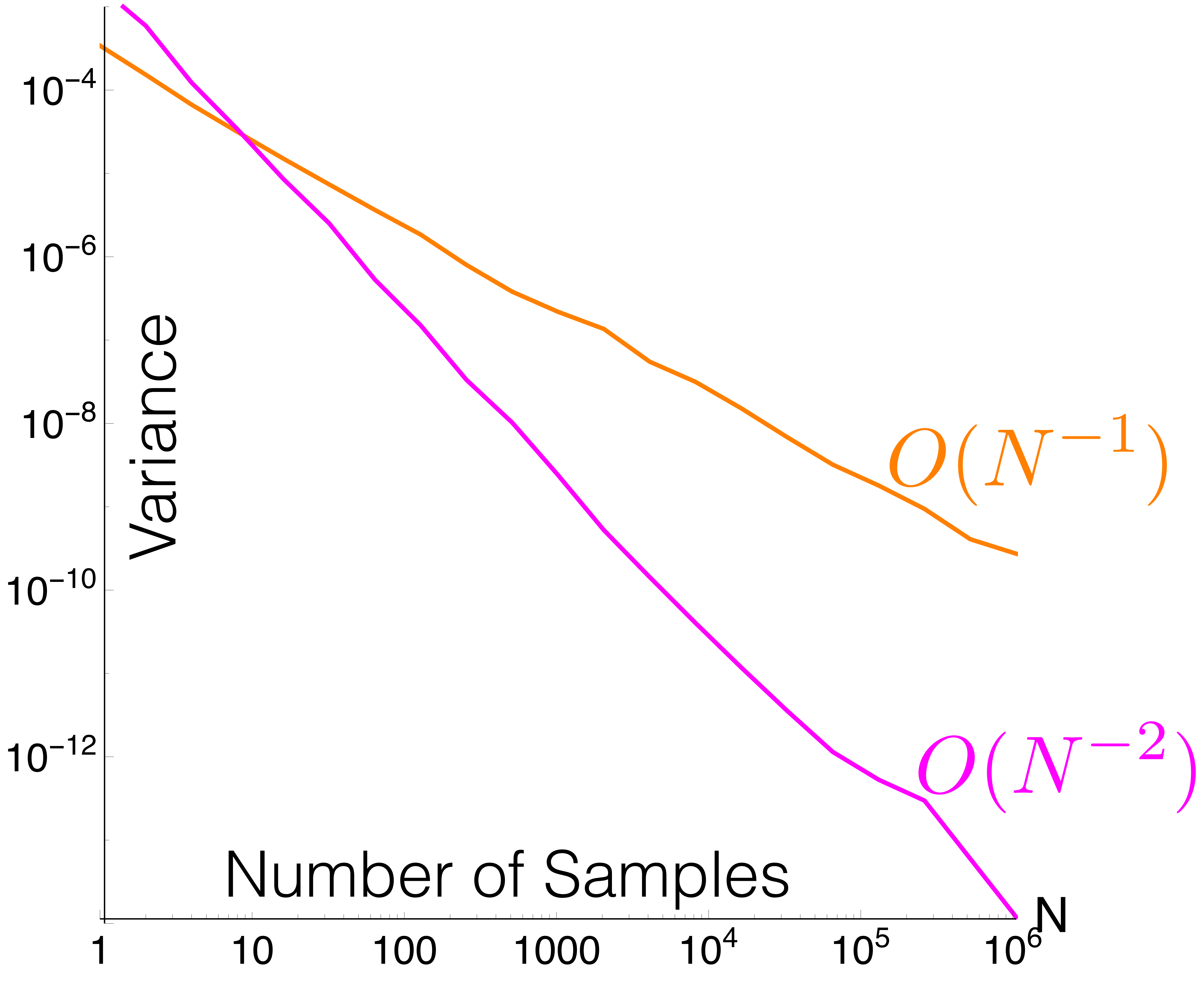
$$d\nu = \int_\Omega$$



$$d\nu$$

Variance Convergence of Latin Hypercube (N-rooks)



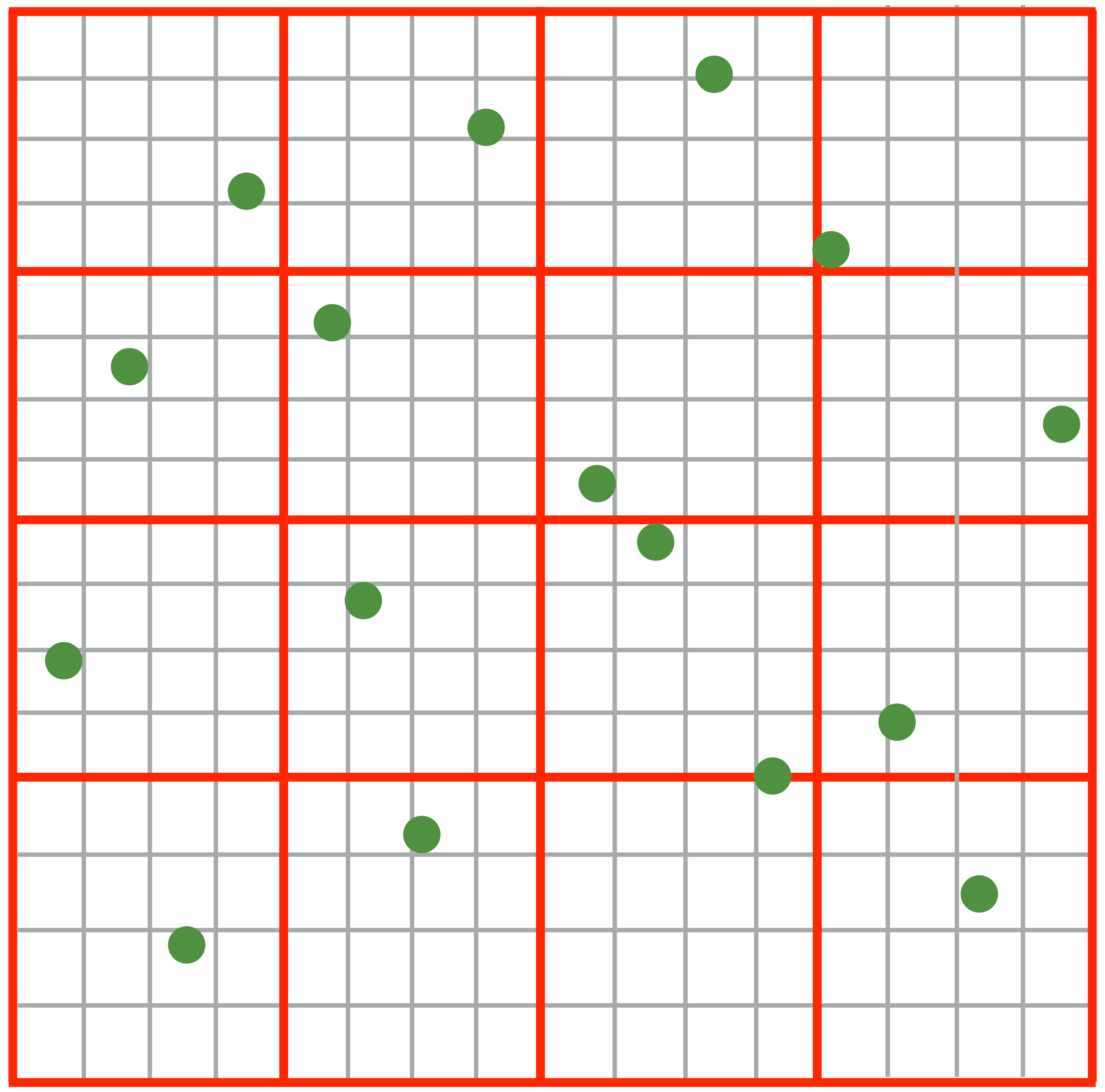


Non-Axis Aligned Integrand Spectra

$\mathcal{P}_f(\nu)$

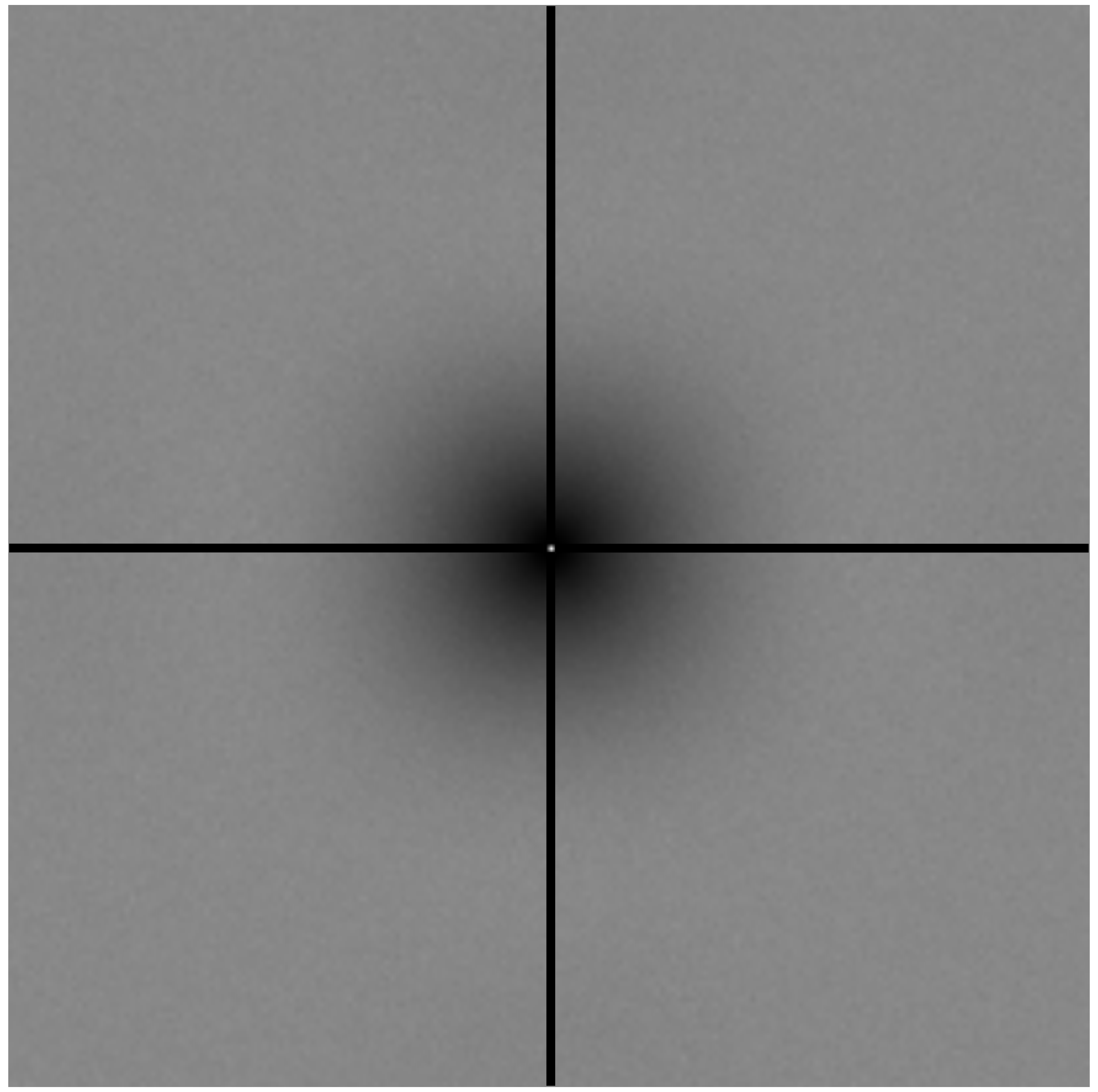
Integrand Spectrum

Non-Axis Aligned Integrand Spectra



Multi-jittered Samples

$\langle \mathcal{P}_{S_N}(\nu) \rangle$

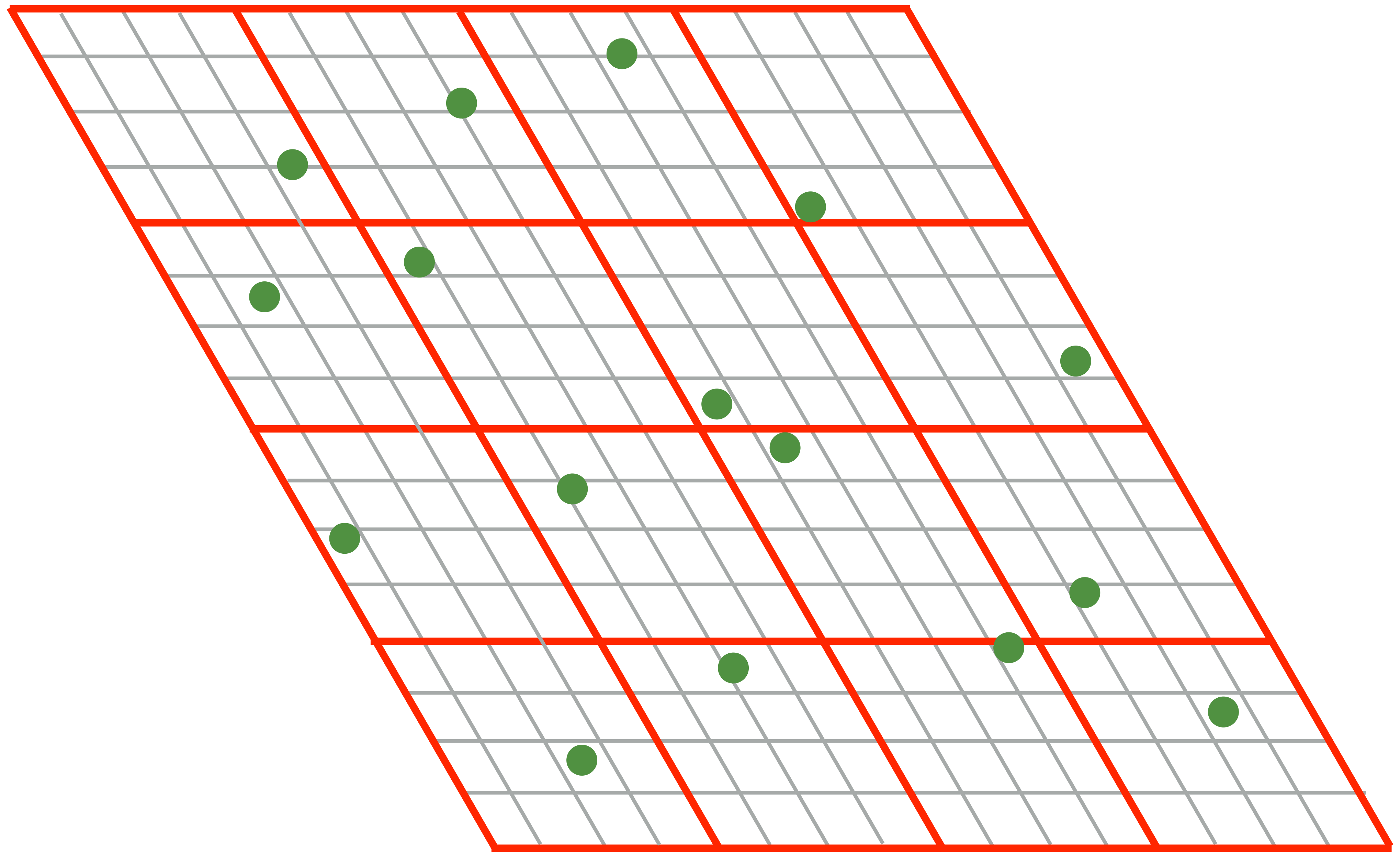


Sampling Spectrum

$\mathcal{P}_f(\nu)$

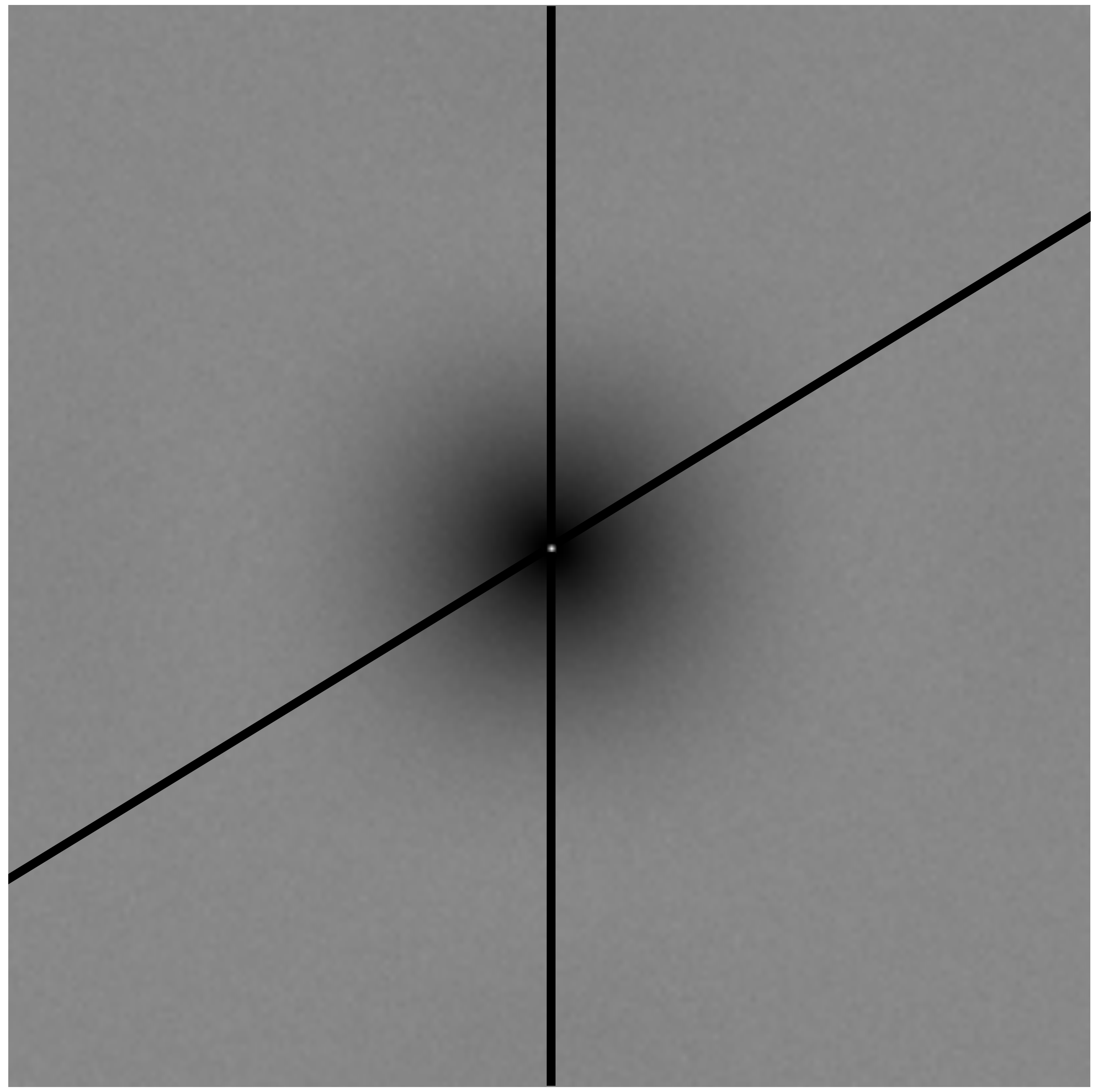
Integrand Spectrum

Shearing Multi-Jittered Samples



Sheared Samples

$\langle \mathcal{P}_{S_N}(\nu) \rangle$

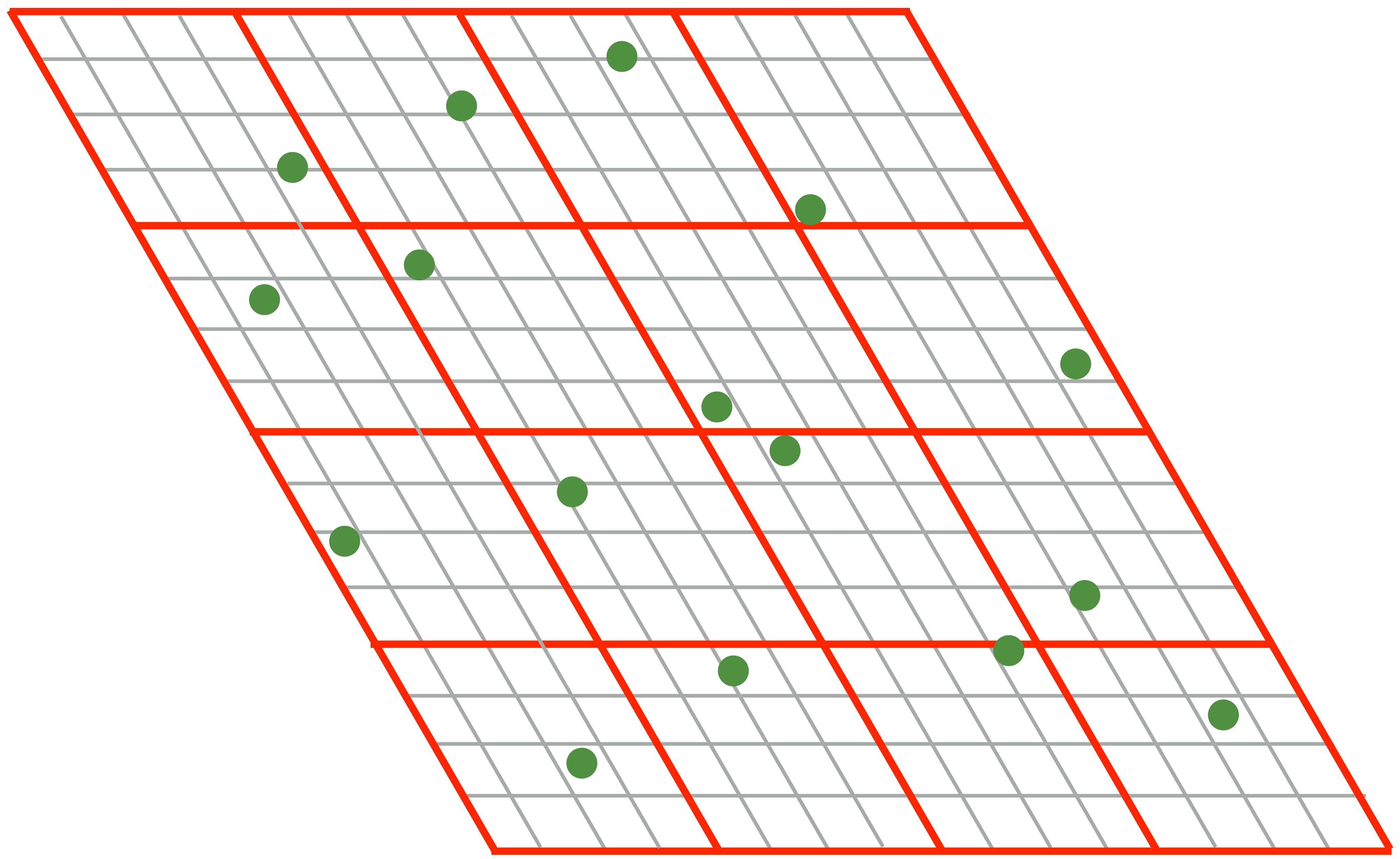


Sheared Spectrum

$\mathcal{P}_f(\nu)$

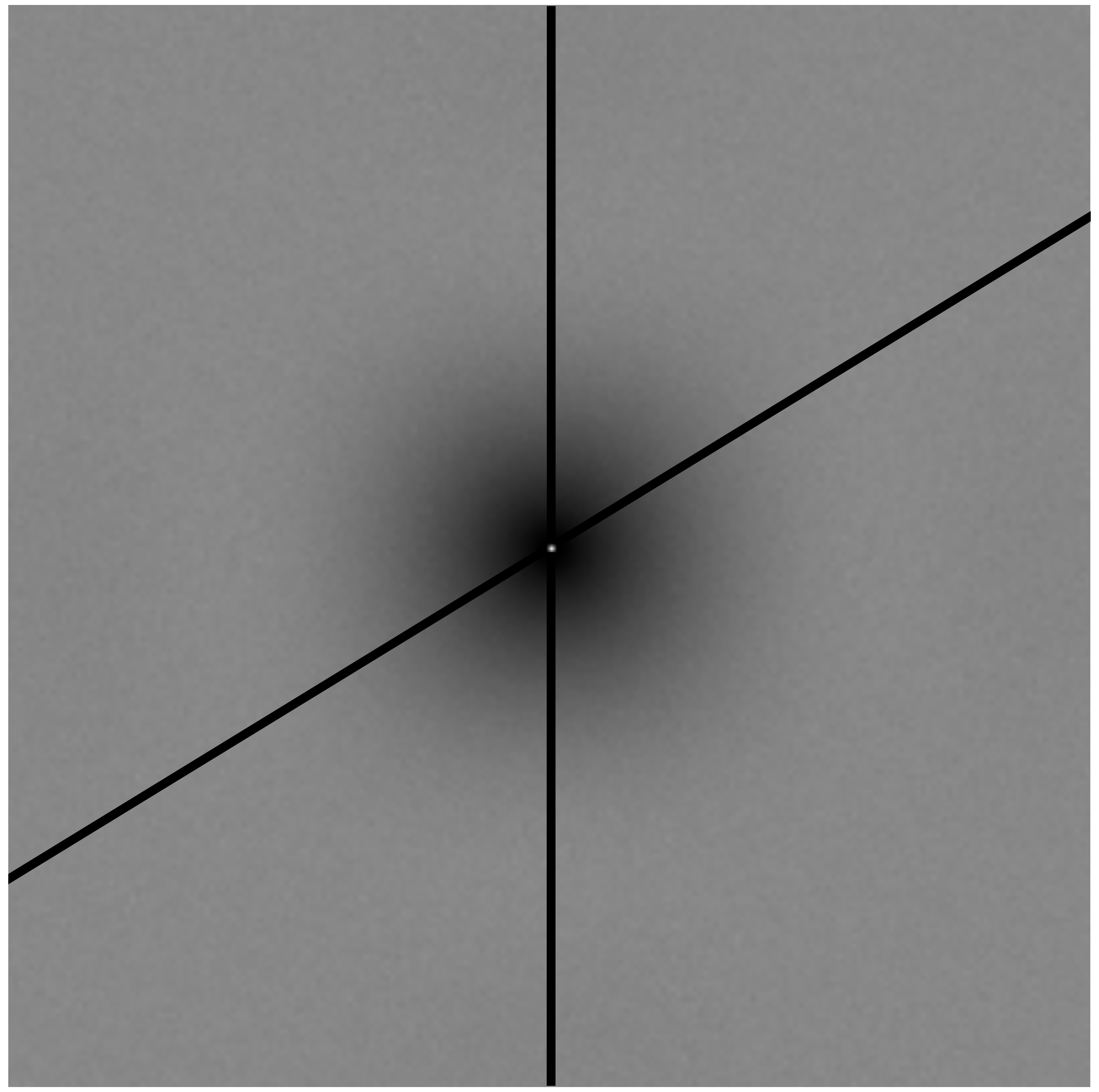
Integrand Spectrum

How can we determine the sample shearing parameters ?



Sheared Samples

$\langle \mathcal{P}_{S_N}(\nu) \rangle$



Sheared Spectrum

$\mathcal{P}_f(\nu)$

Integrand Spectrum

Our Algorithm

1) Develop an oracle using the Frequency Analysis of Light Transport

2) Use this oracle to shear the samples

3) Perform Monte Carlo integration using the sheared samples

Frequency Analysis of Light Transport

Related Work

- Frequency Analysis of Light Transport Durand et al. [2005]
- Depth of Field Soler et al. [2009]
- Motion Blur Egan et al. [2009]
- Ambient Occlusion Egan et al. [2011] and more…

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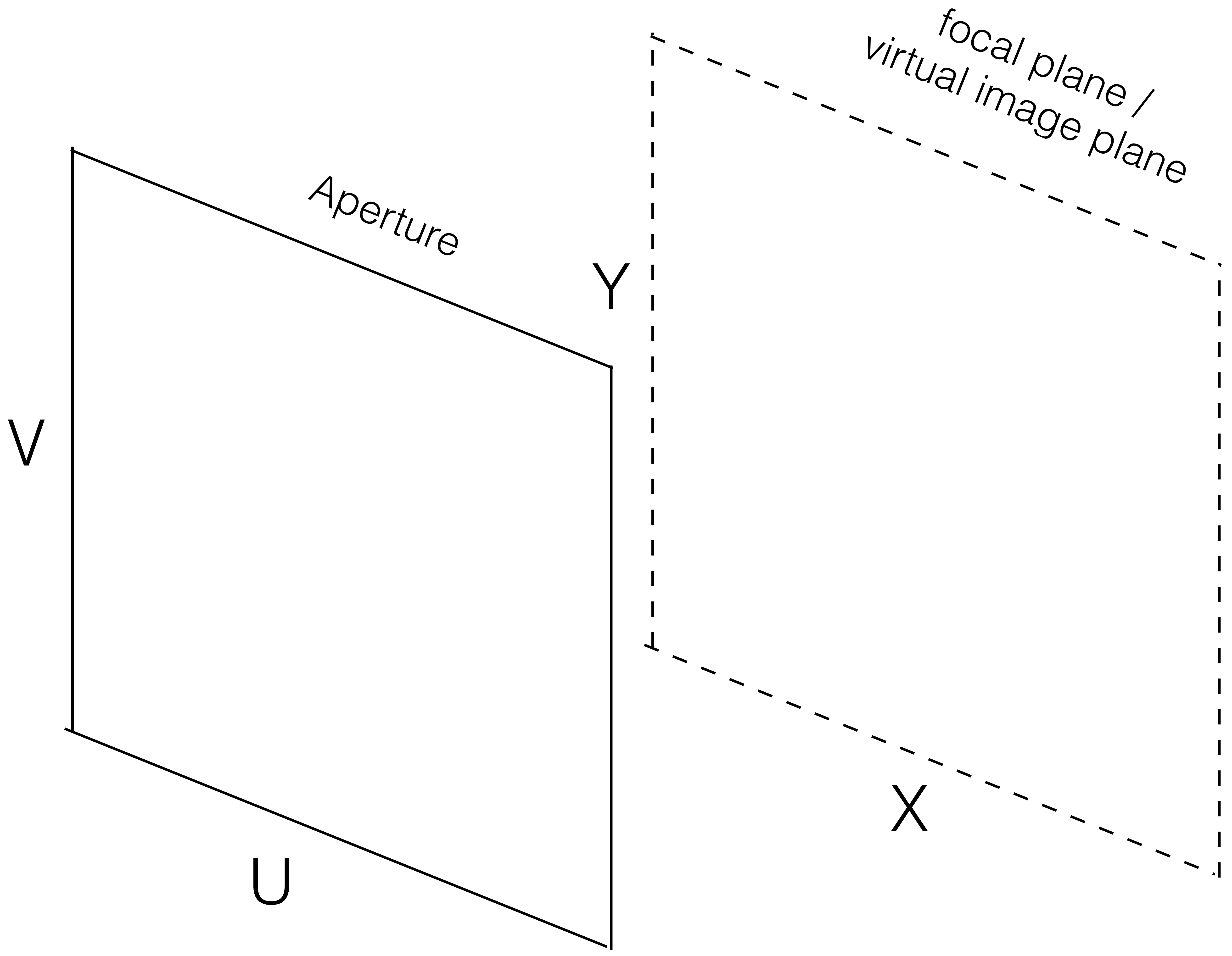
Reconstruction

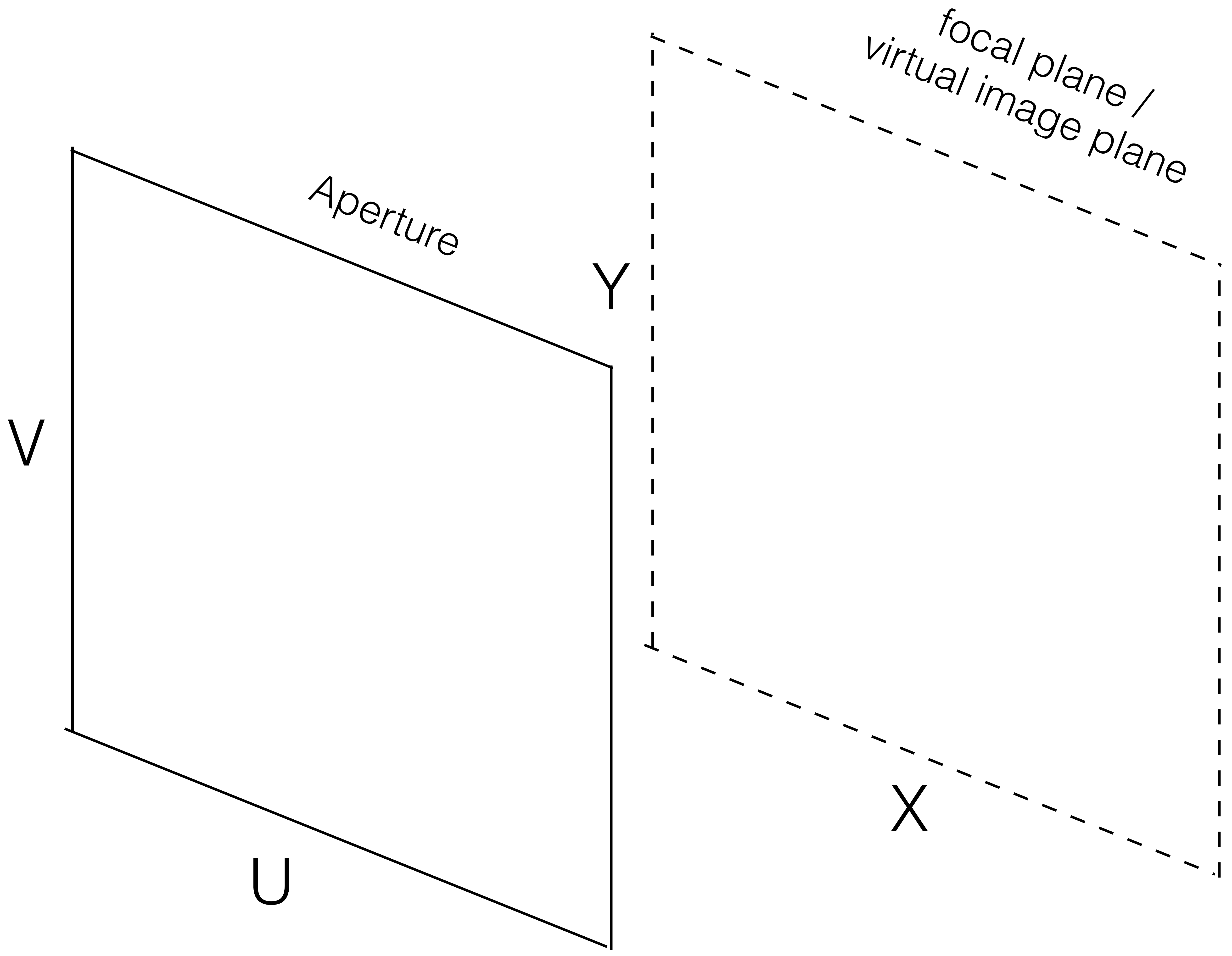
Our Work

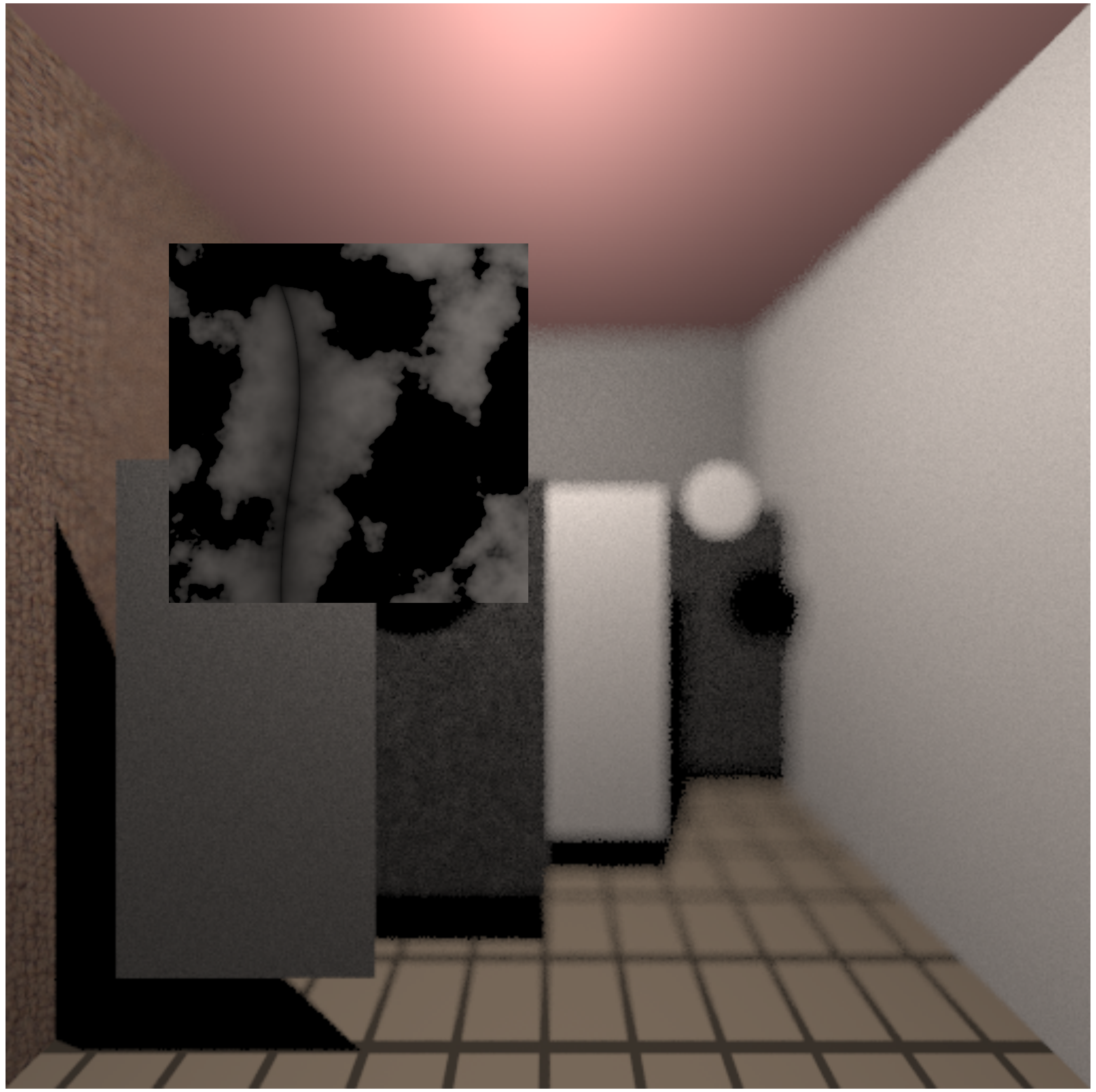
Integration

Depth of Field Analysis

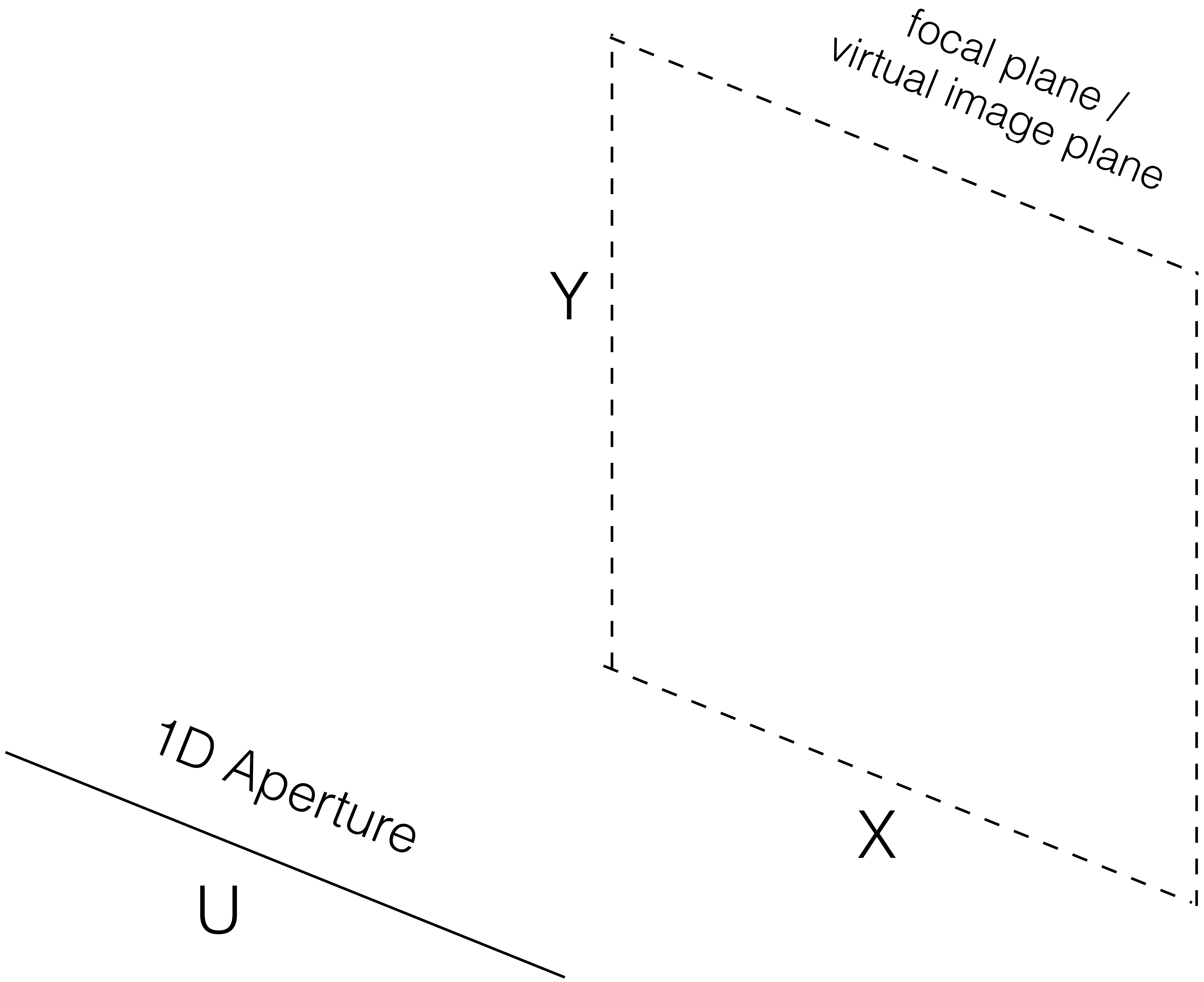


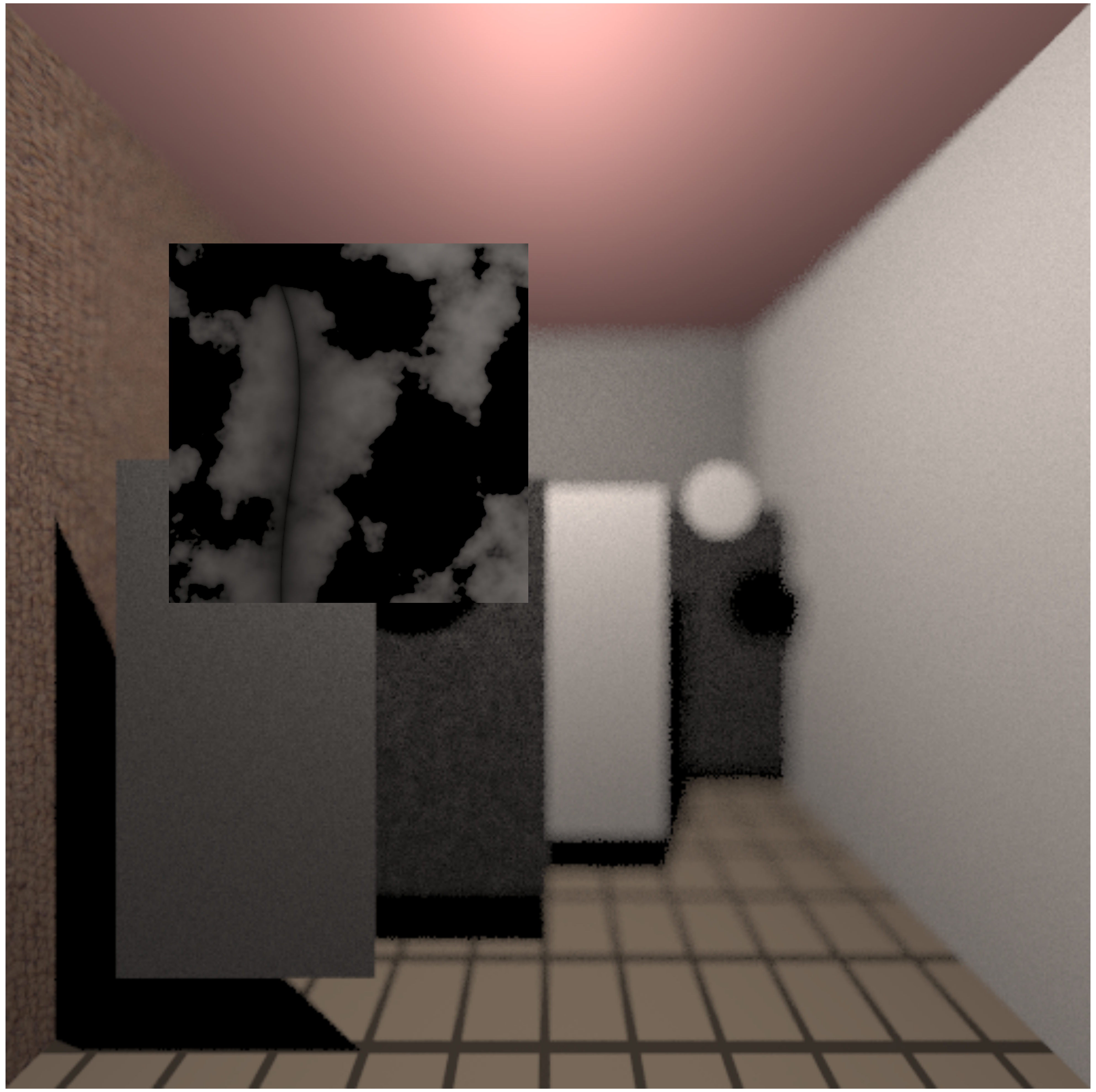
Depth of Field Analysis



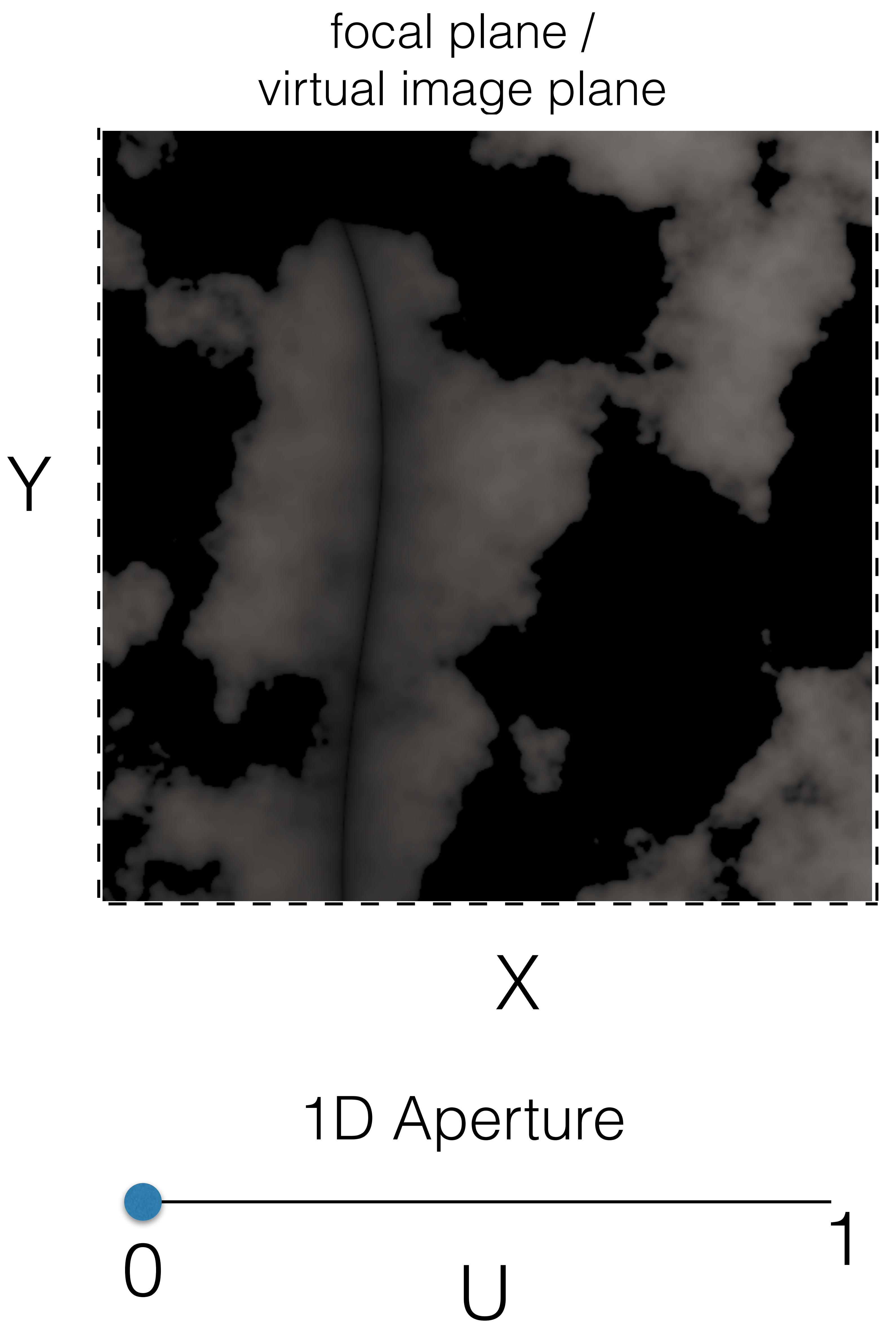


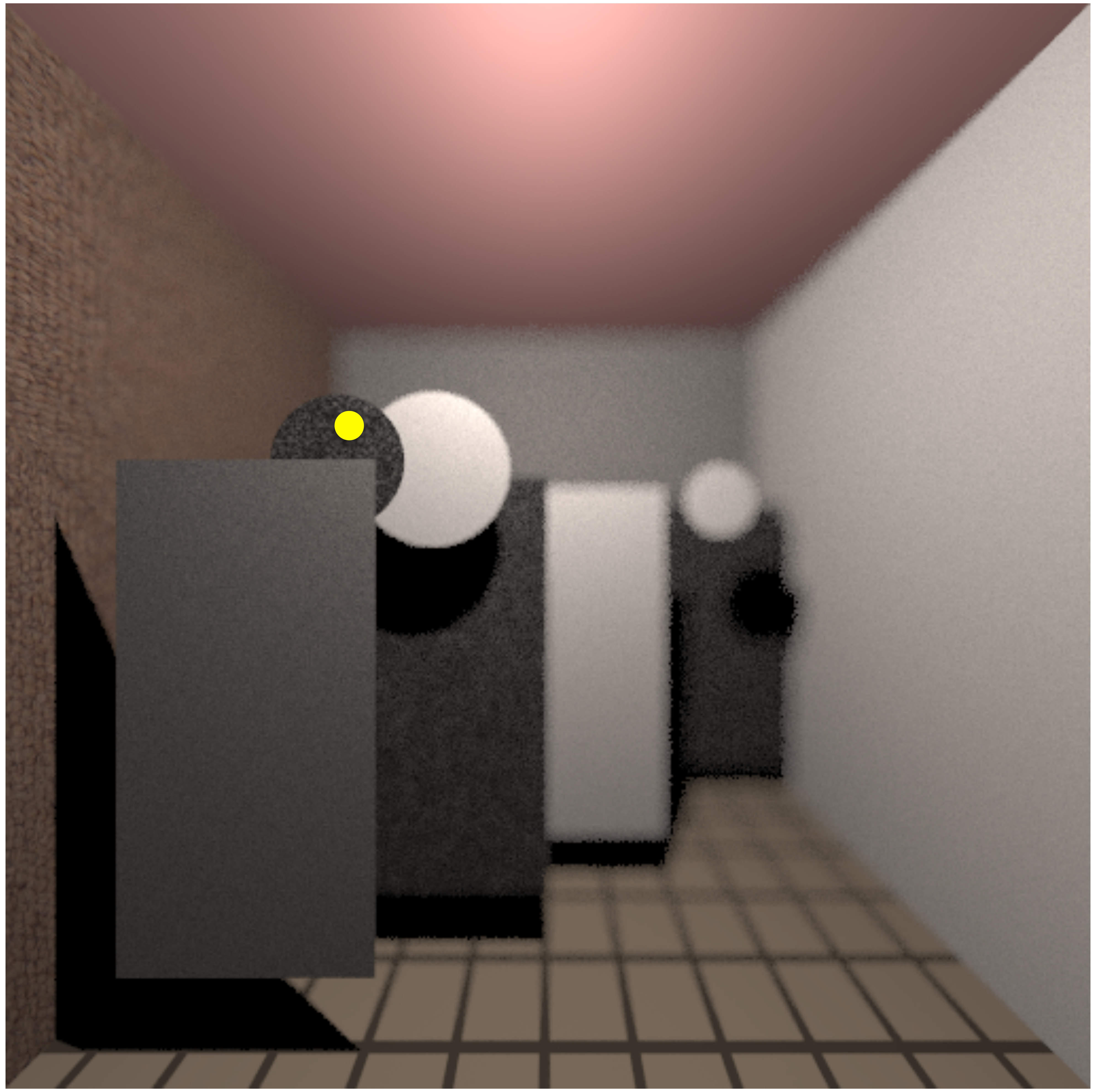
Depth of Field Analysis



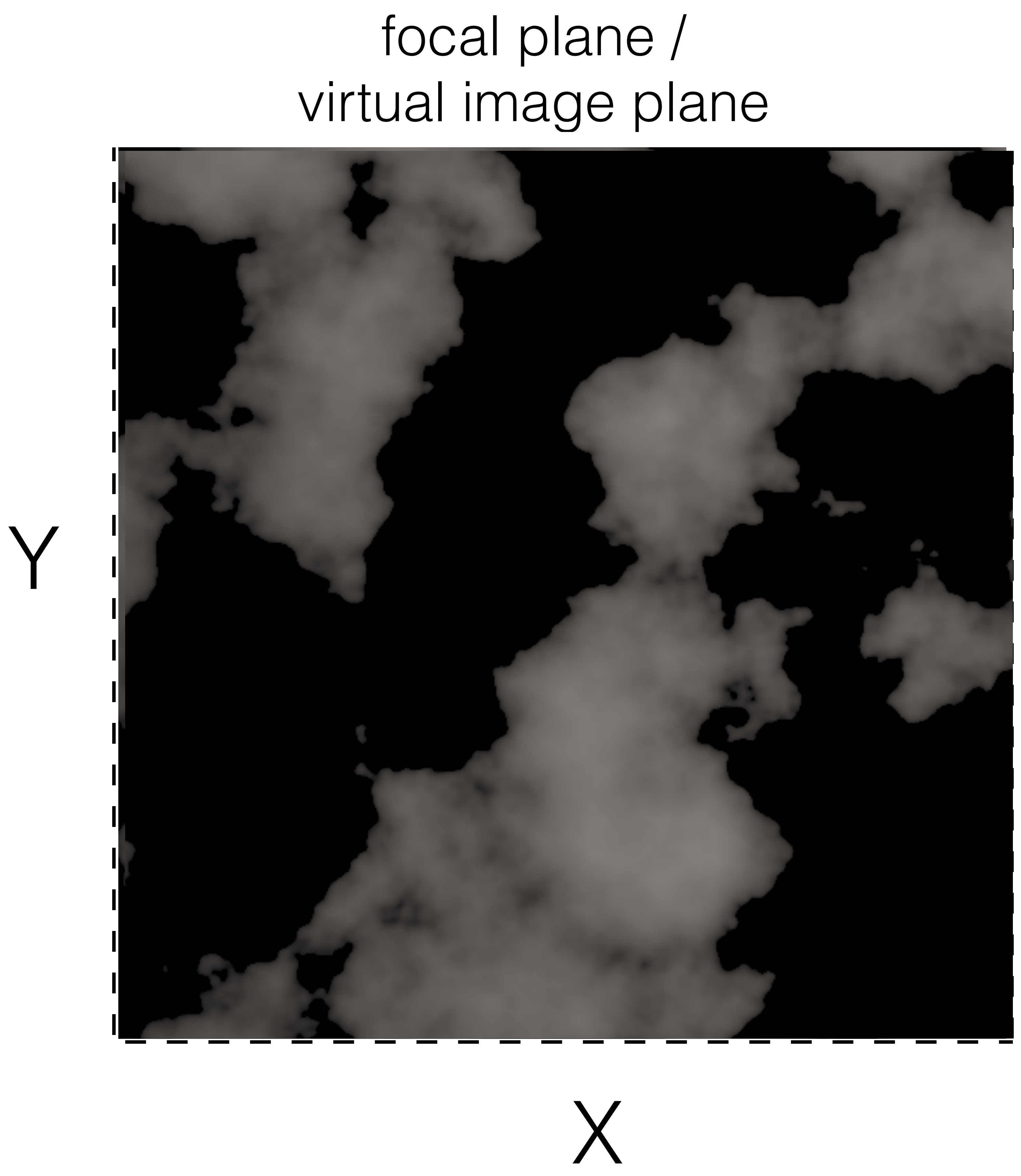


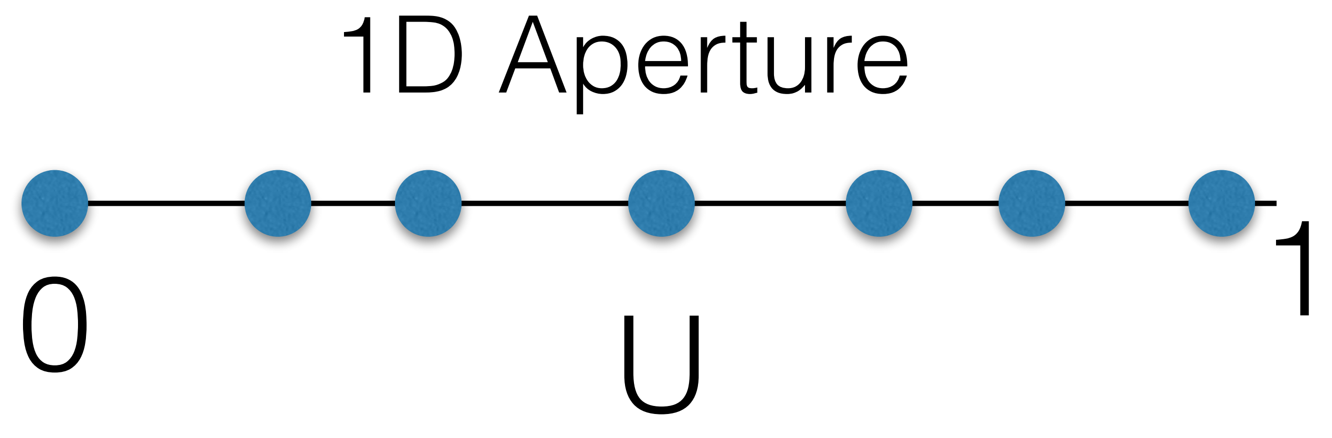
Depth of Field Analysis

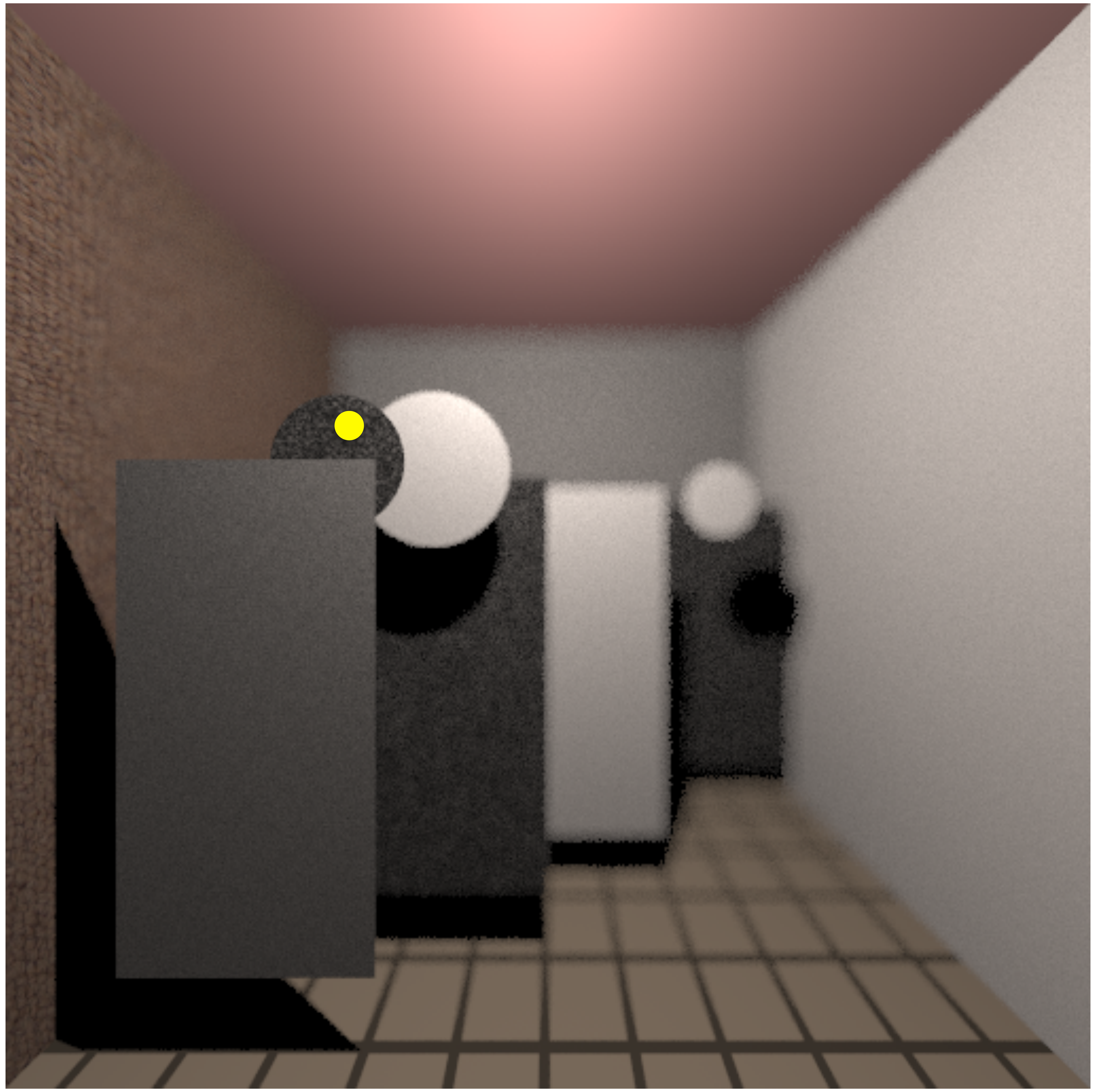




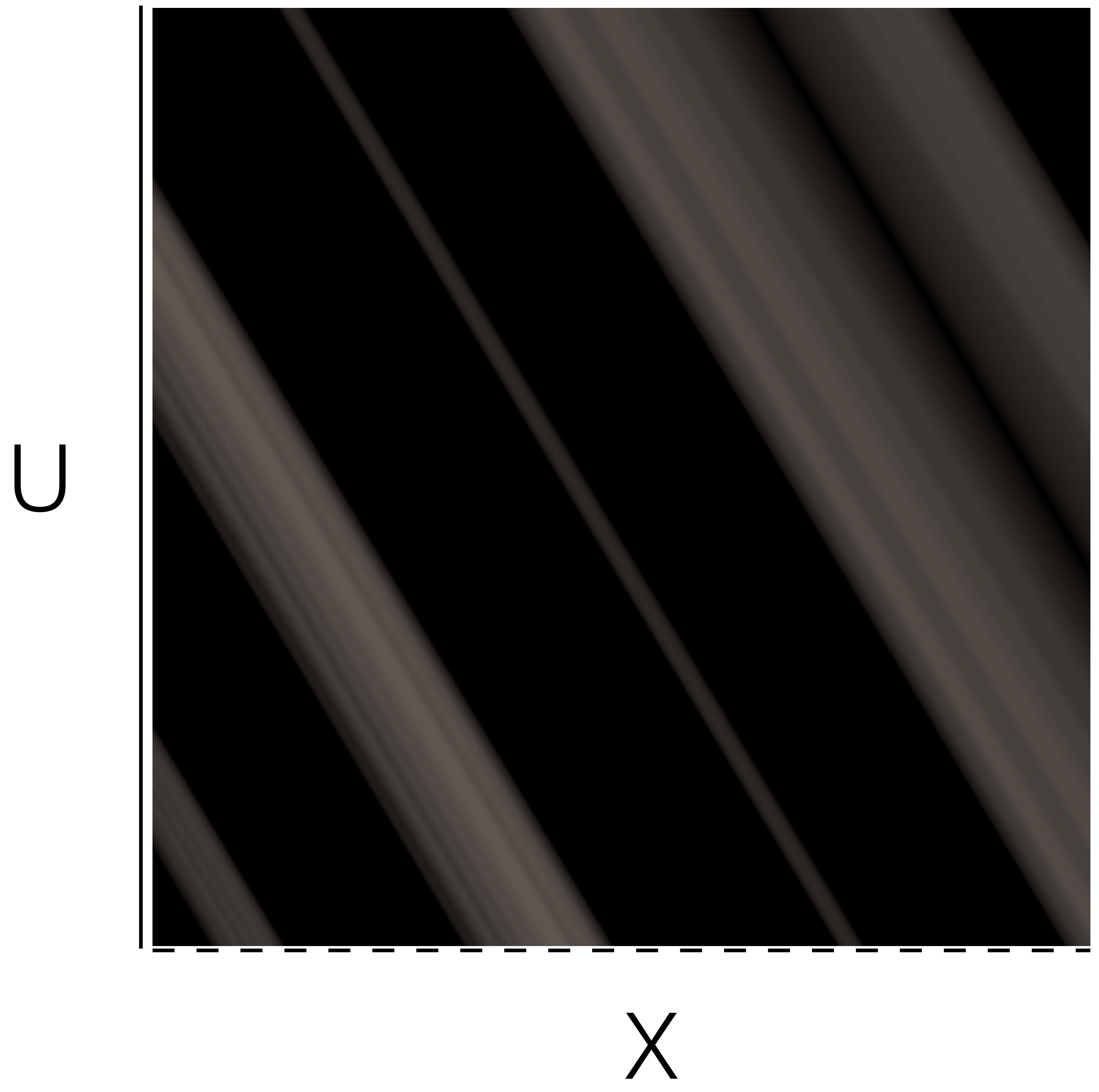
Depth of Field Analysis



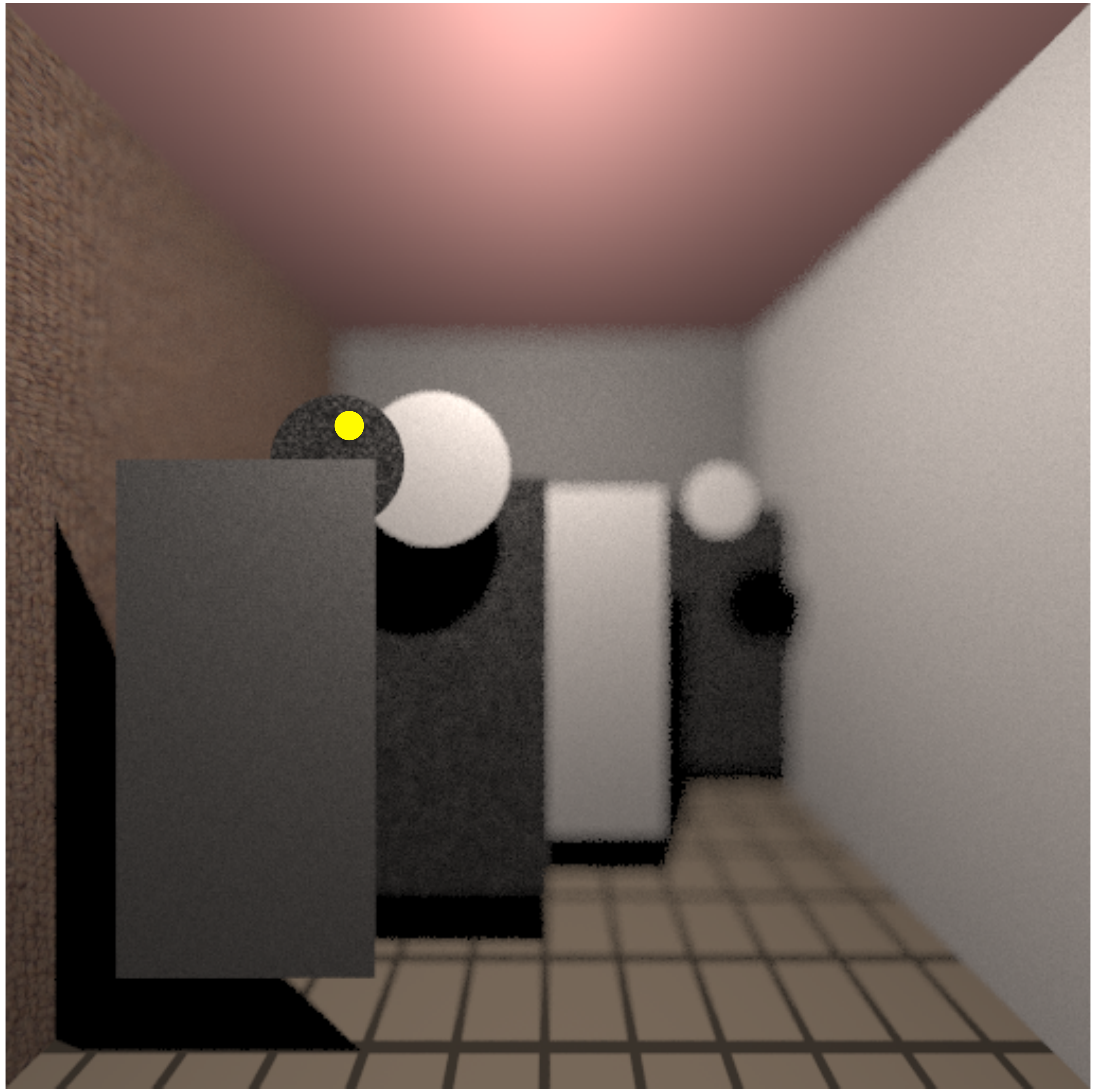




Depth of Field Analysis

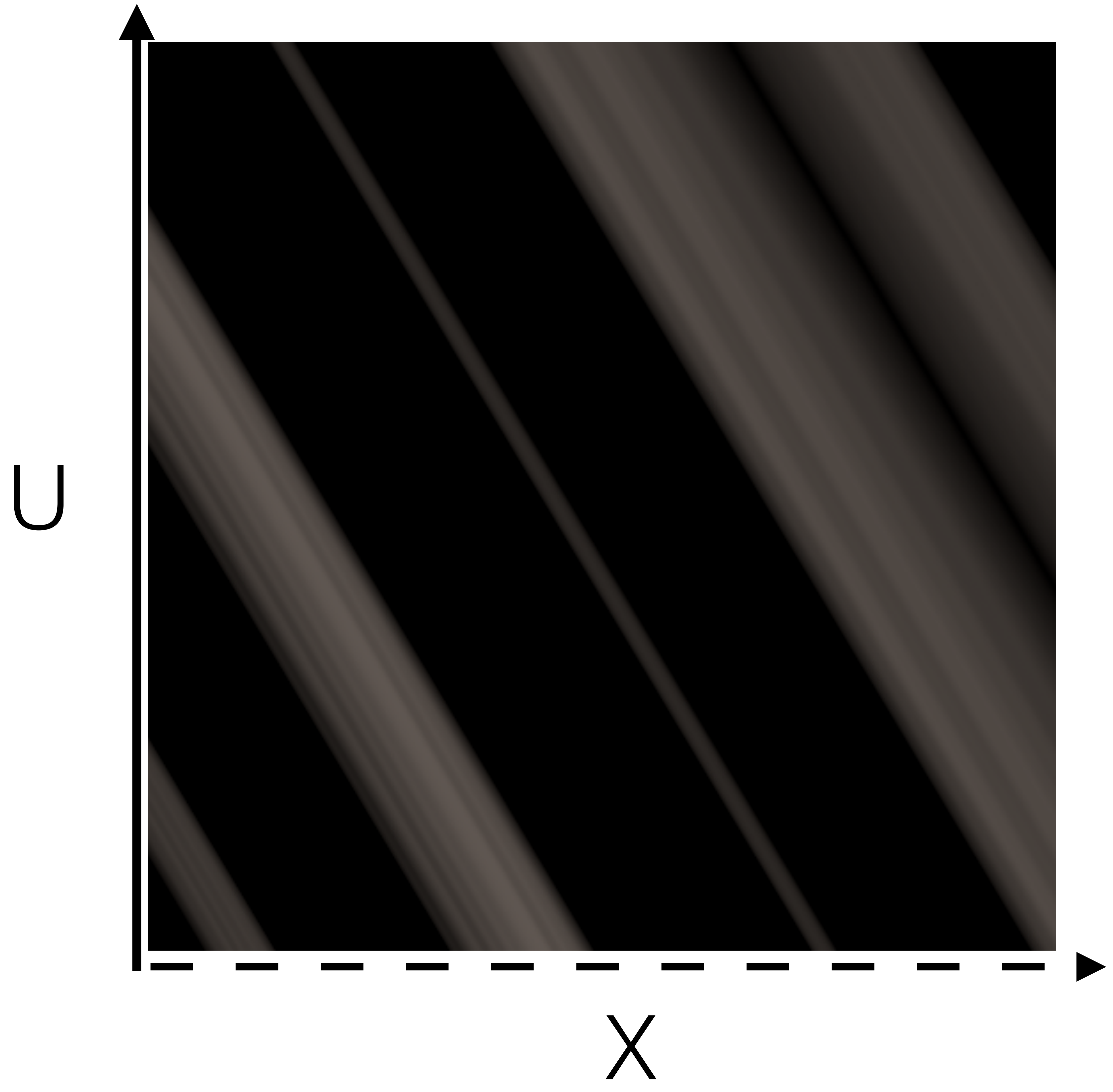


XU Slices

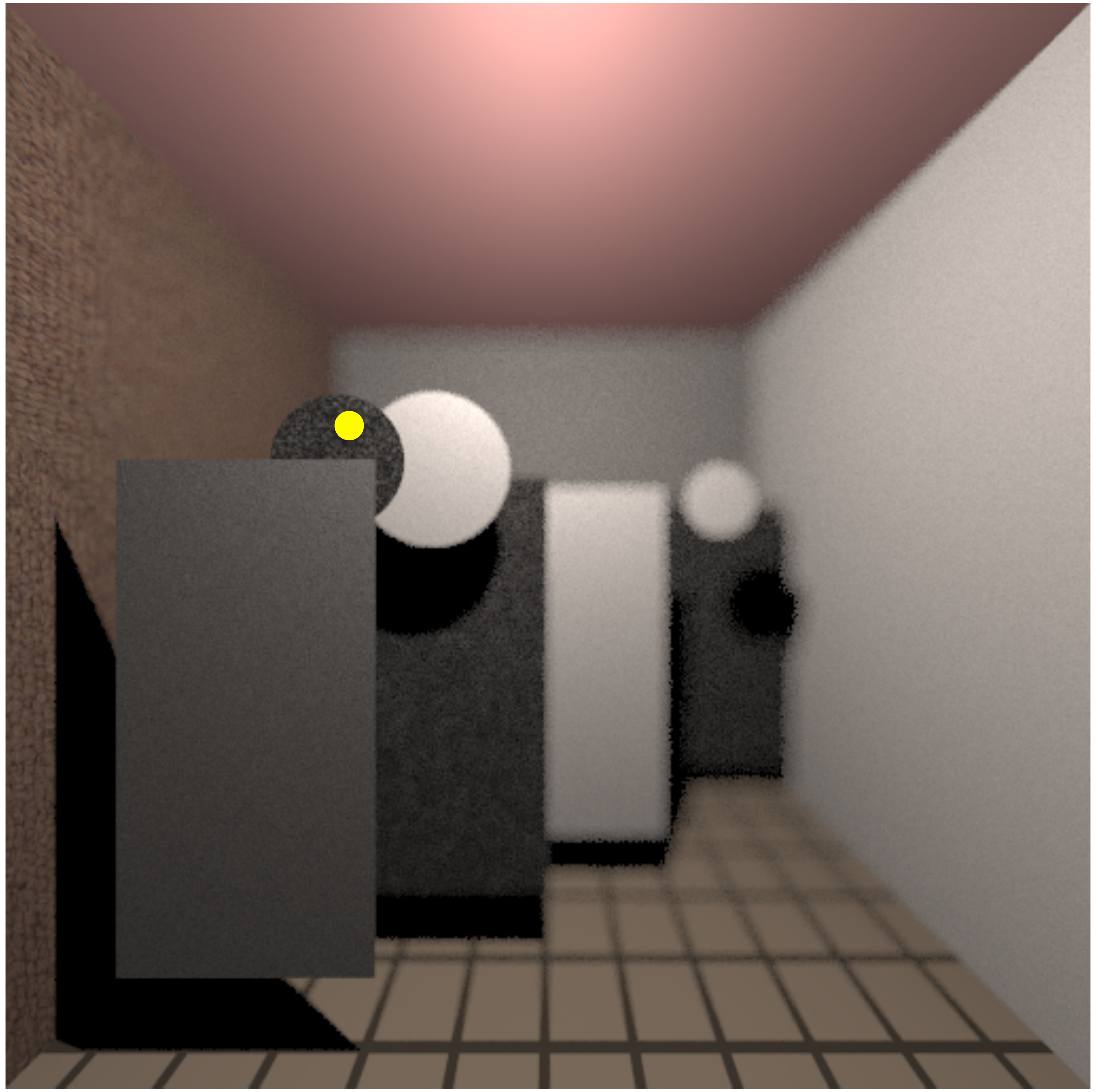


Durand et al. [2005]

Depth of Field Analysis

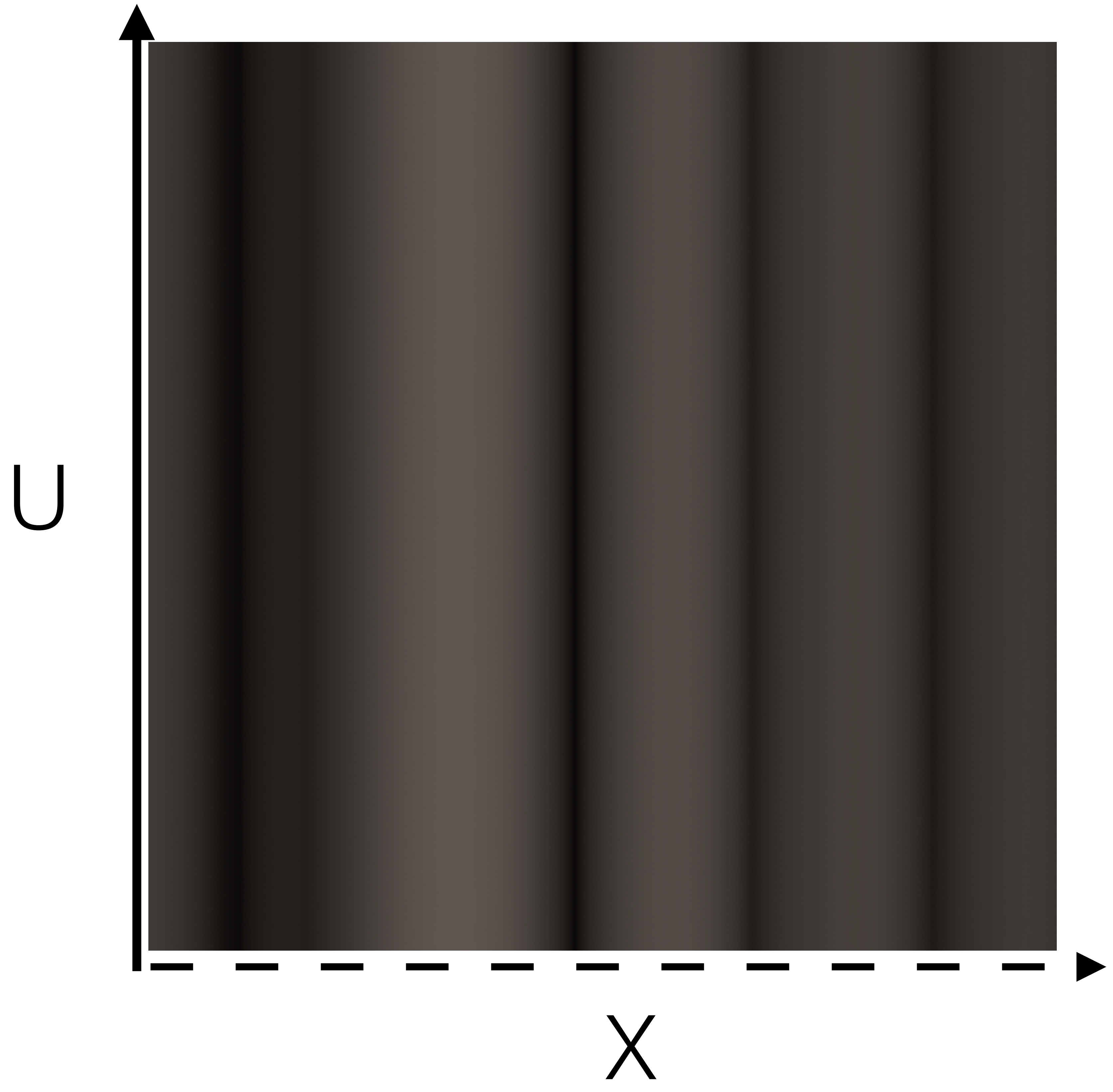


XU Slices

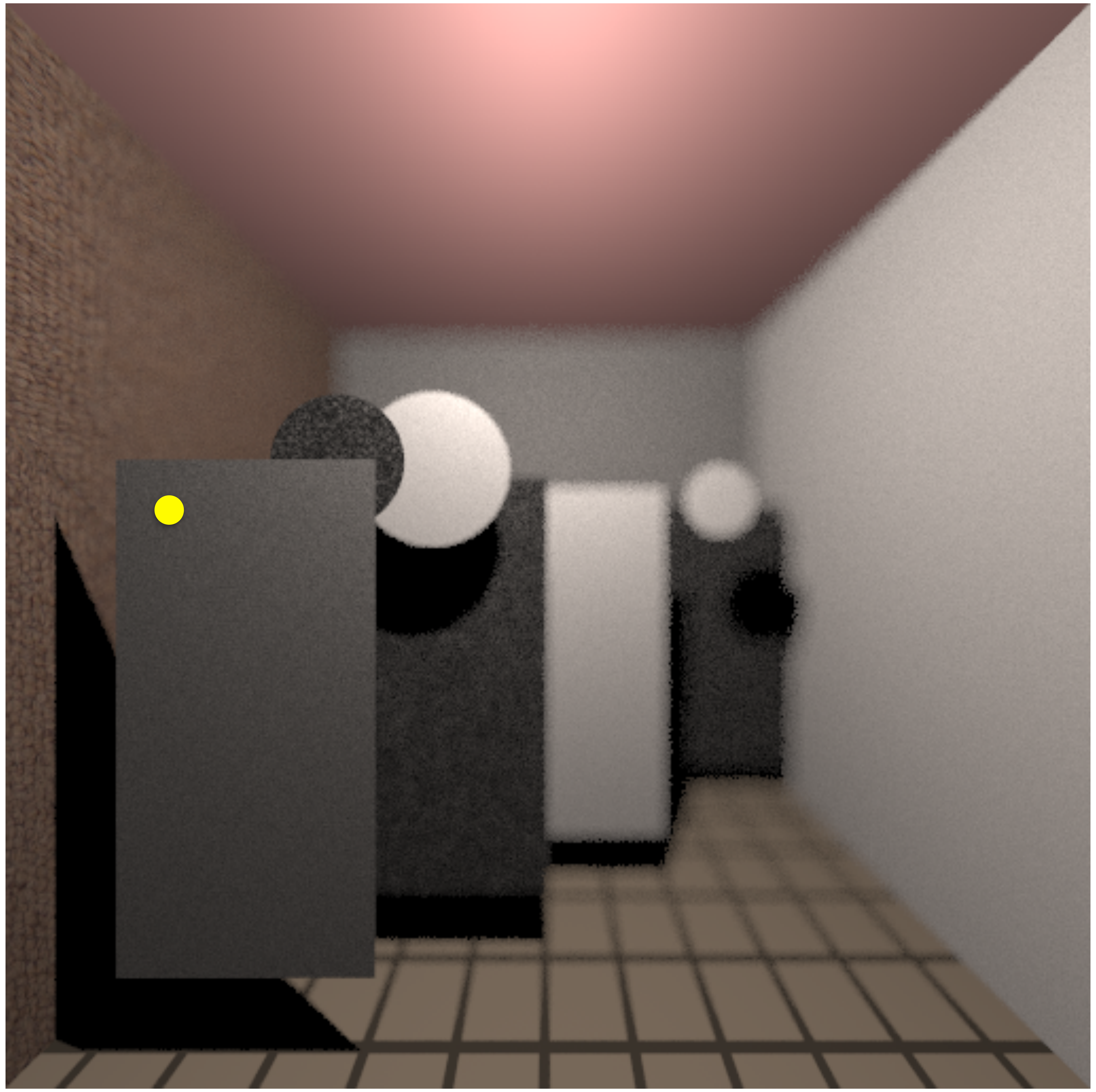


Durand et al. [2005]

Depth of Field Analysis



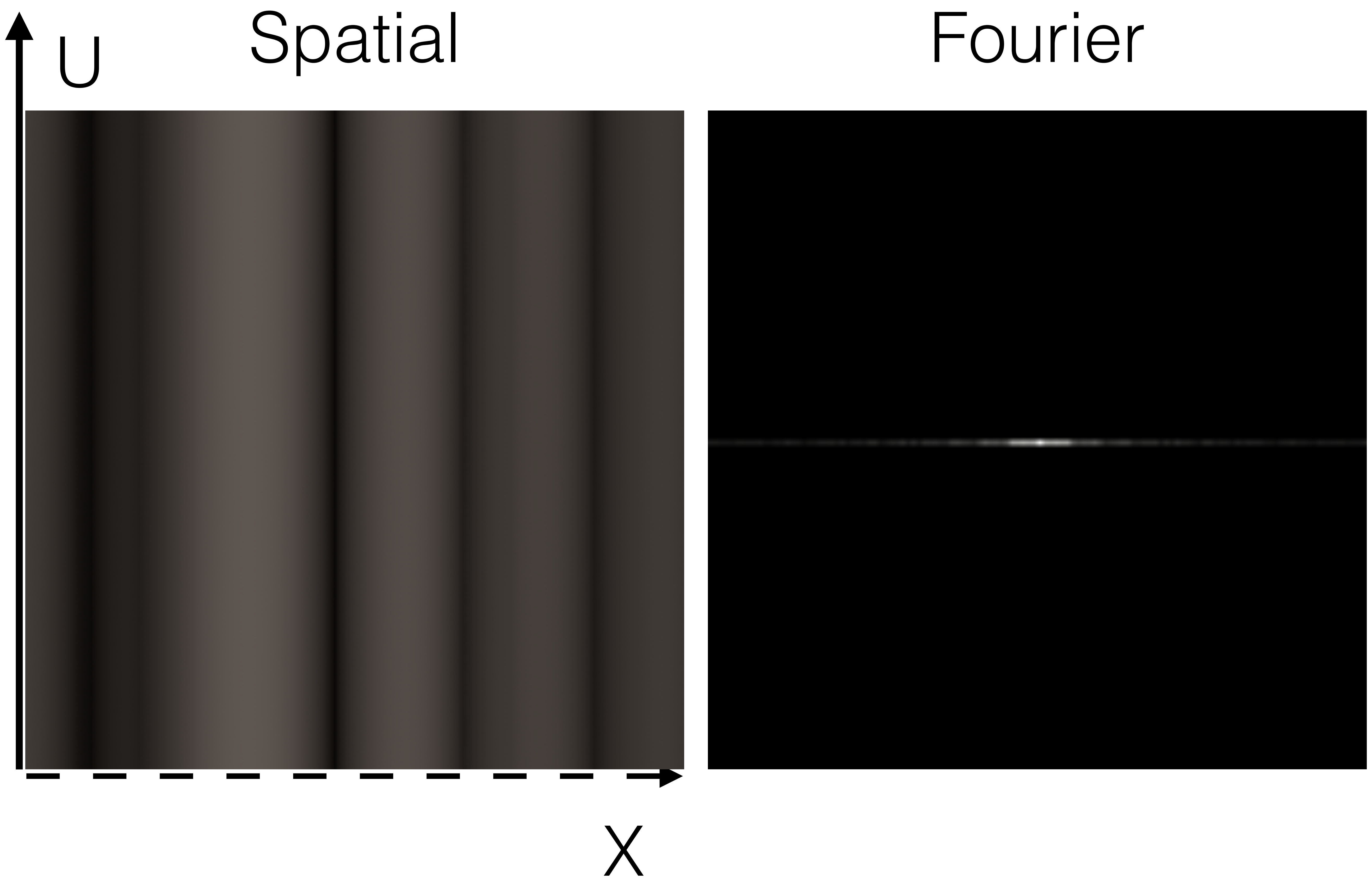
XU Slices



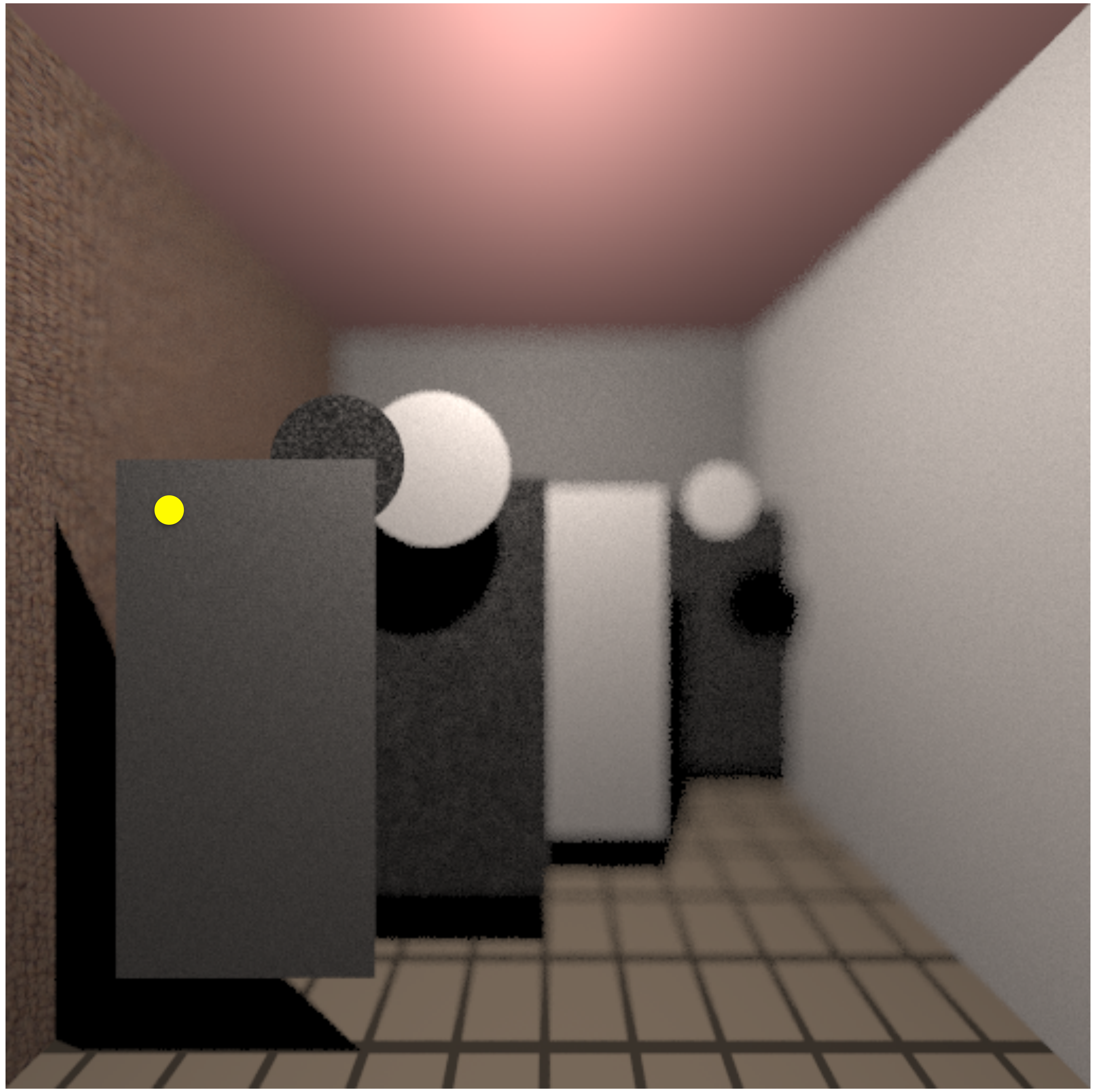
Durand et al. [2005]

Depth of Field Analysis

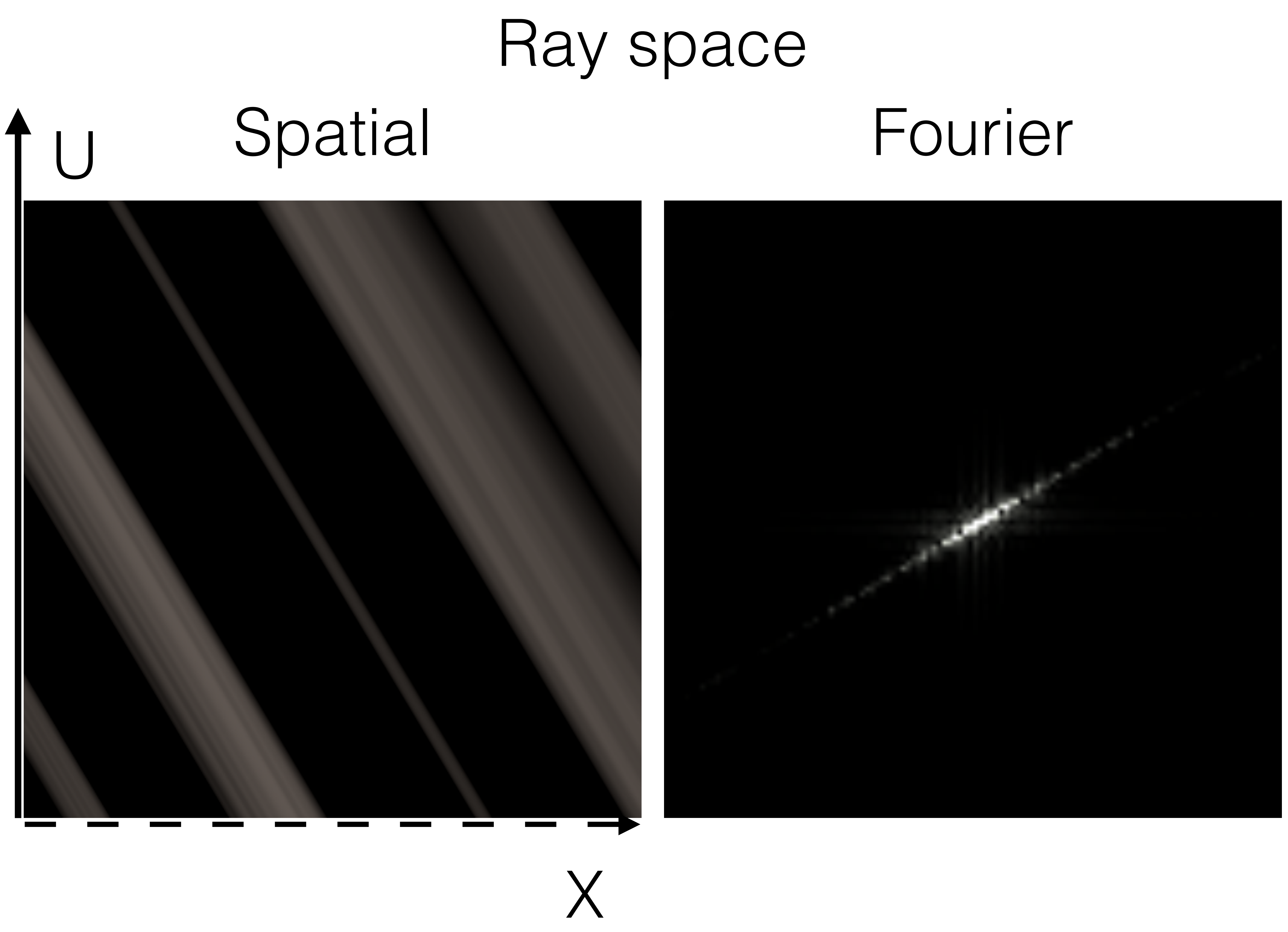
Ray space



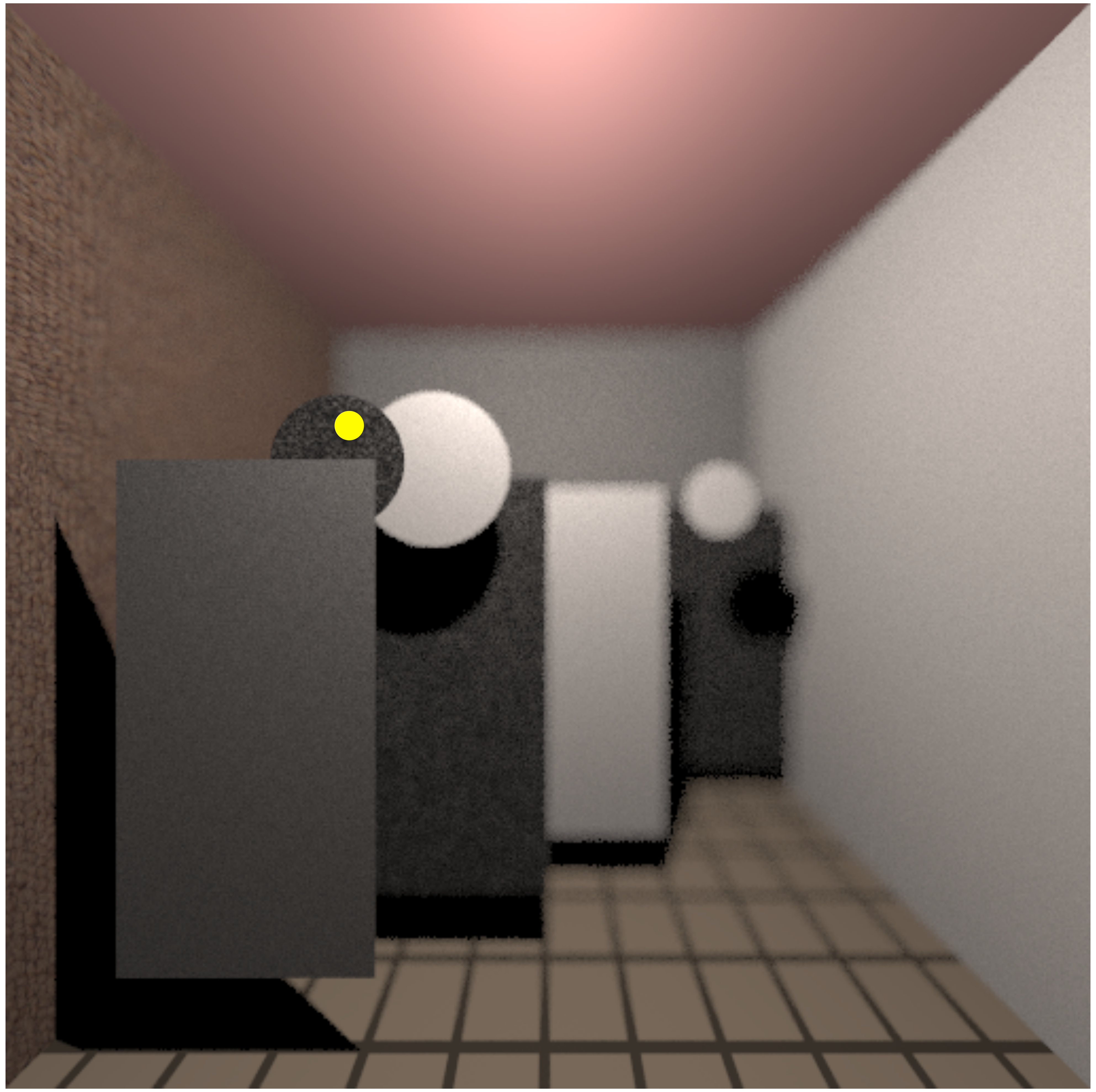
XU Slices



Depth of Field Analysis



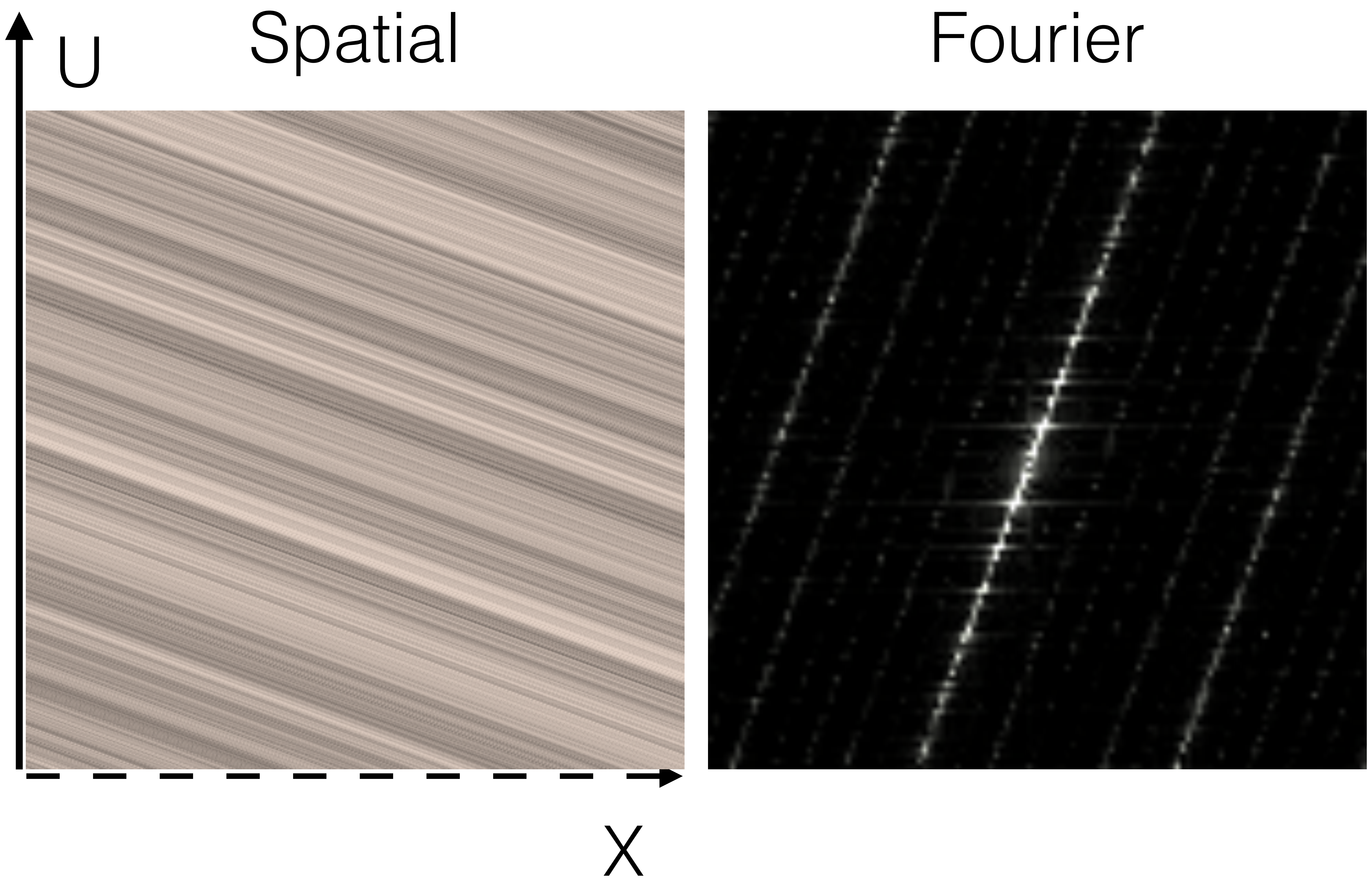
XU Slices



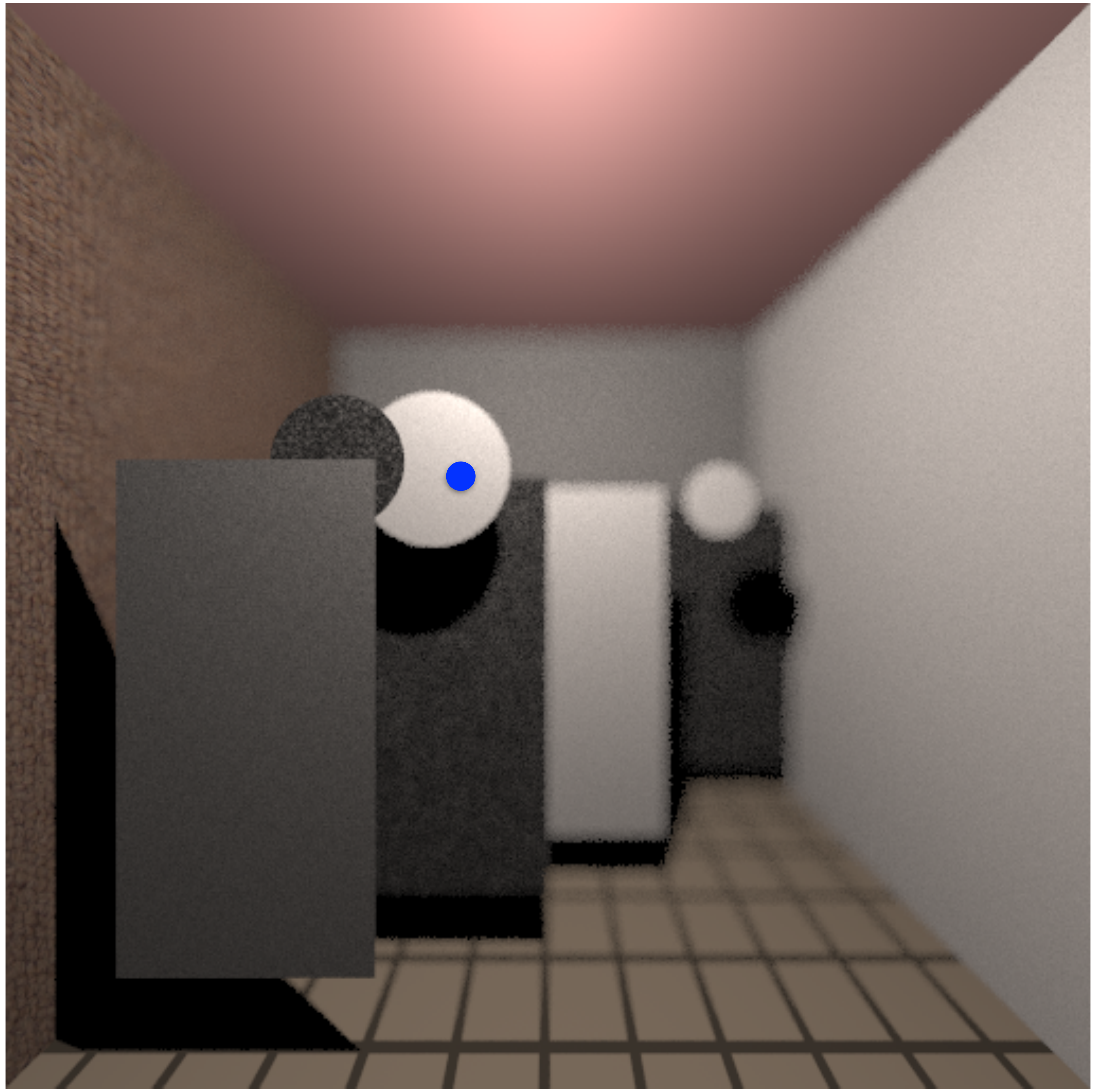
Durand et al. [2005]

Depth of Field Analysis

Ray space



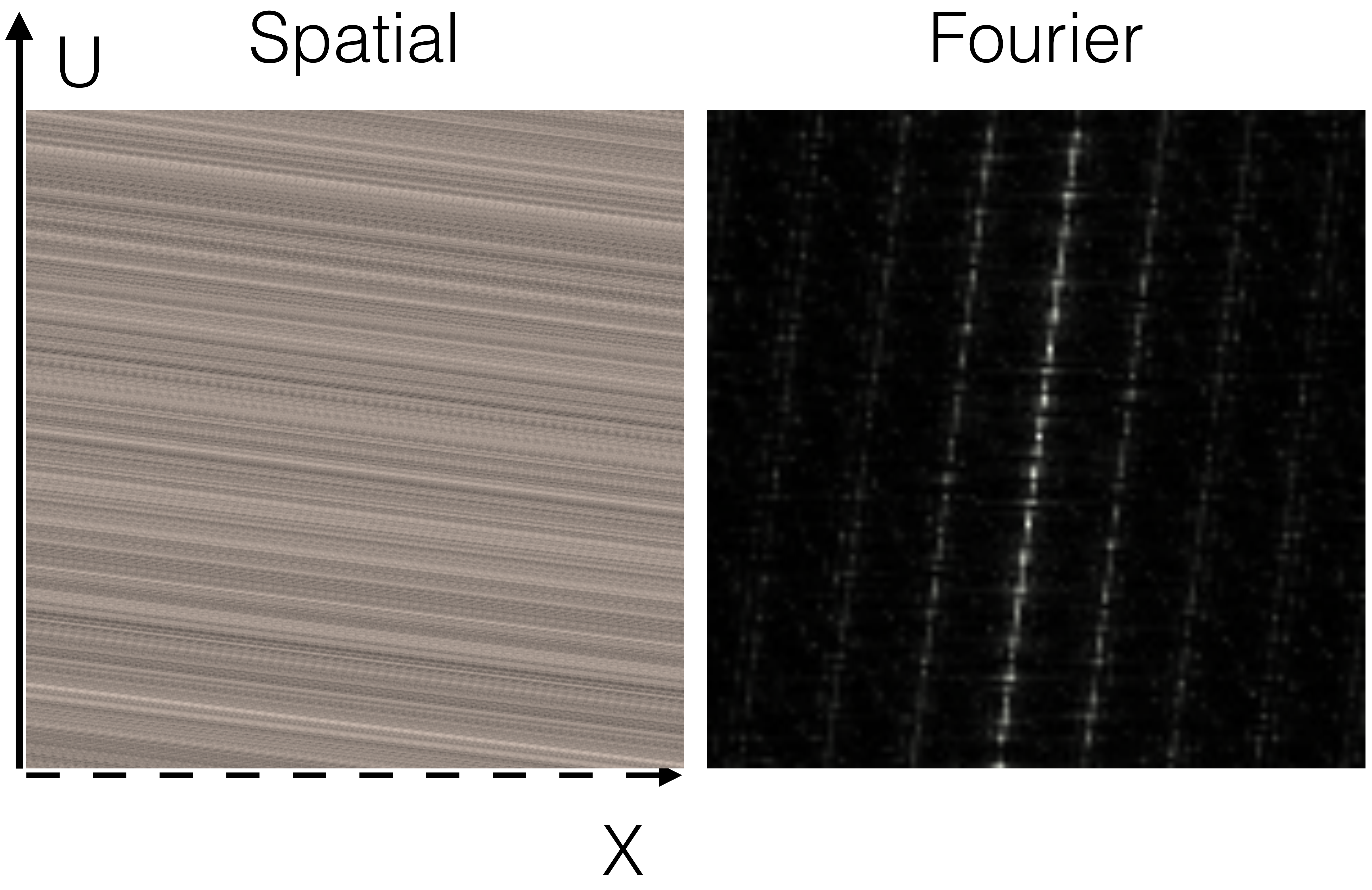
XU Slices



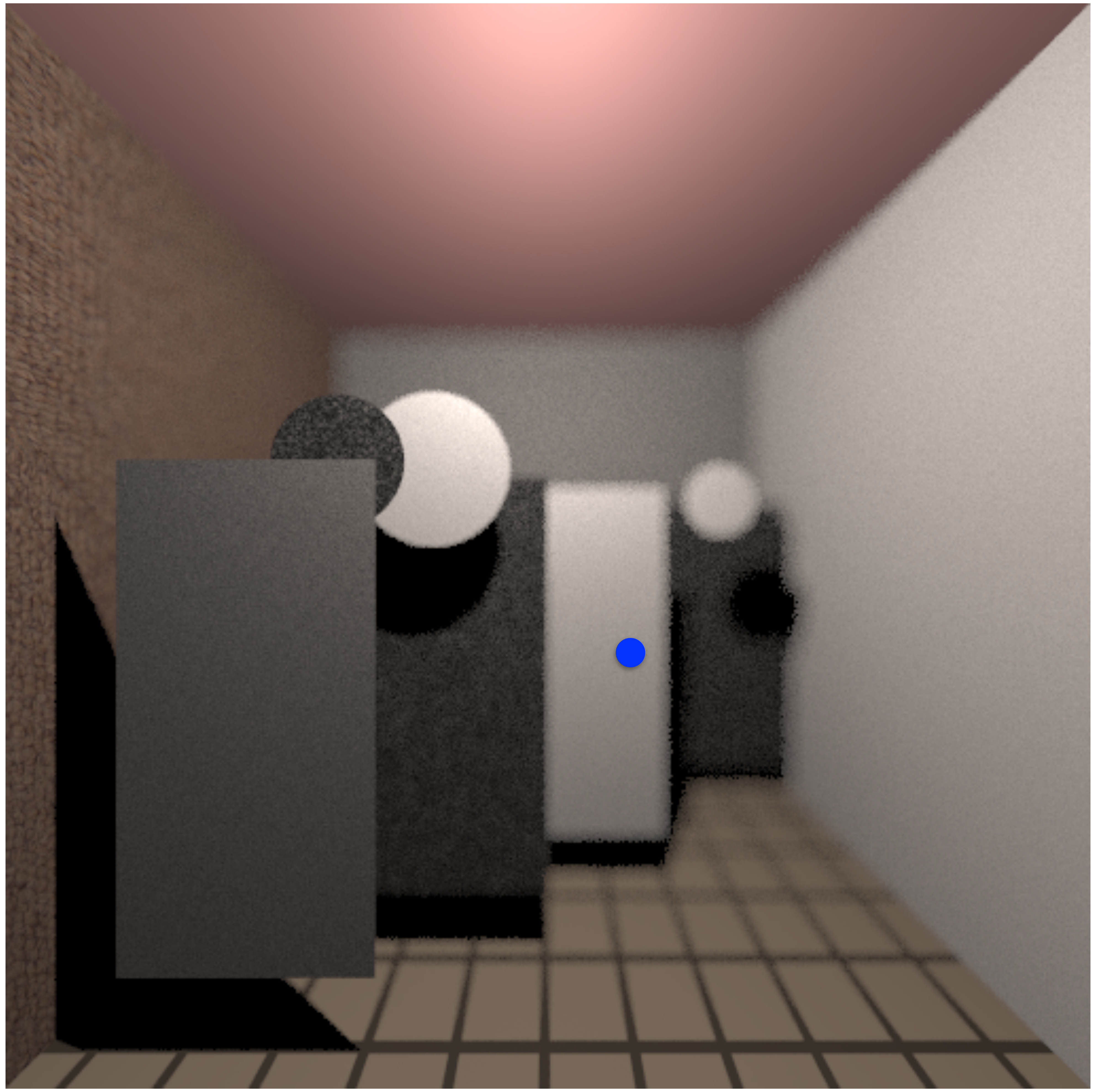
Durand et al. [2005]

Depth of Field Analysis

Ray space



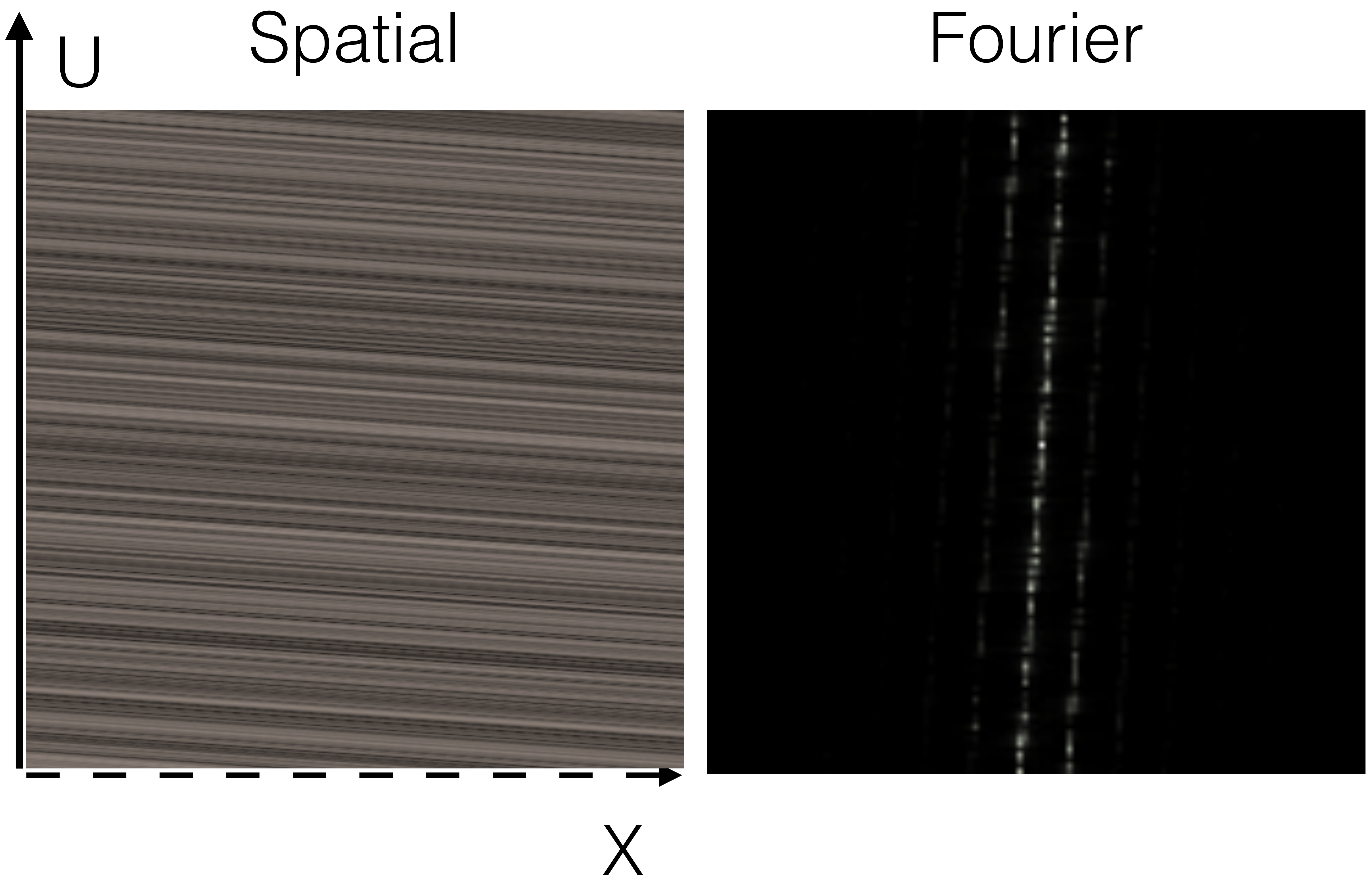
XU Slices



Durand et al. [2005]

Depth of Field Analysis

Ray space



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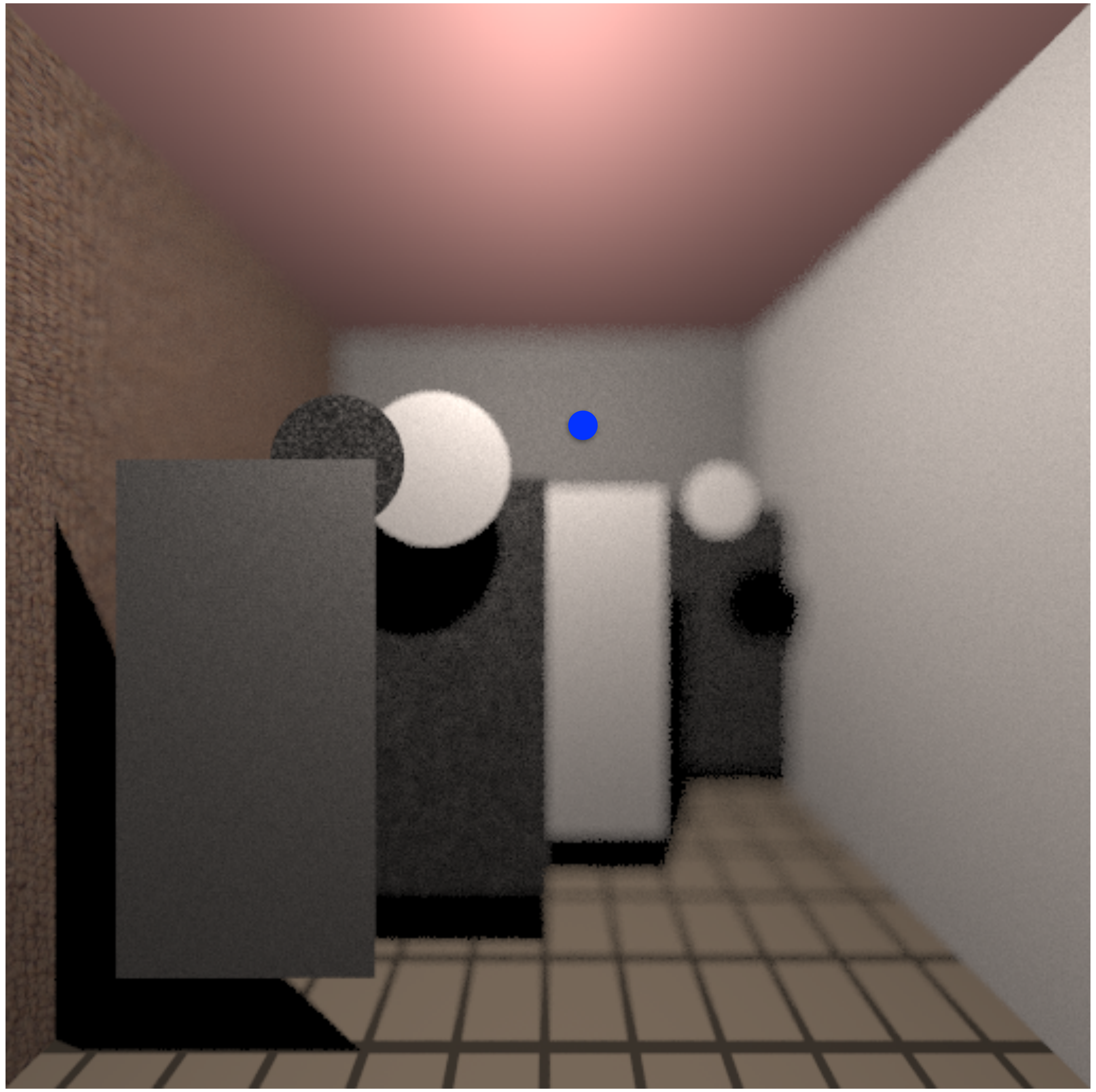


Durand et al. [2005]

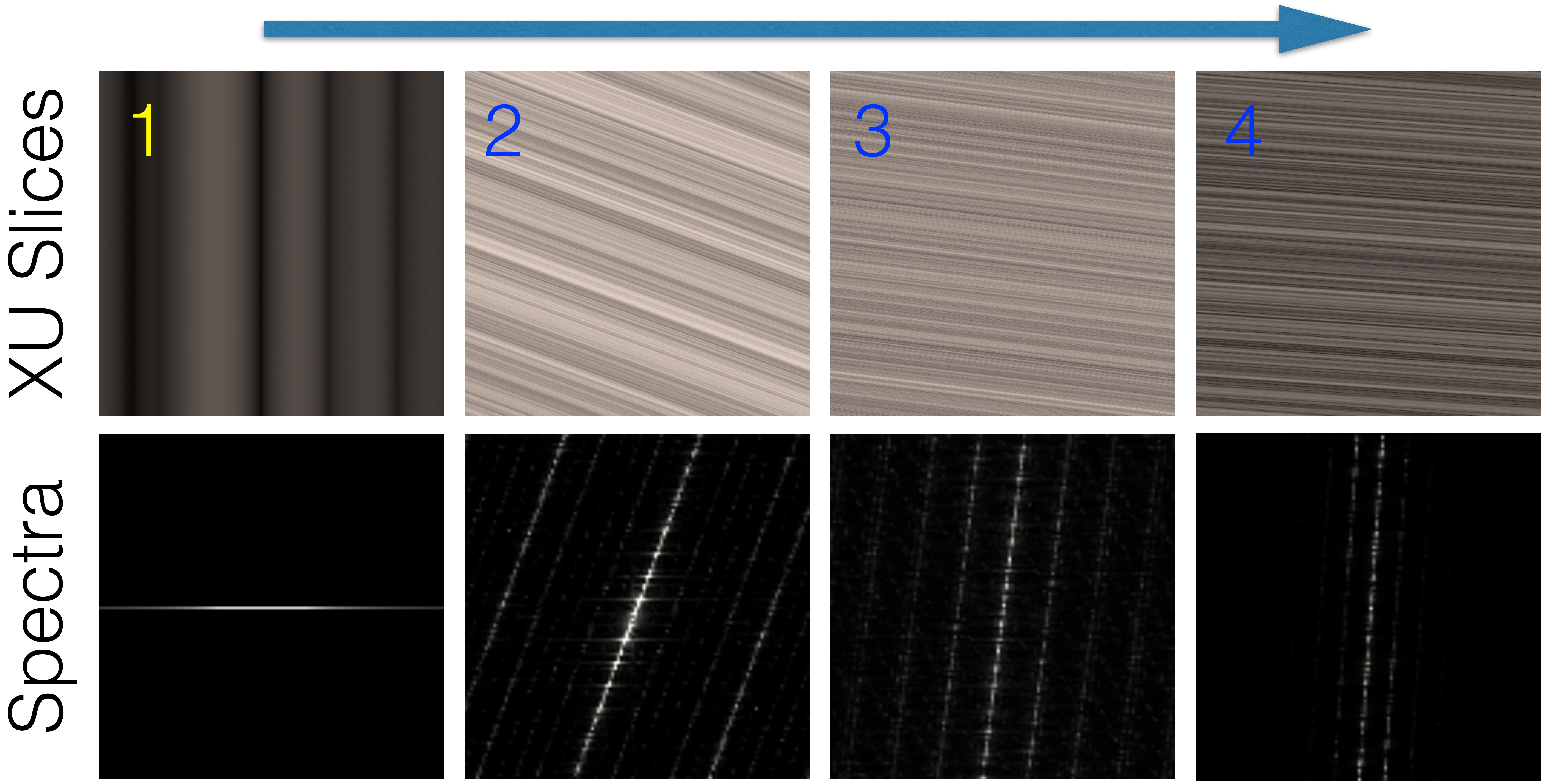
Light Field gets Sheared

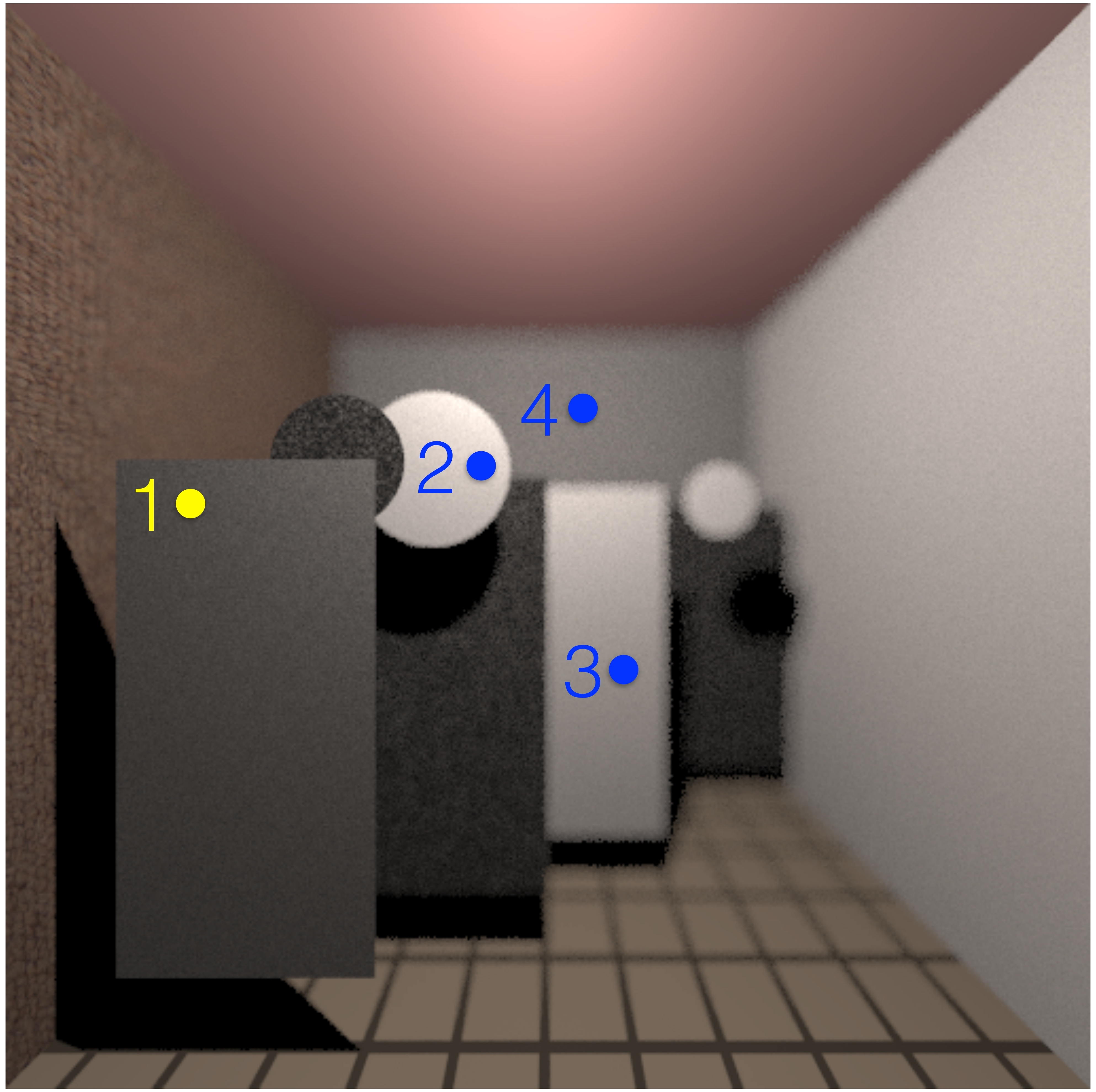
$$x = x + u\frac{F-d}{d},$$

F: focal distance

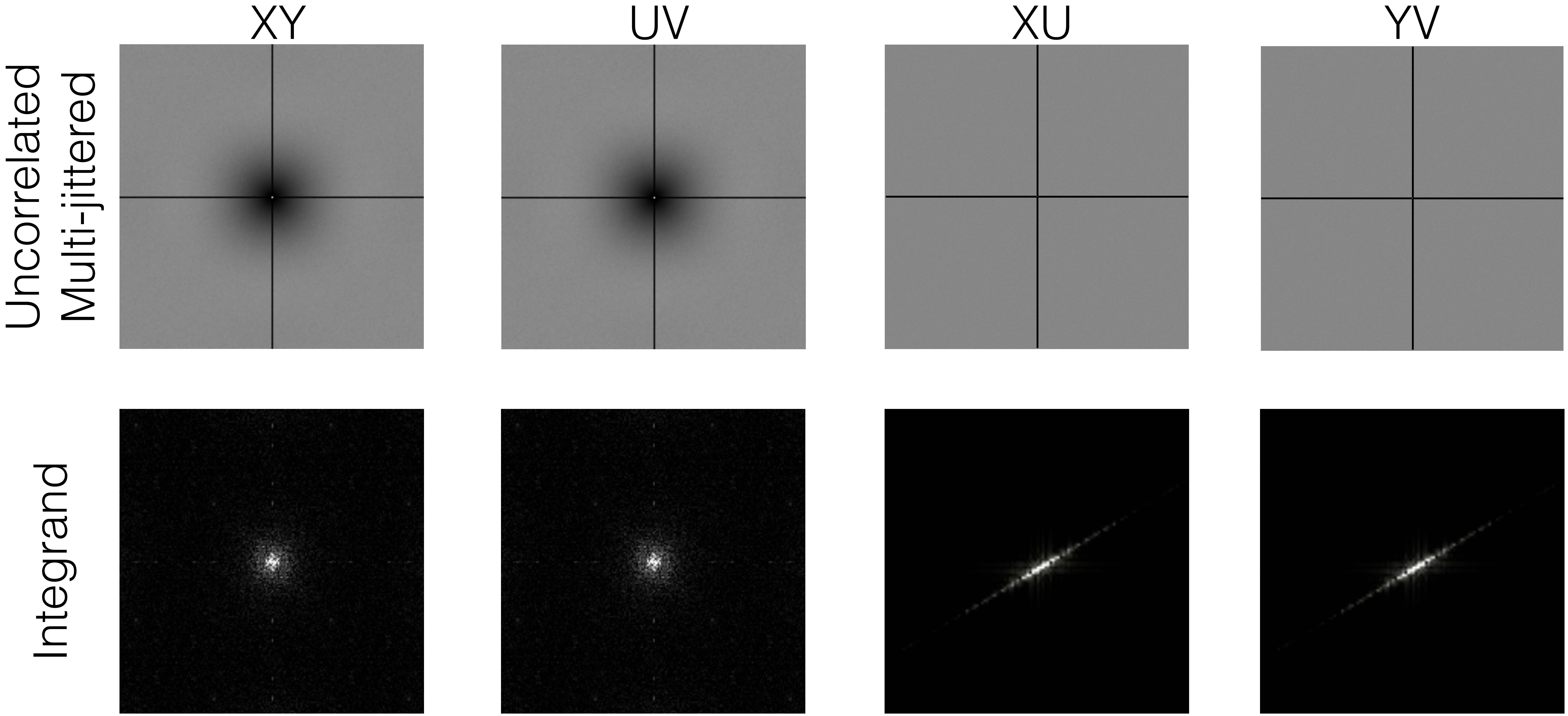
d: depth of the hit object

Shear increases with object depth





Spectra along Different Projections

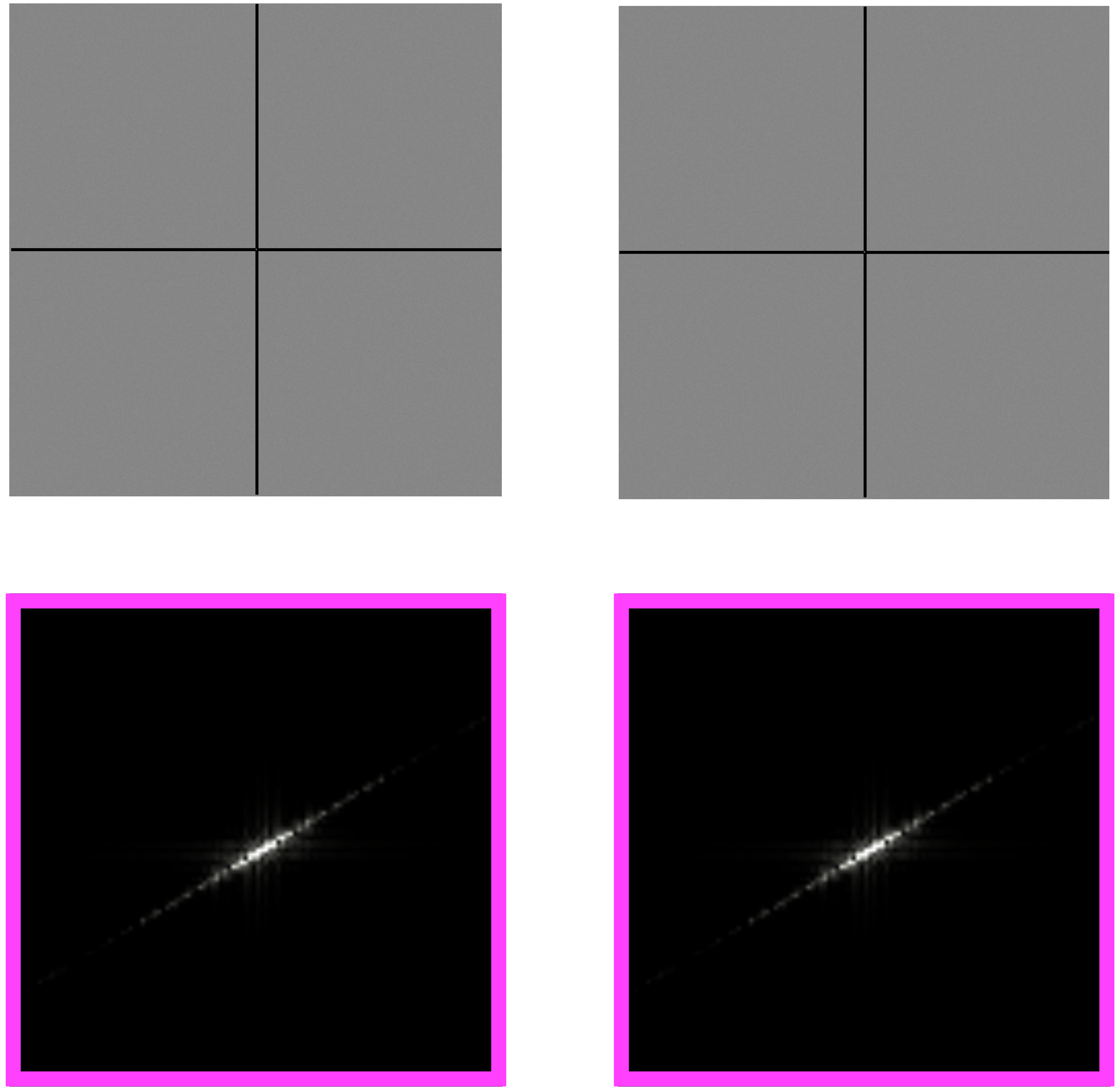


Spectra along Different Projections

XU YV

Uncorrelated Multi-jittered

Integrand



Spectra along Different Projections

XU YV

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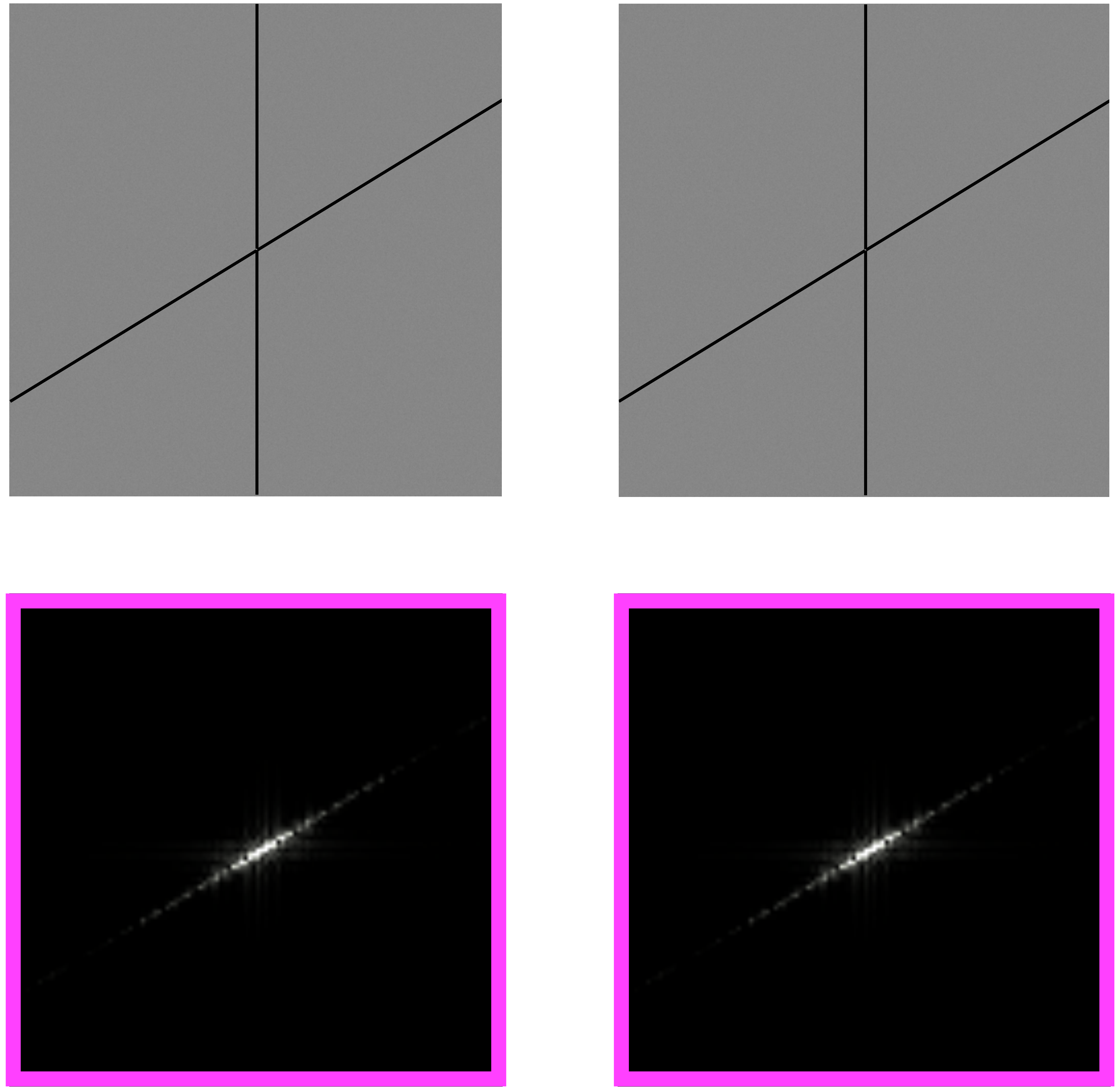
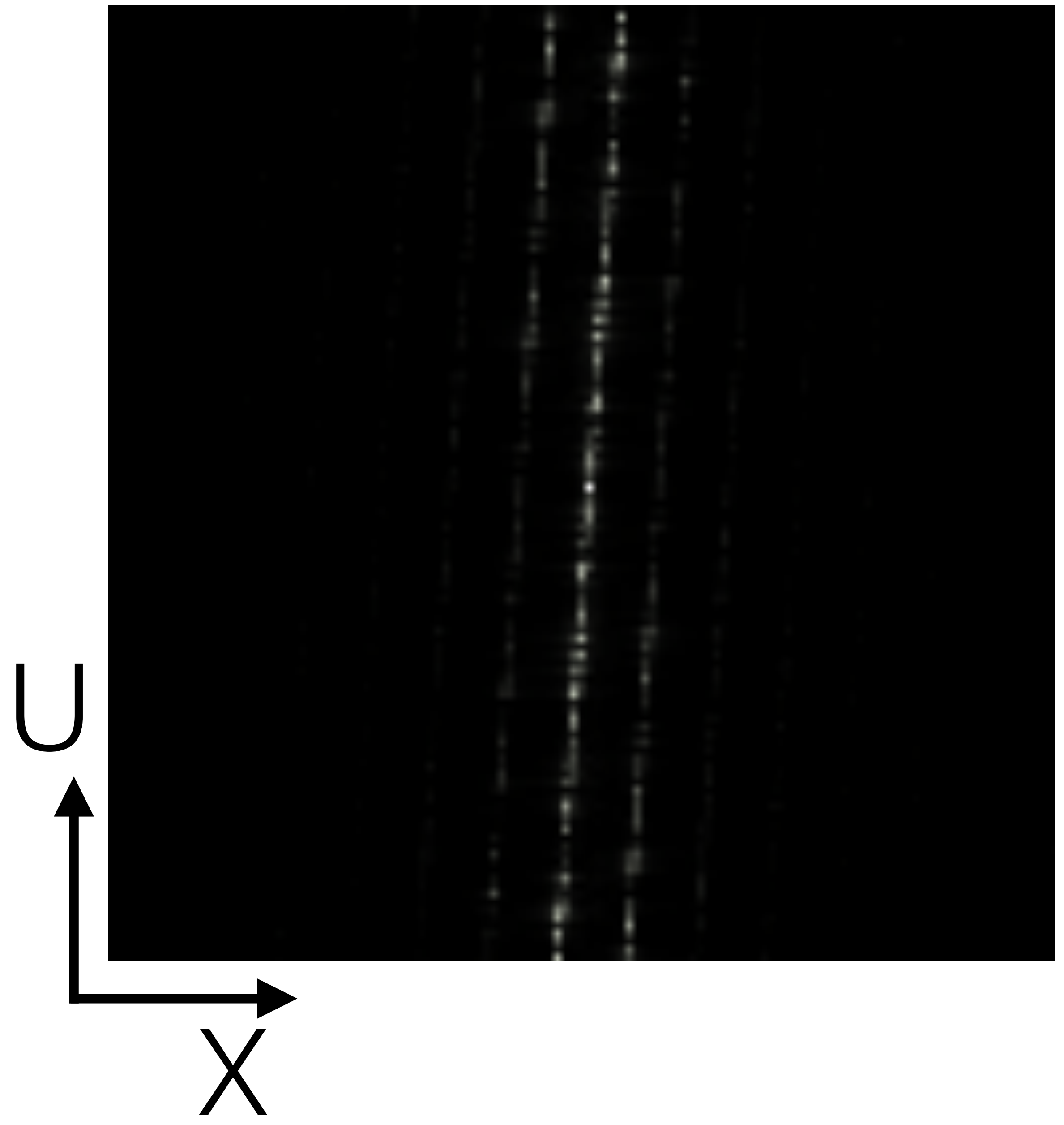


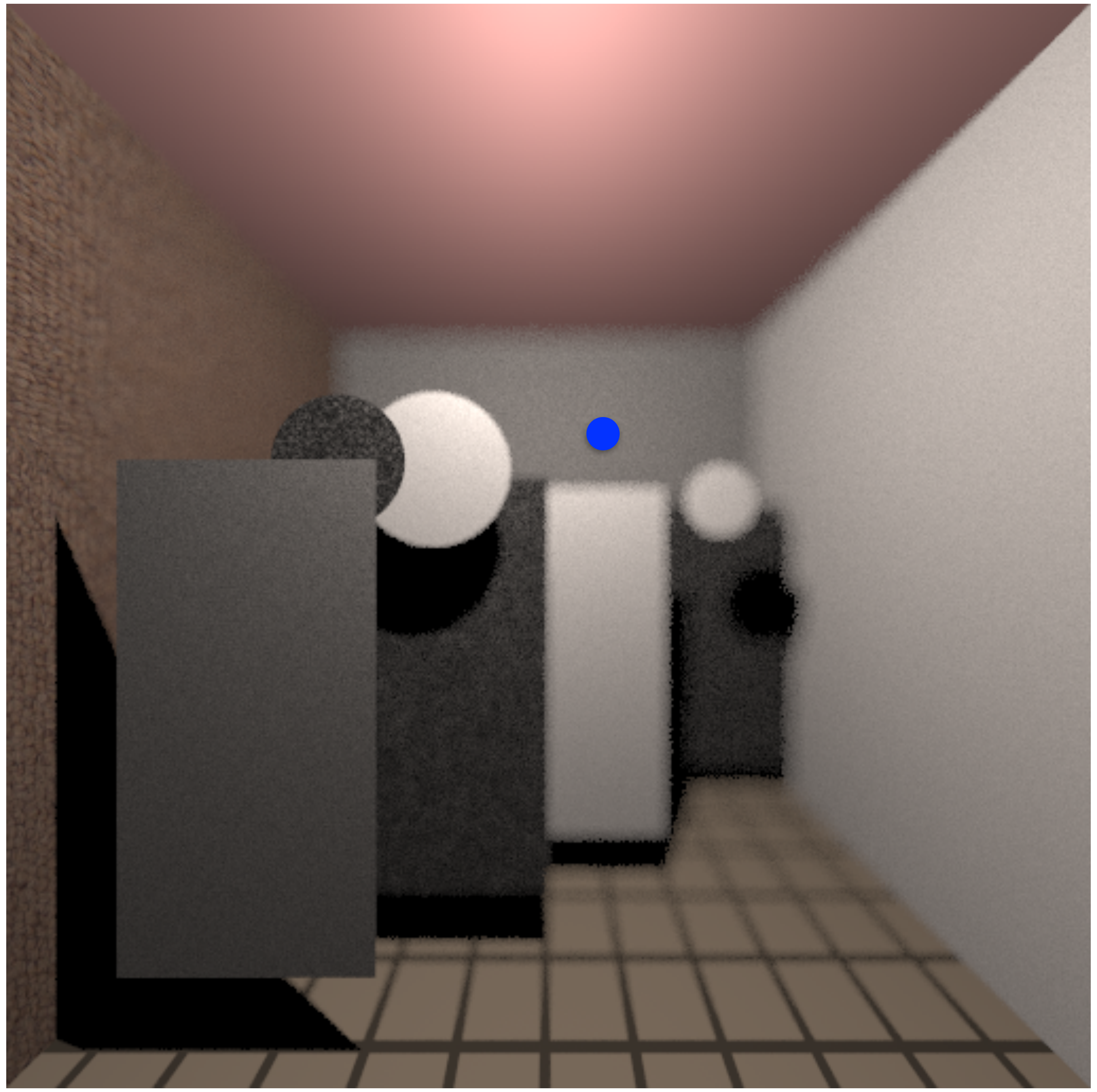
Variance & Convergence Analysis with Sheared Samples

Cornell Box Scene

XU Projection



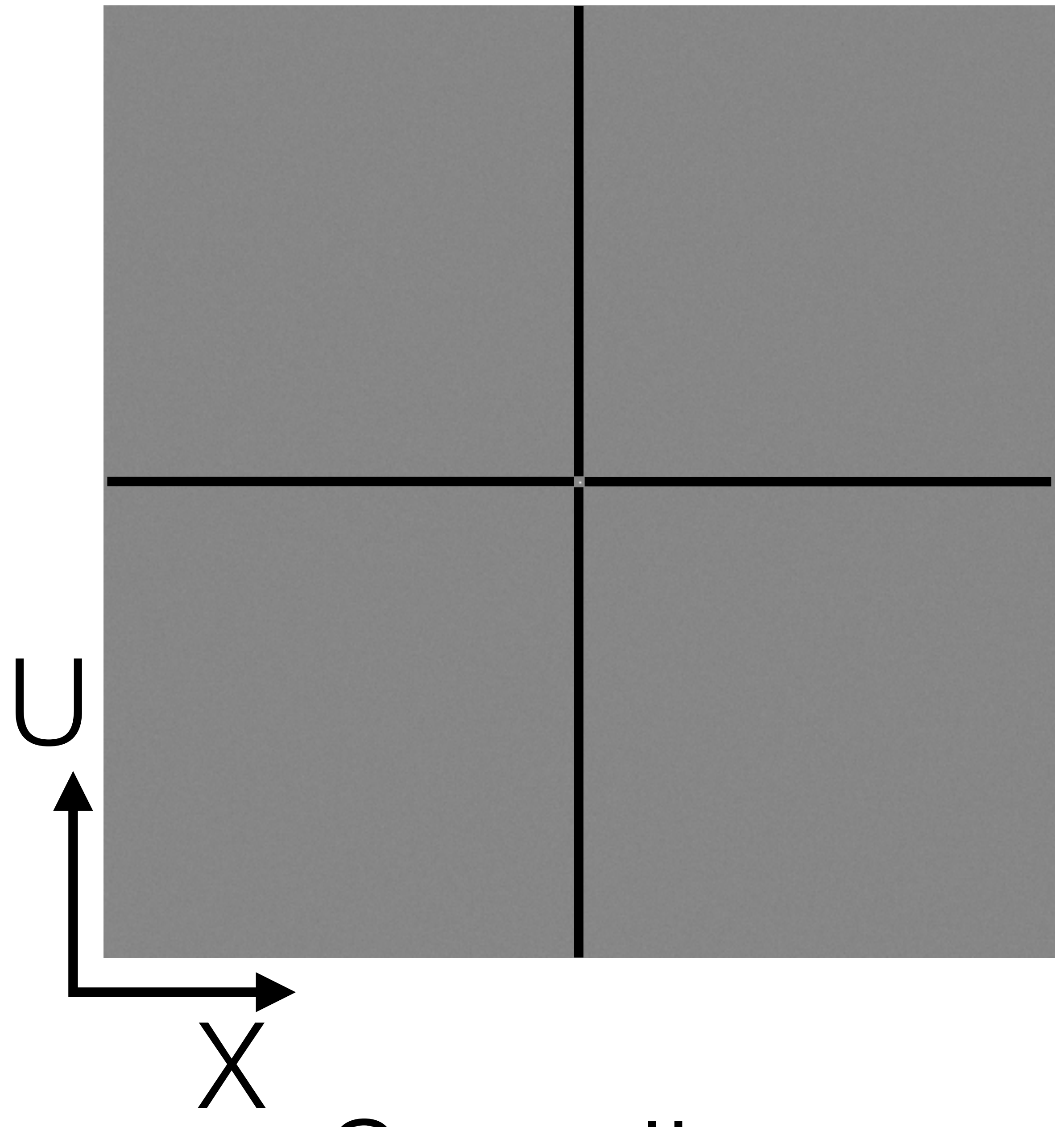
Integrand Spectrum



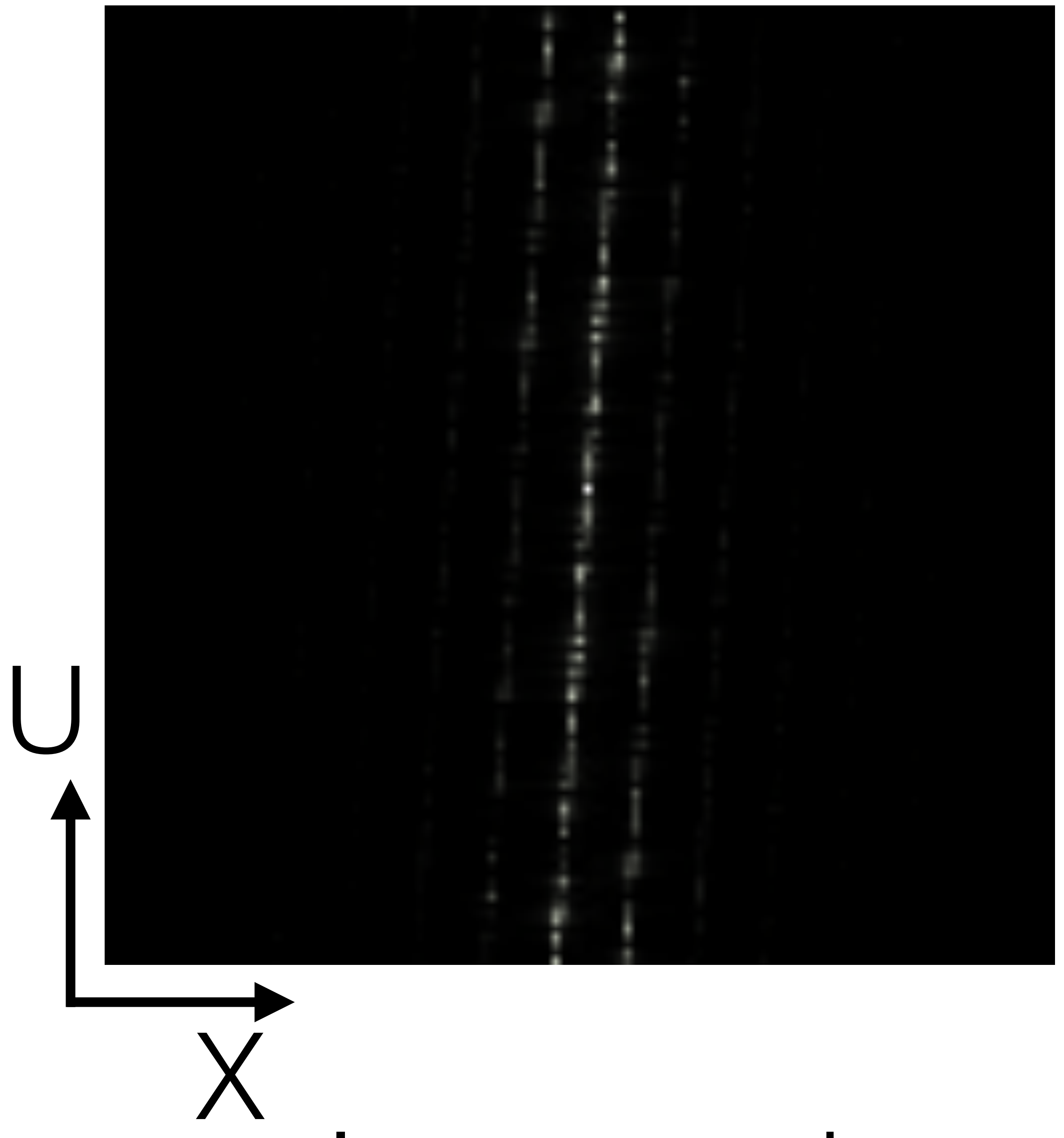
$$\int_x \int_y \int_u \int_v f(x, y, u, v)\, dv\, du\, dy\, dx$$

Original Uncorrelated-MultiJittered Samples

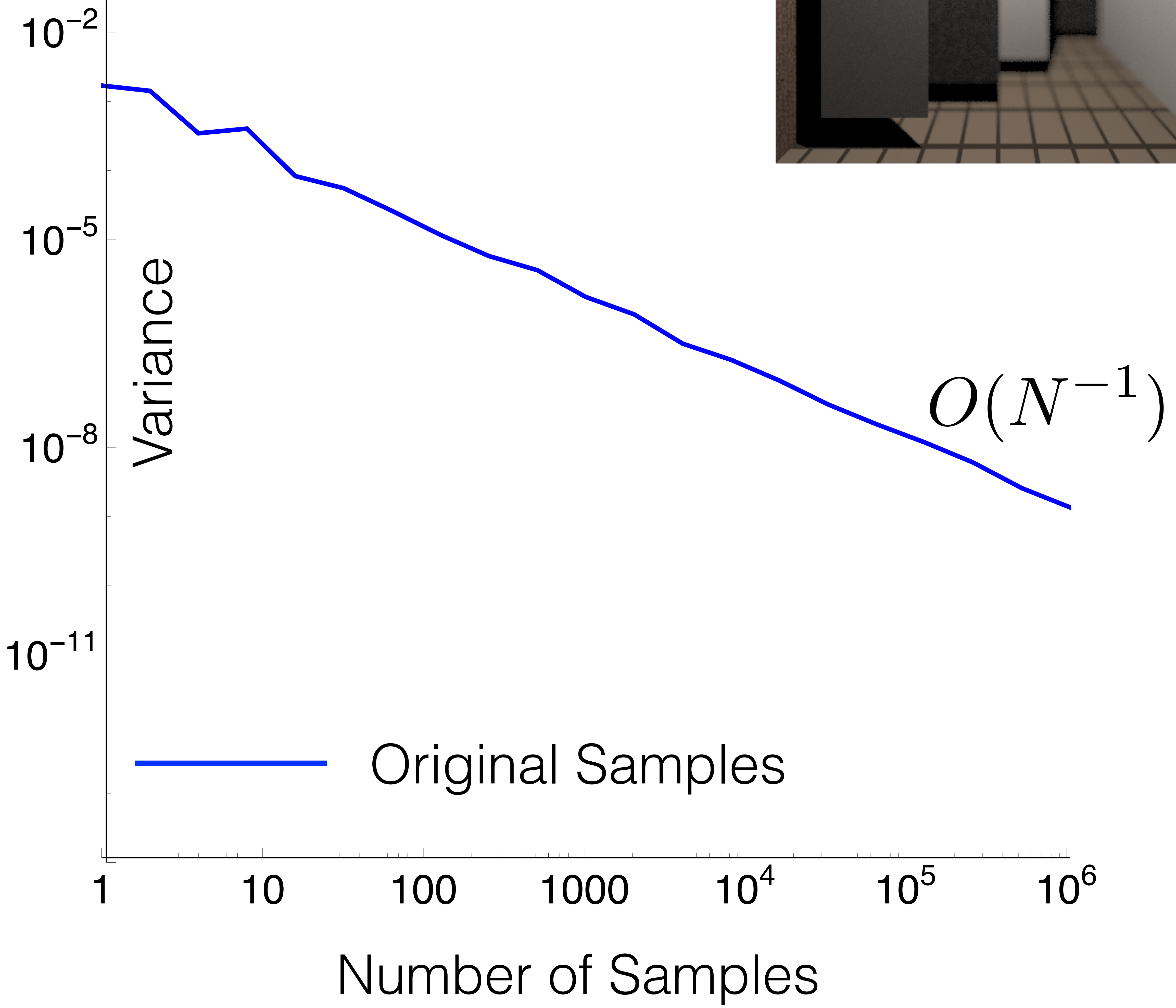
XU Projection

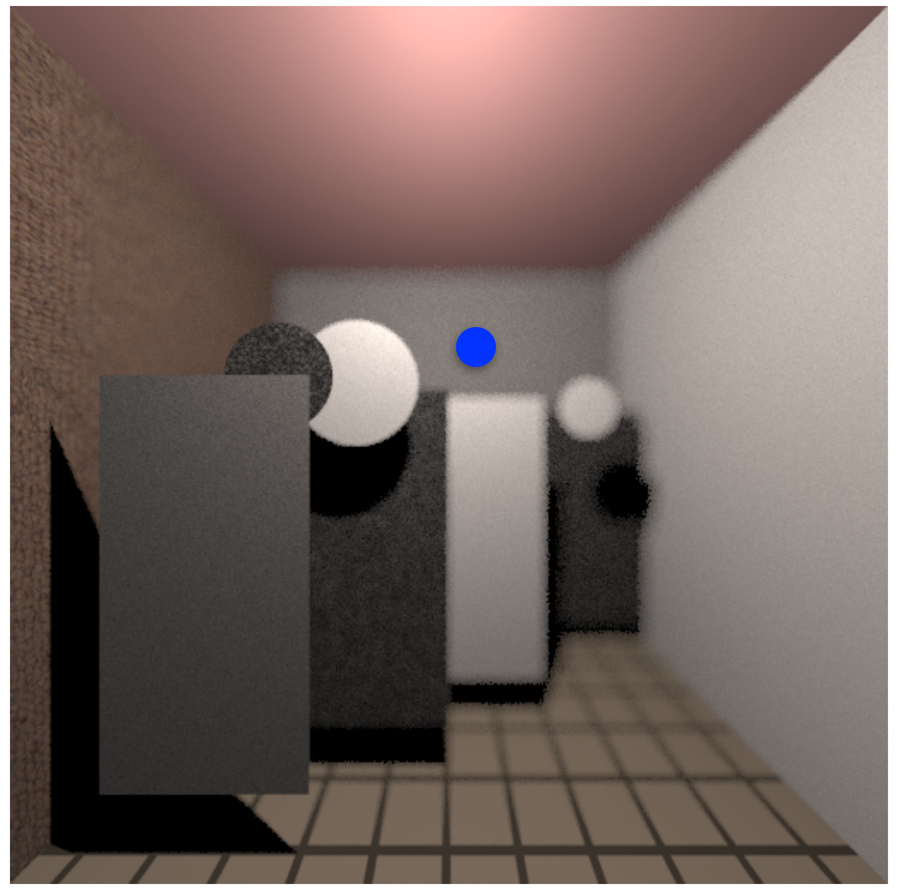


Sampling Spectrum

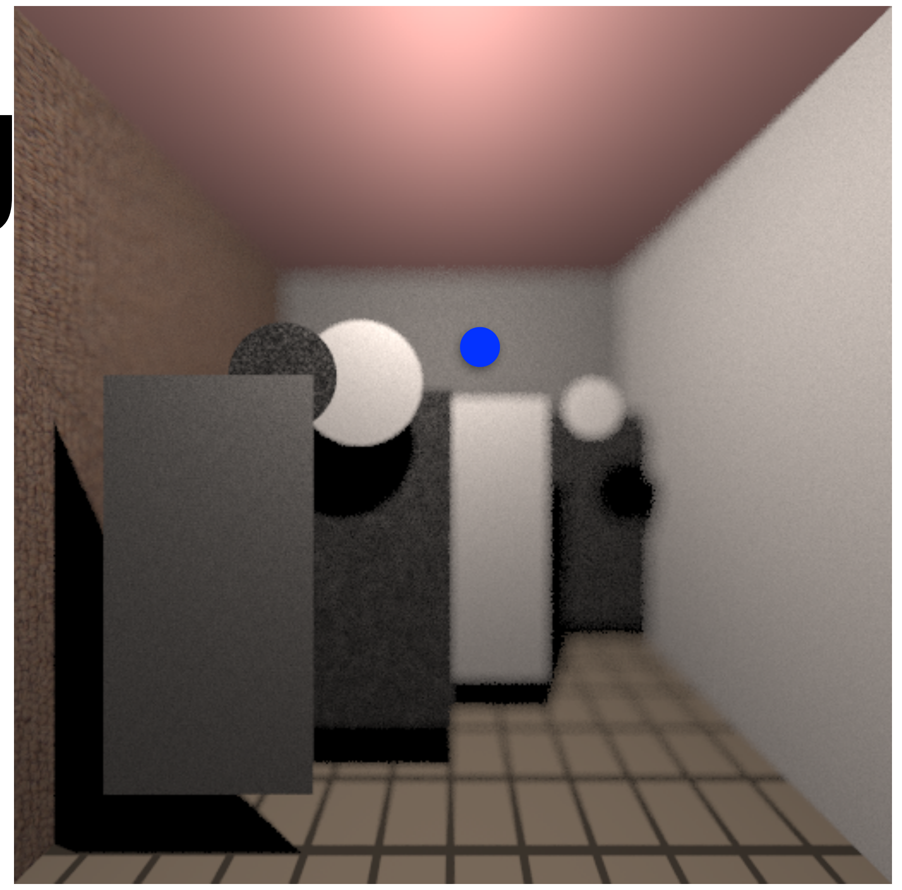


Integrand Spectrum

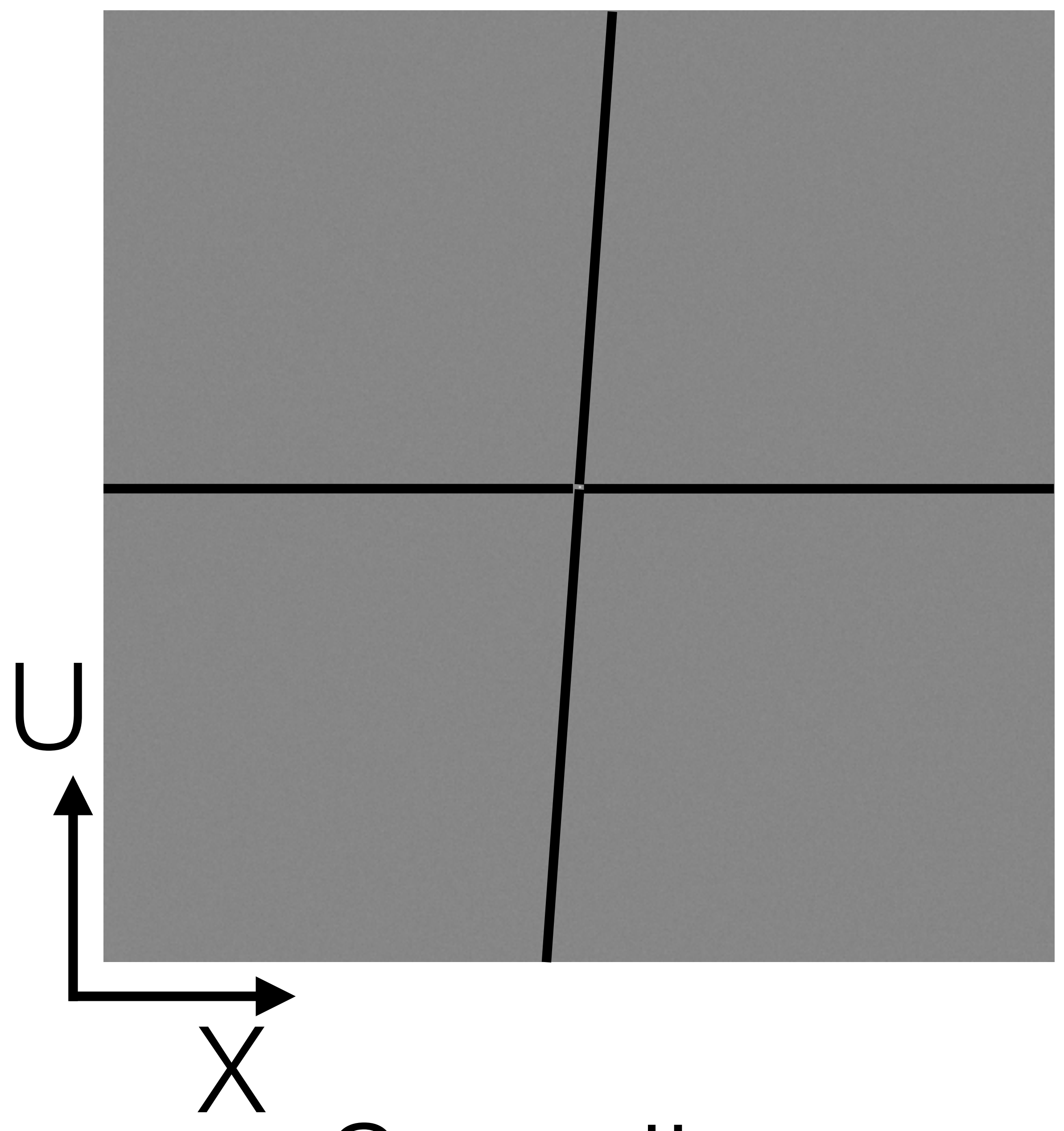




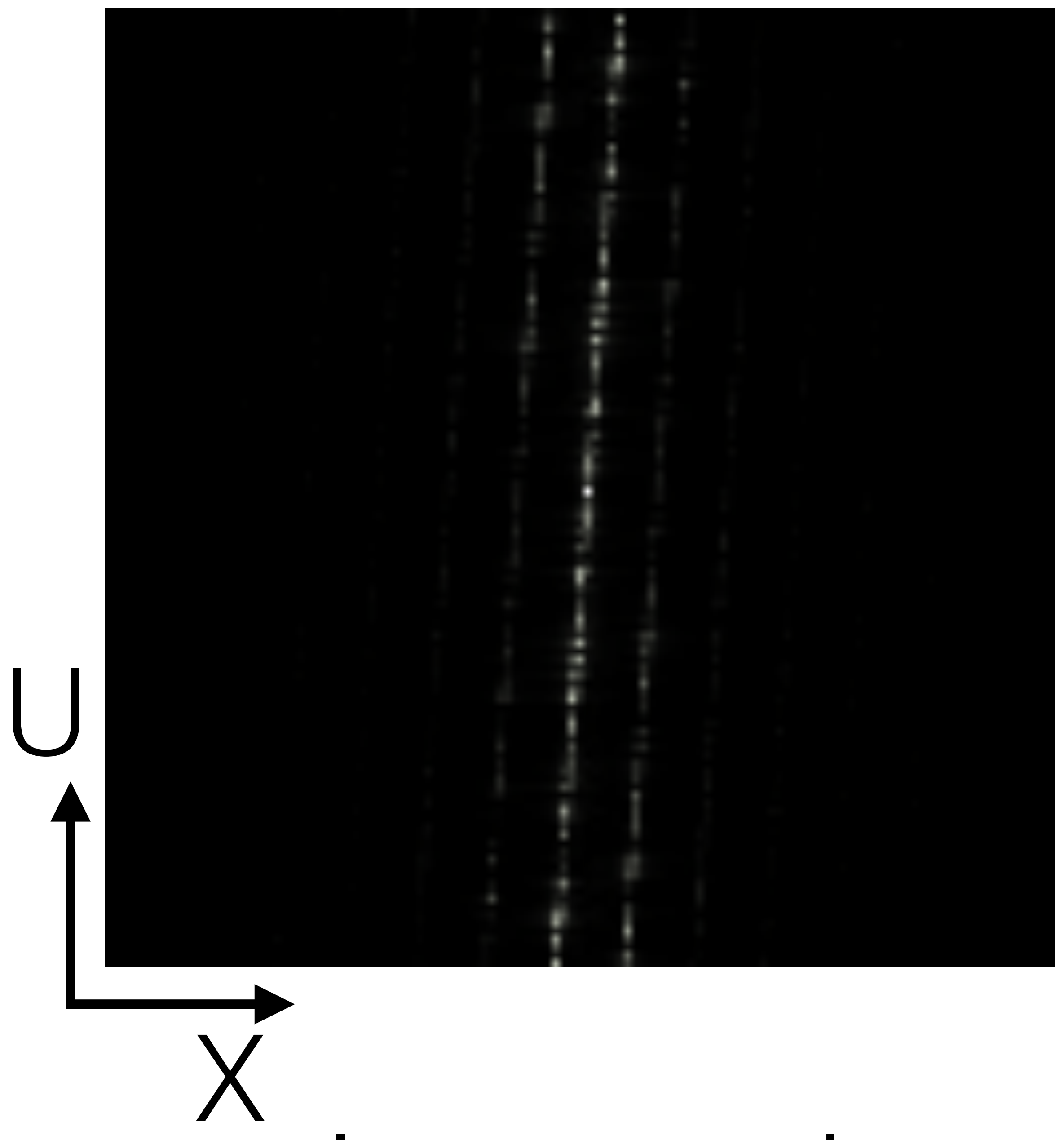
Convergence improvement after Shearing



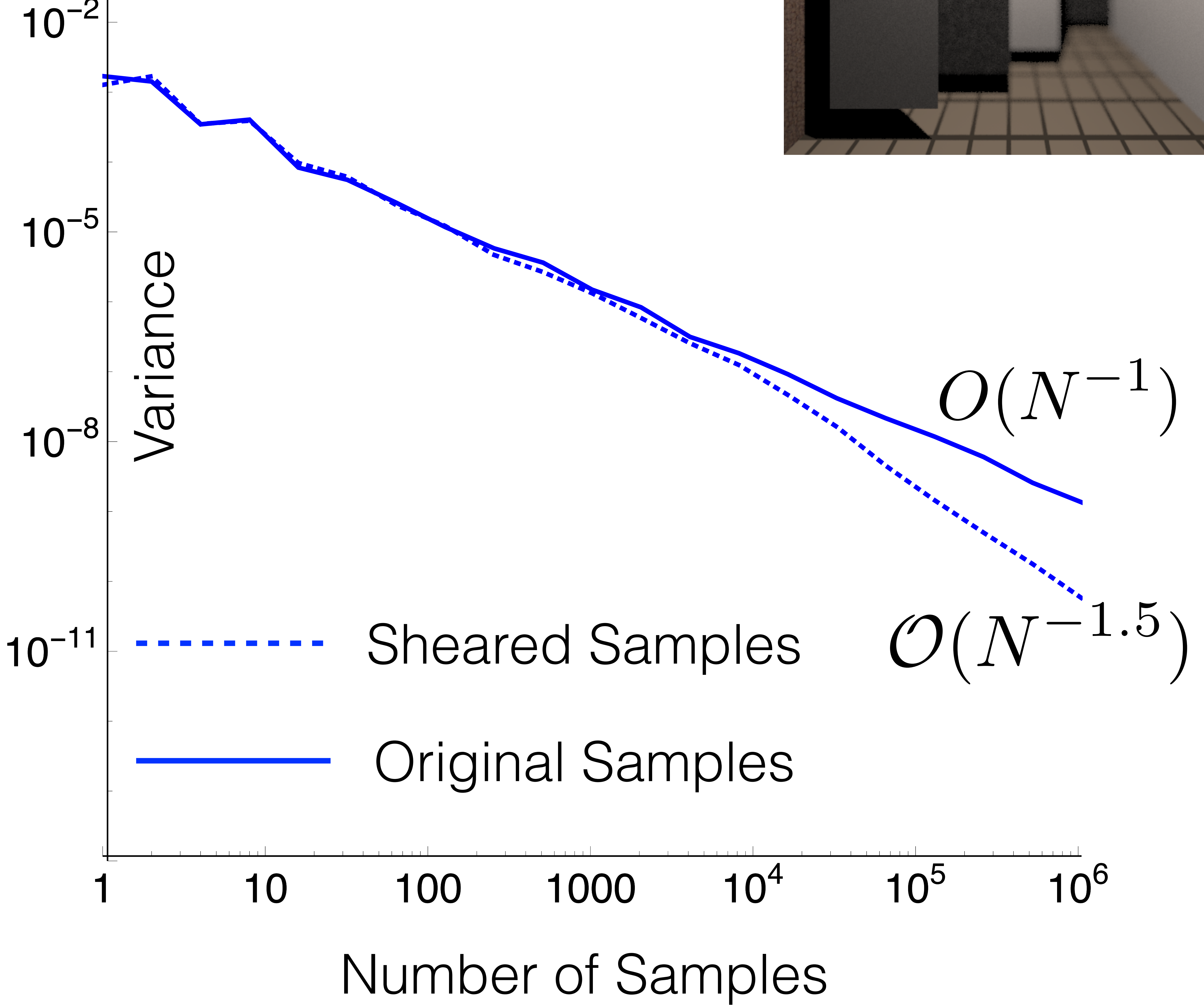
XU Subspace



Sampling Spectrum

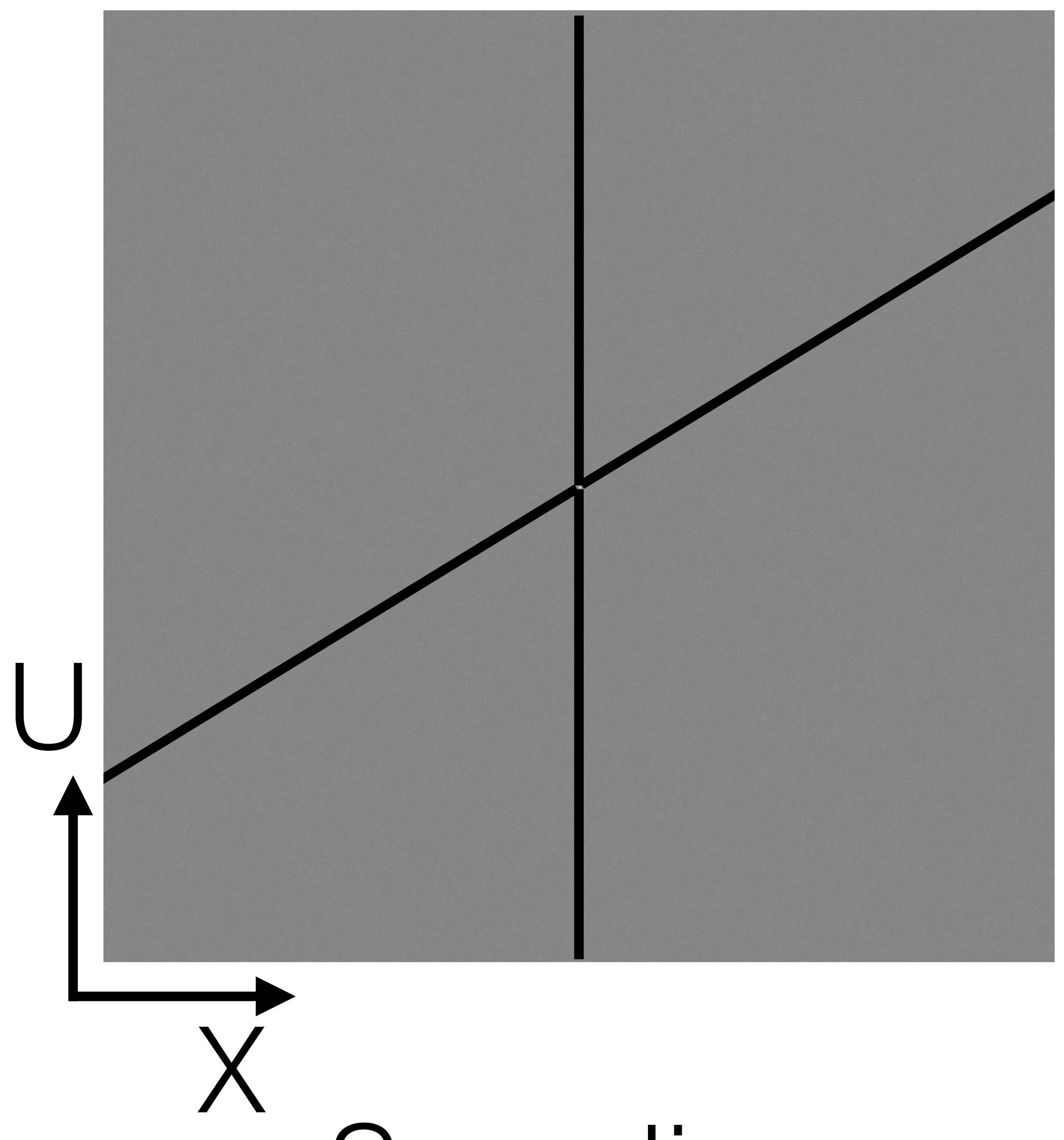


Integrand Spectrum

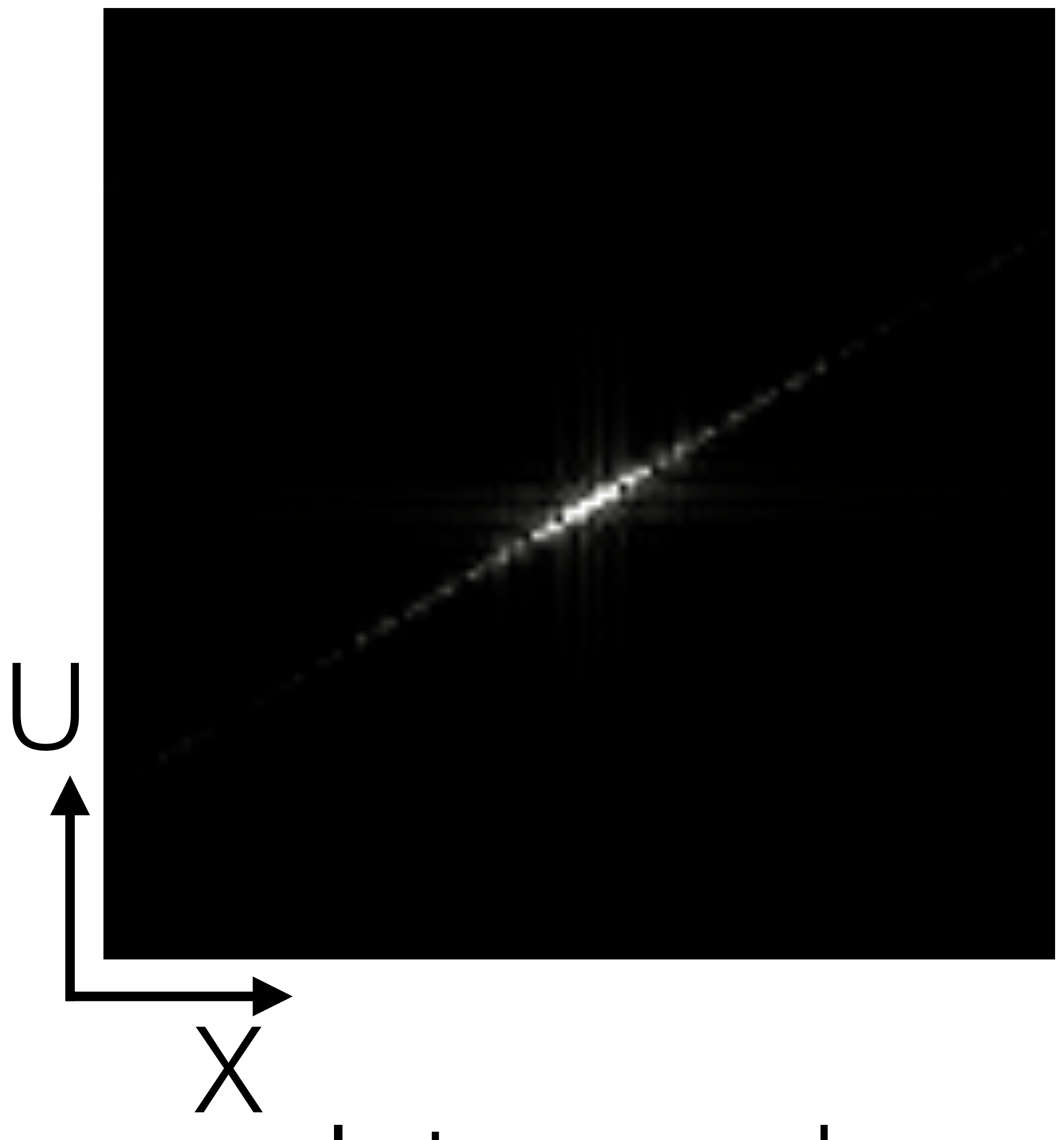


Sheared Uncorrelated Multi-jittered Samples

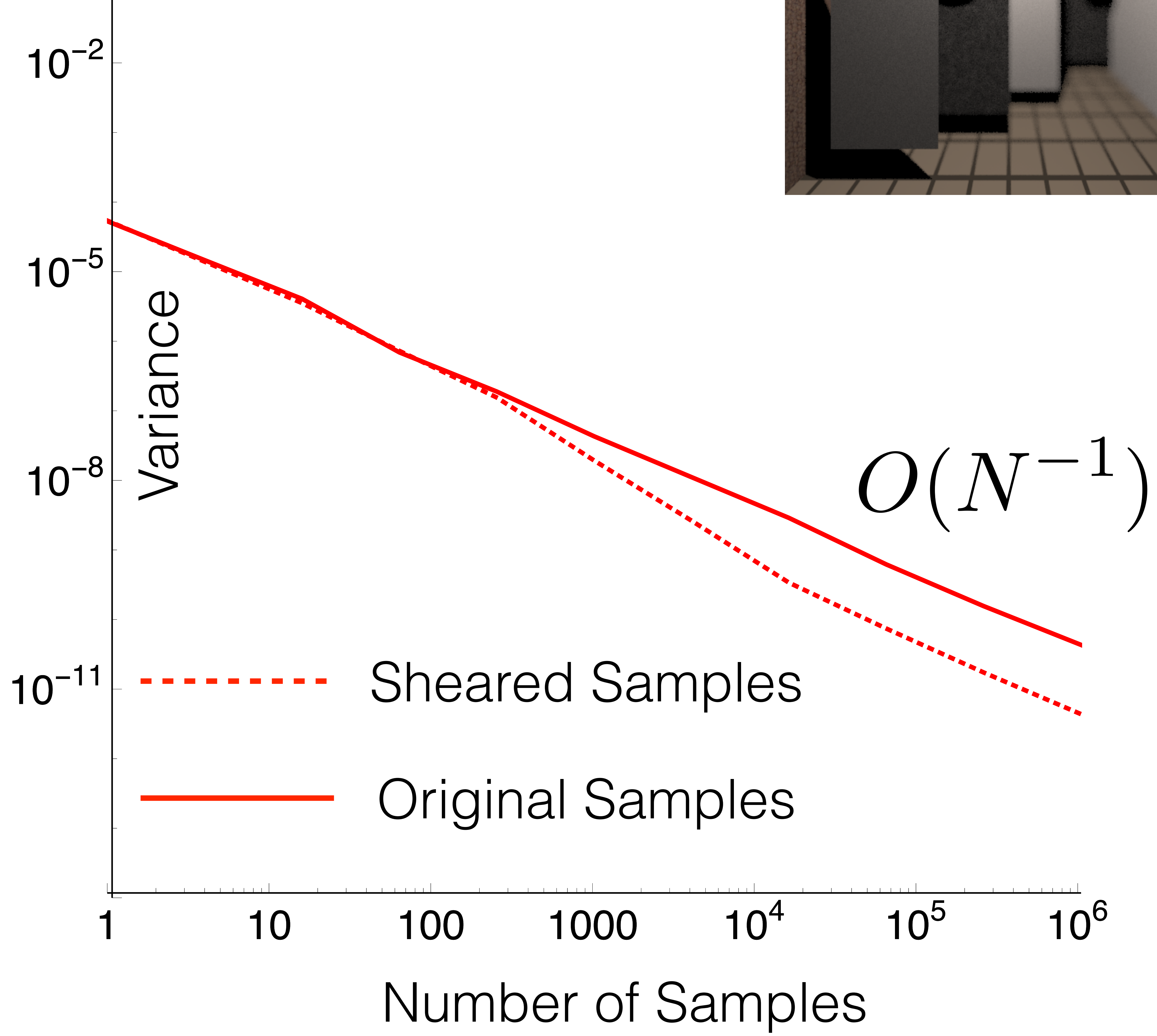
XU Subspace

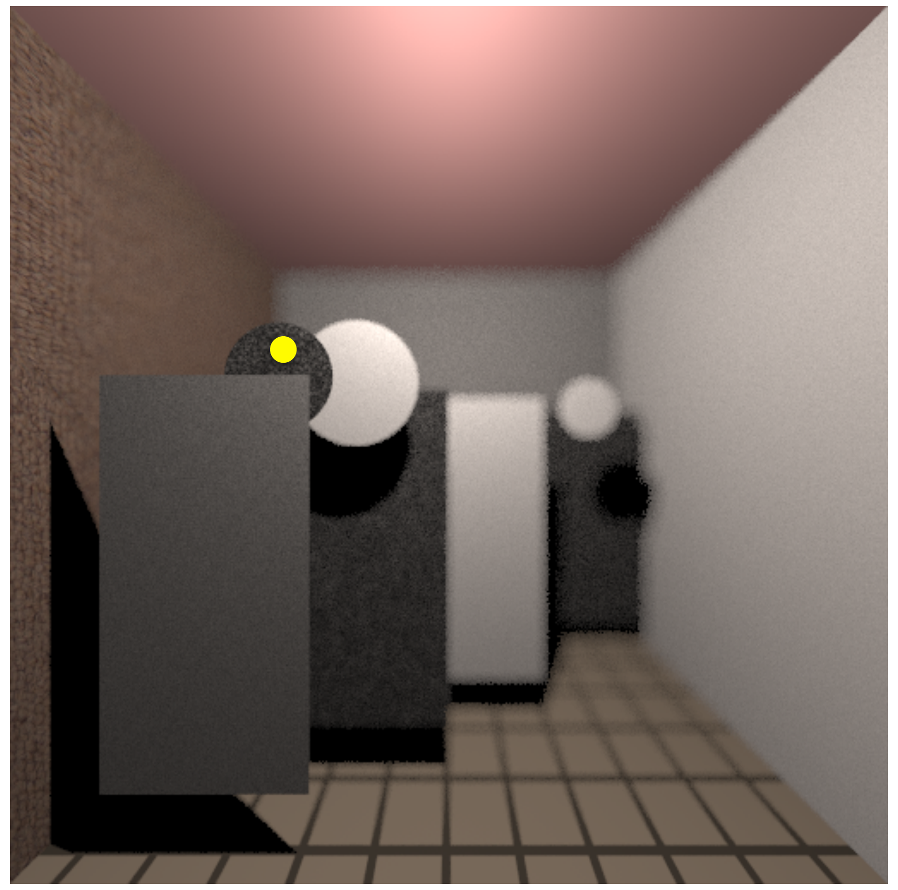


Sampling Spectrum

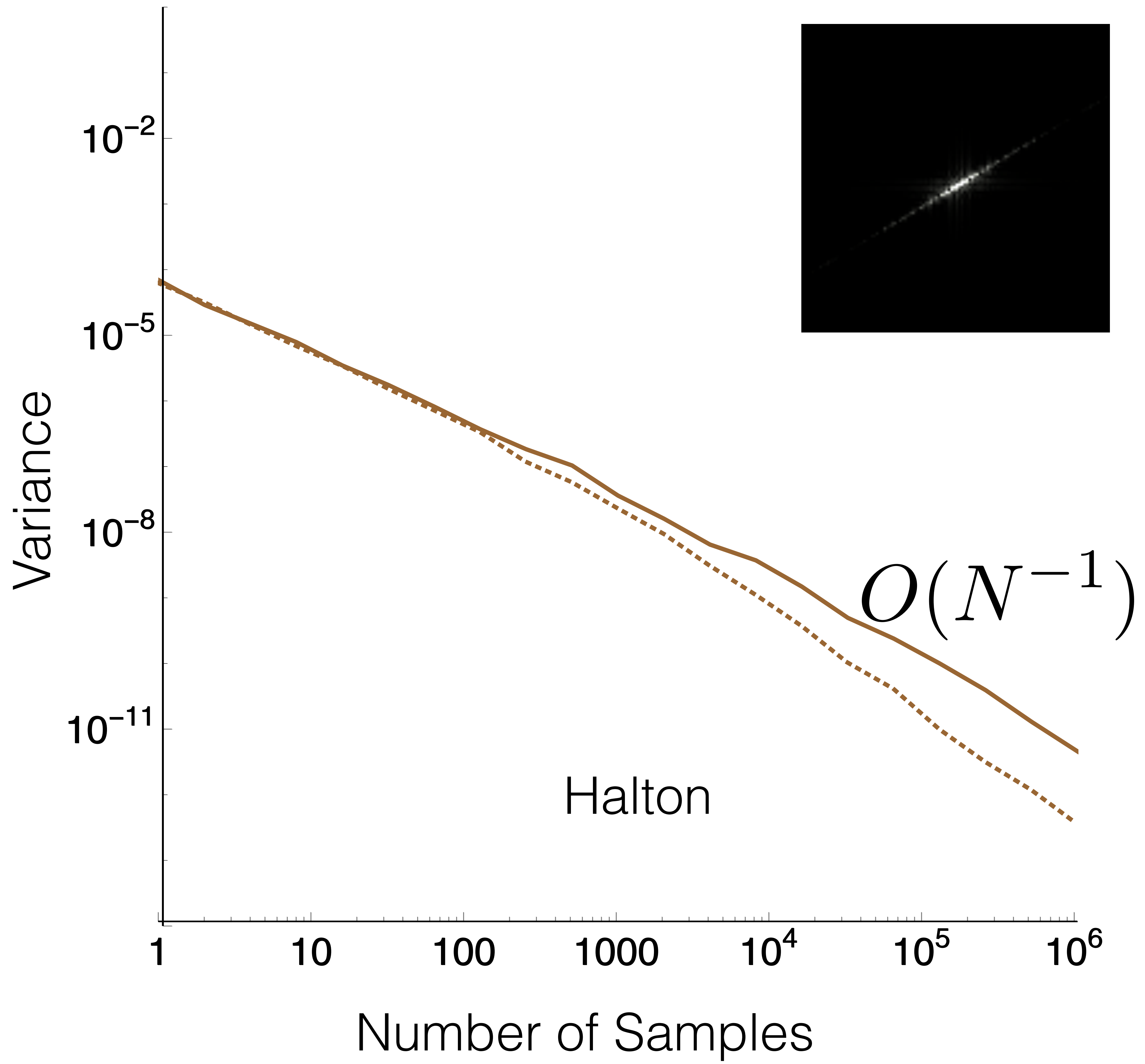


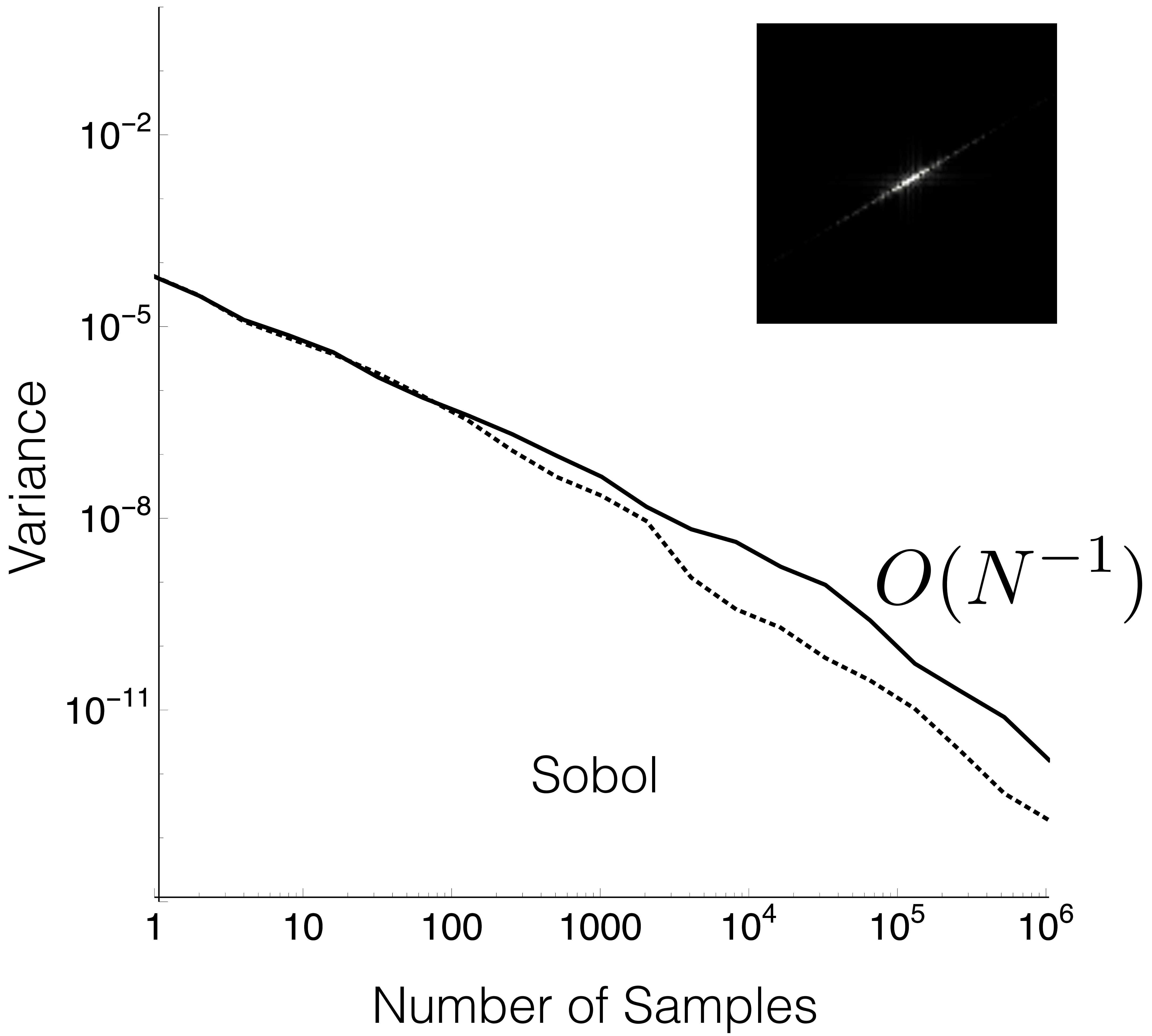
Integrand Spectrum





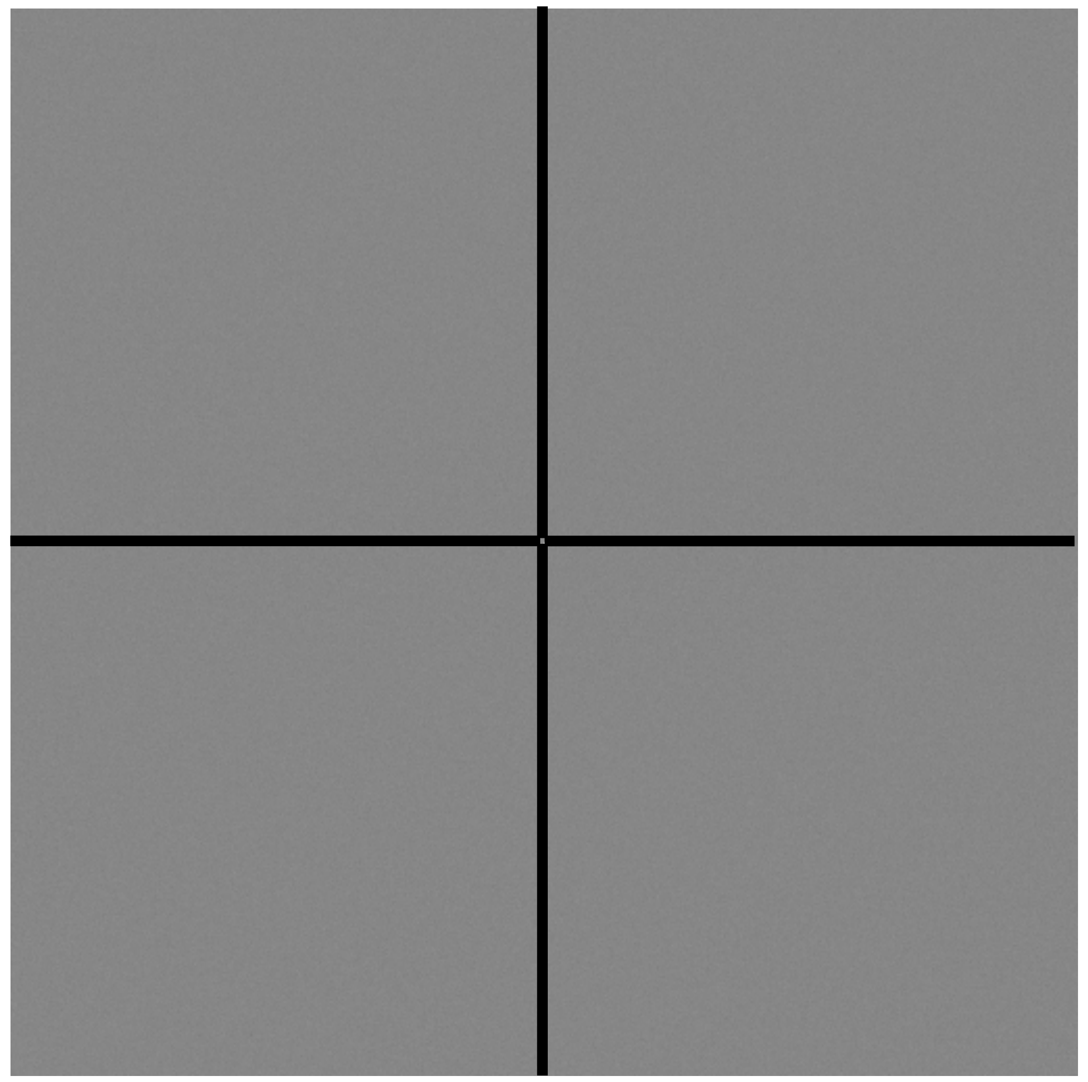
Variance improvement after Shearing



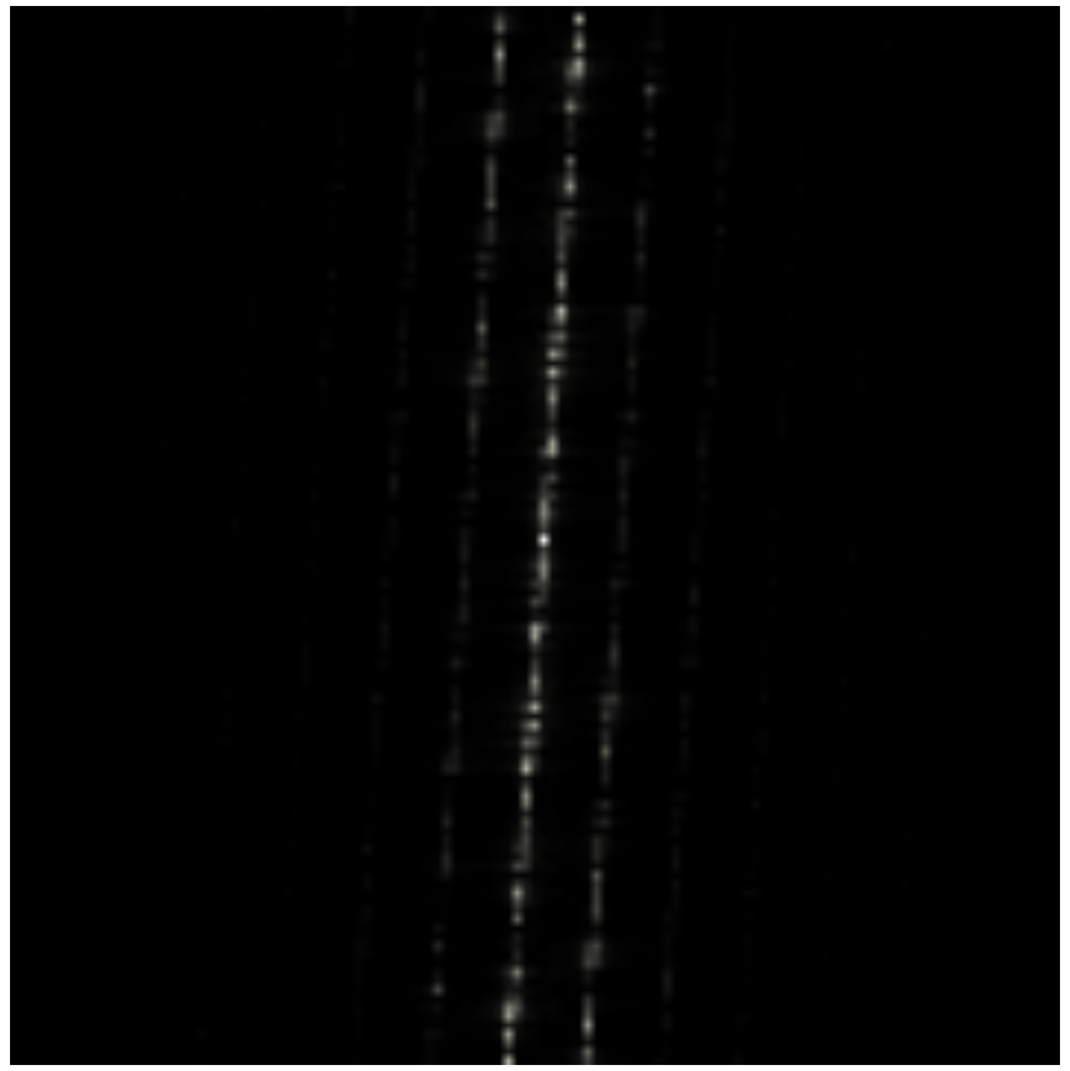


Challenging Cases: XU & YV Projections

Hairline Anisotropy

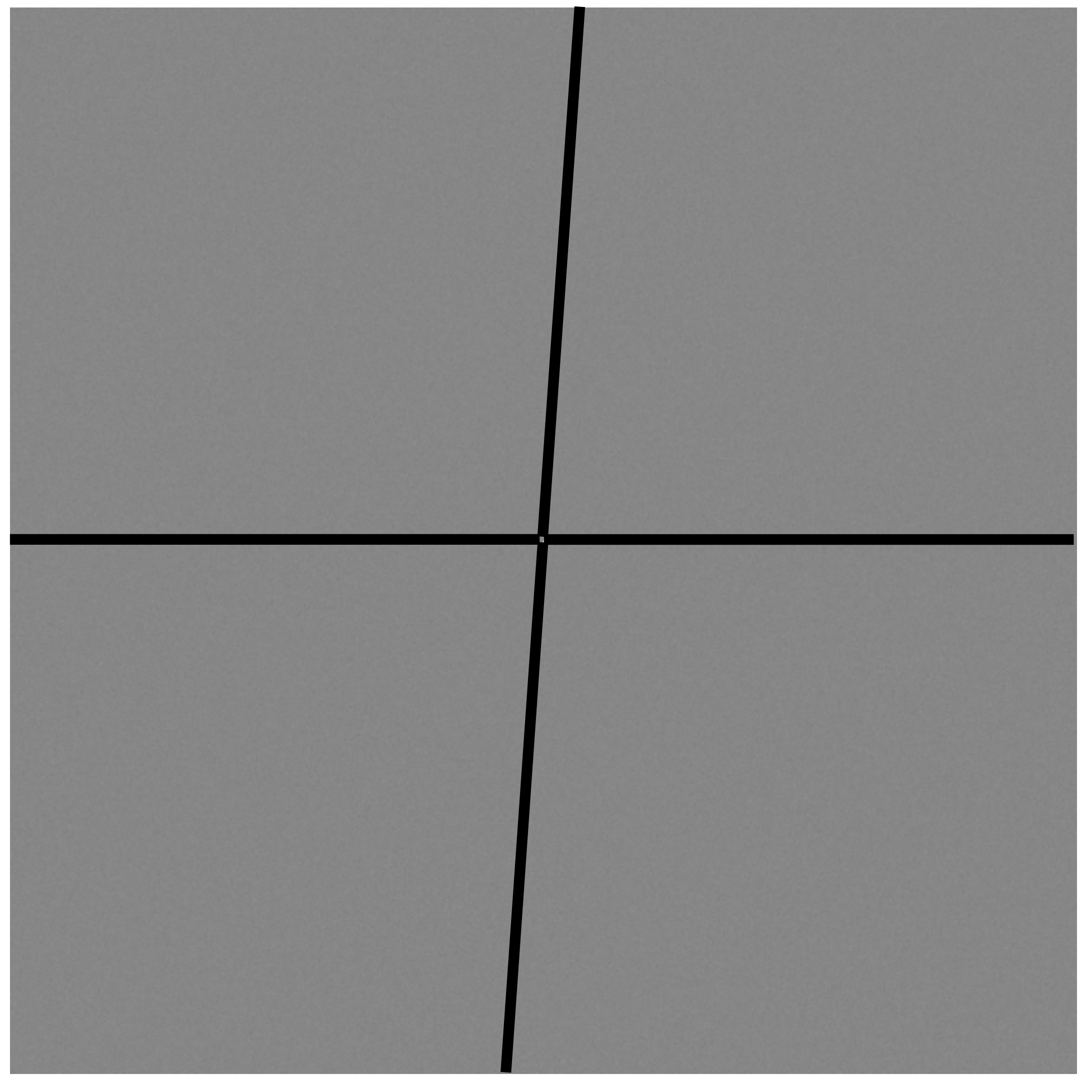


Sampling
XU Spectrum



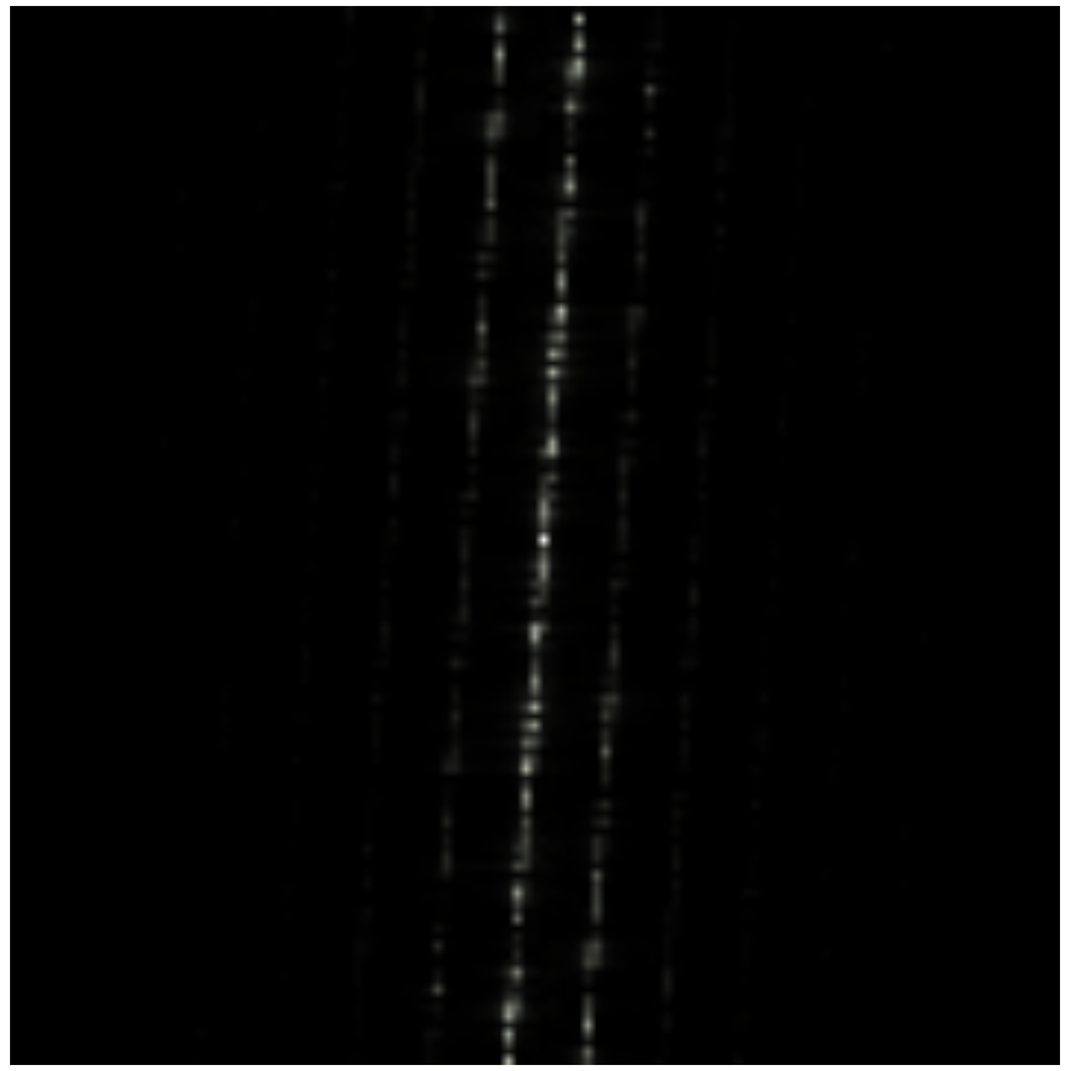
Pixel A
XU Spectrum

Challenging Cases: XU & YV Projections



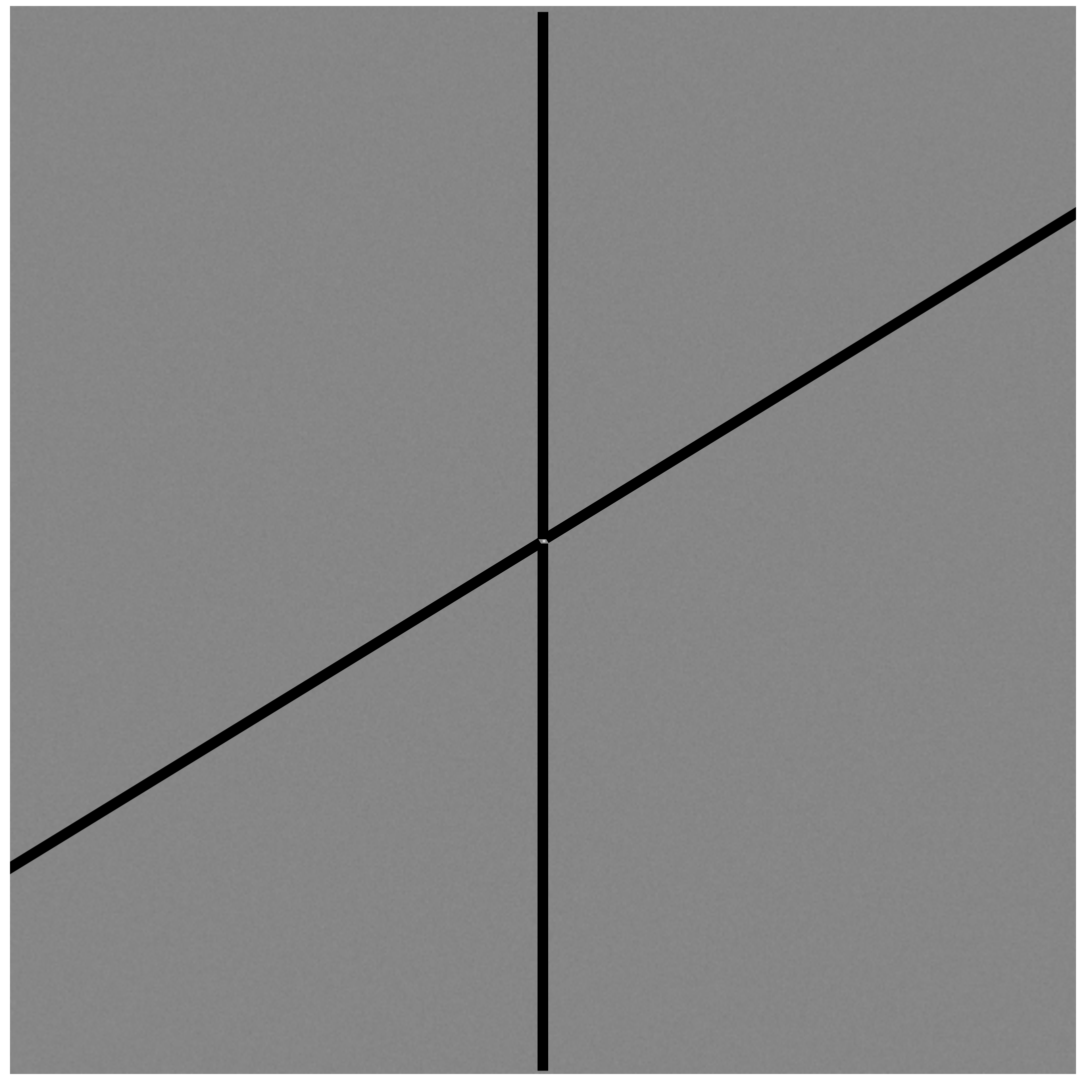
Sampling
XU Spectrum

Hairline Anisotropy



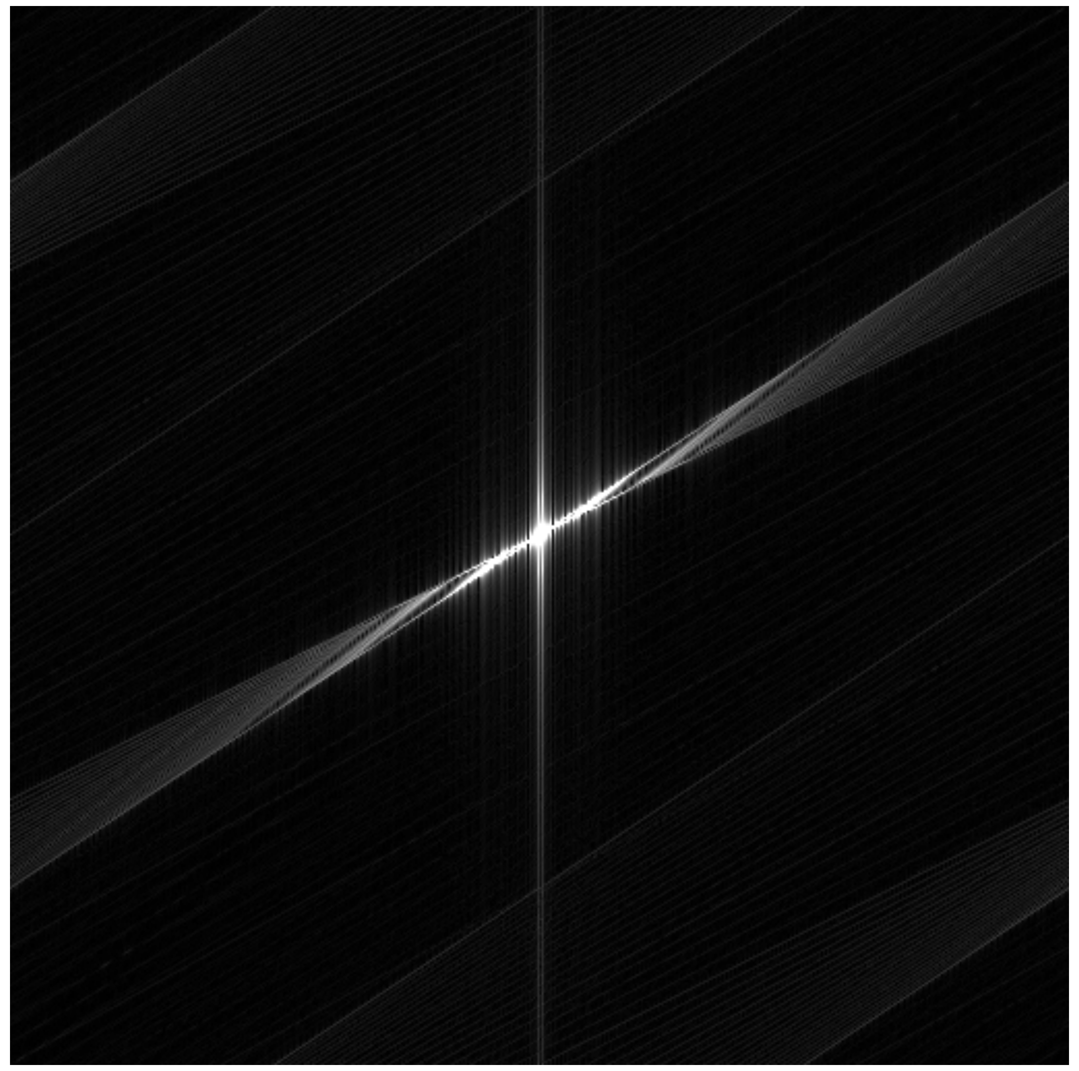
Pixel A
XU Spectrum

Oracle Accuracy



Sampling
XU Spectrum

Double-wedge Anisotropy



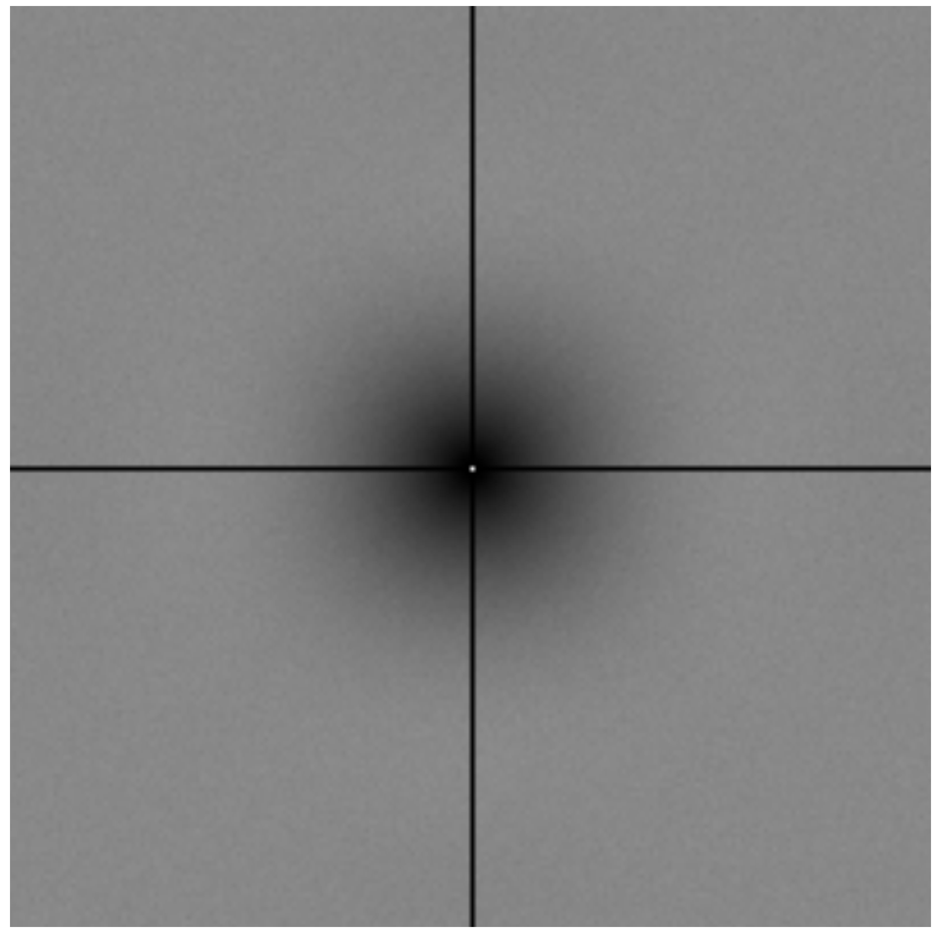
Pixel B
XU Spectrum

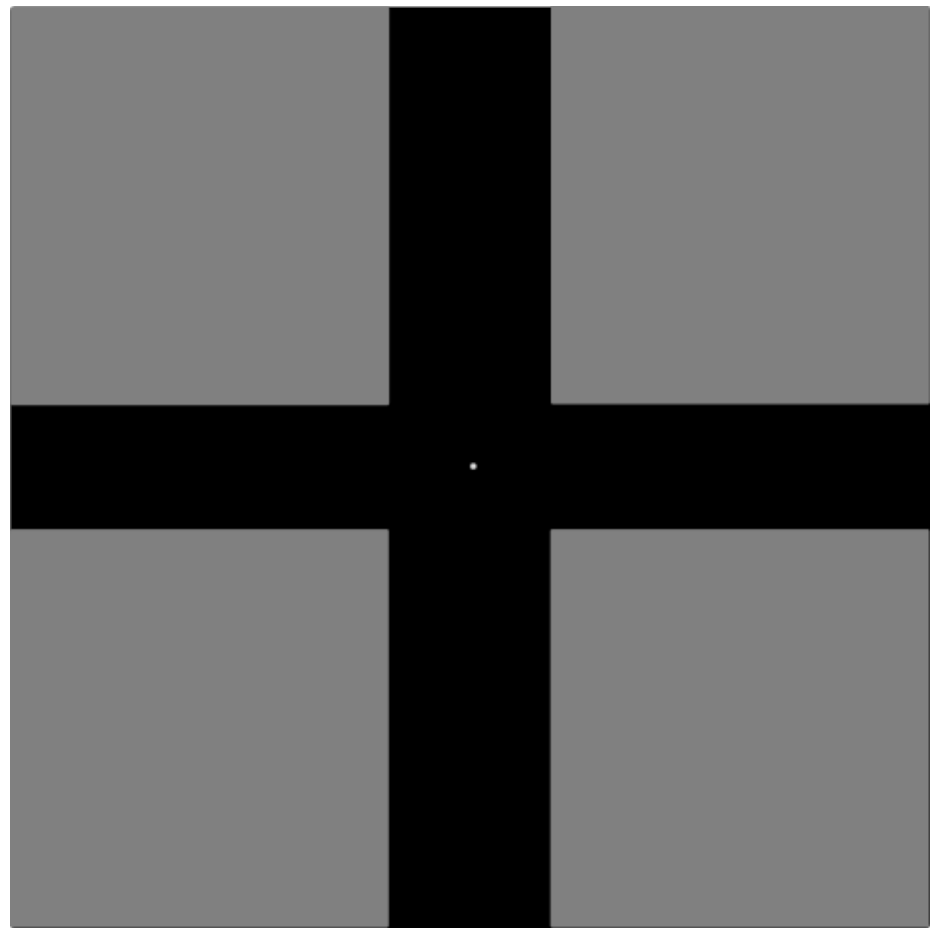
Double-wedge Spectrum

Design Principles for New Sampling Patterns

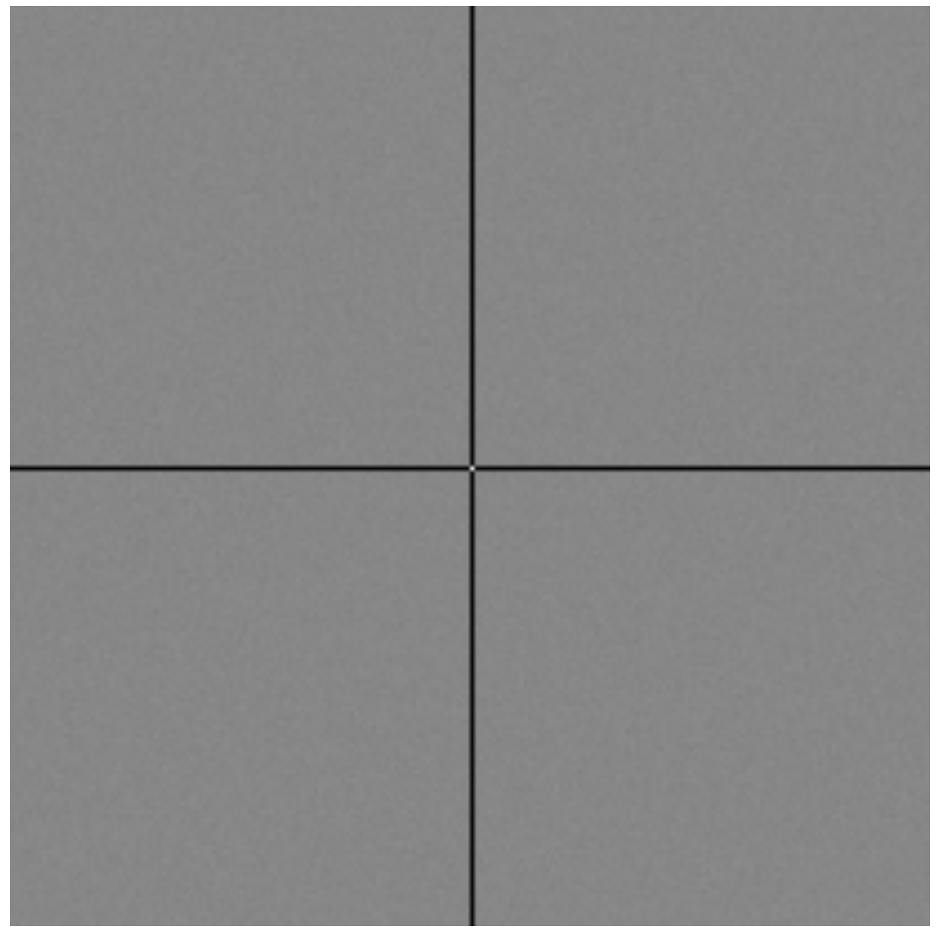
Multi-Jittered Spectra | Desired Sampling Spectra

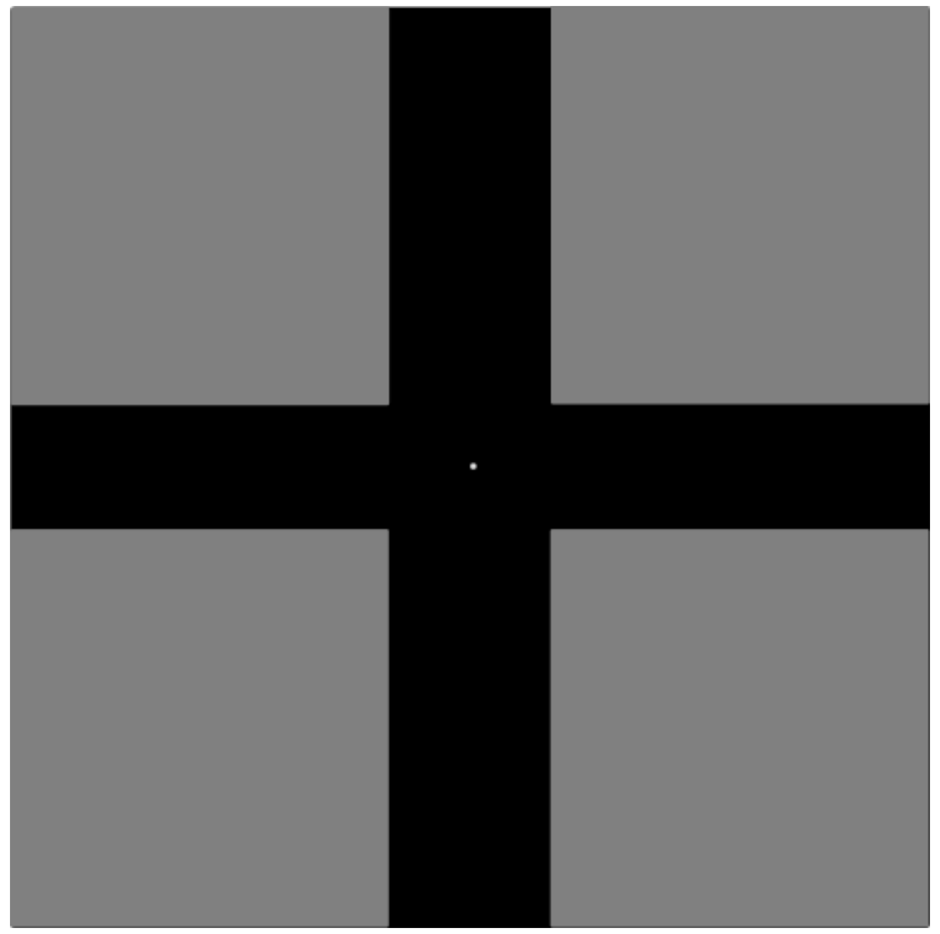
XY





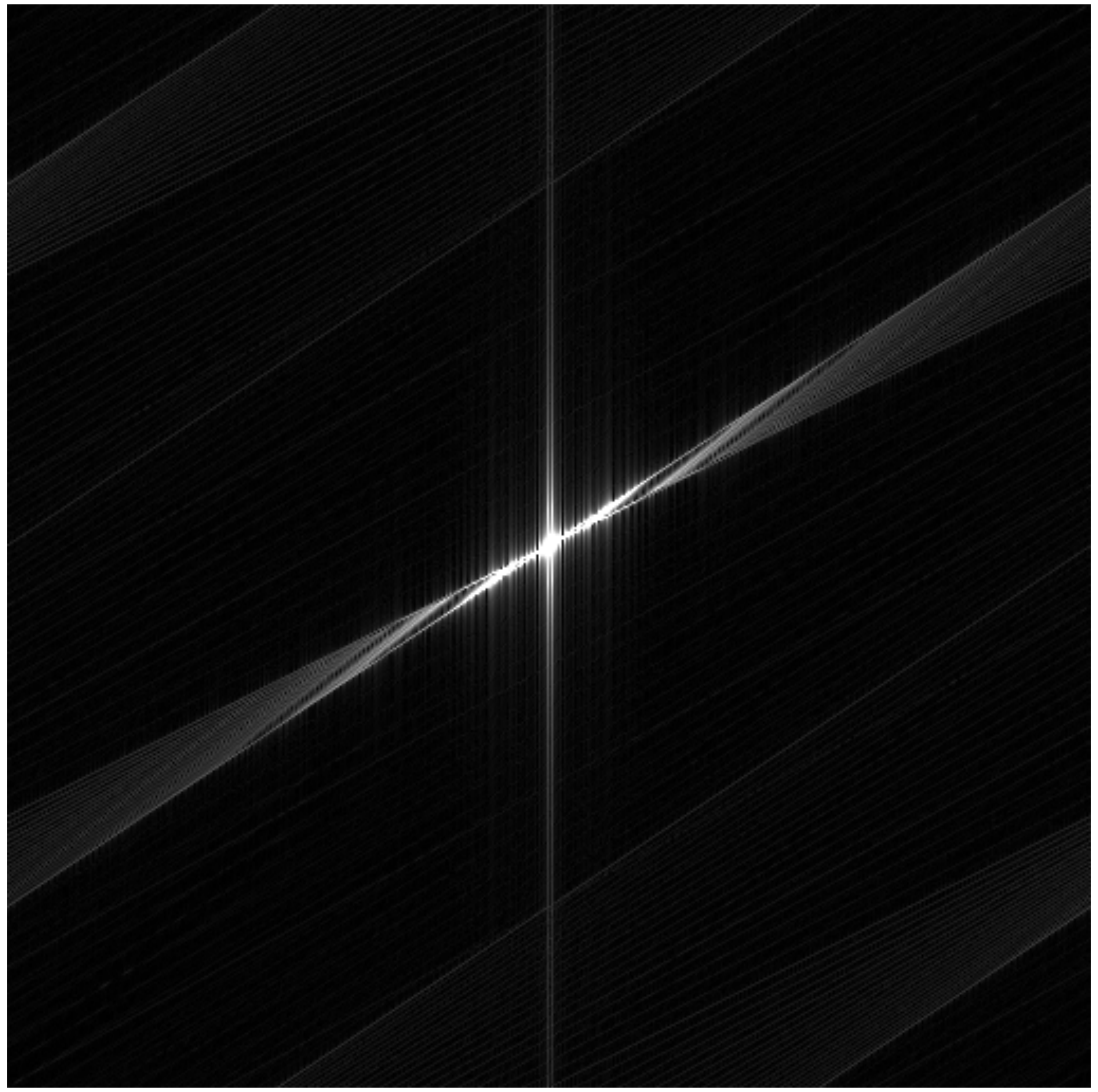
XU



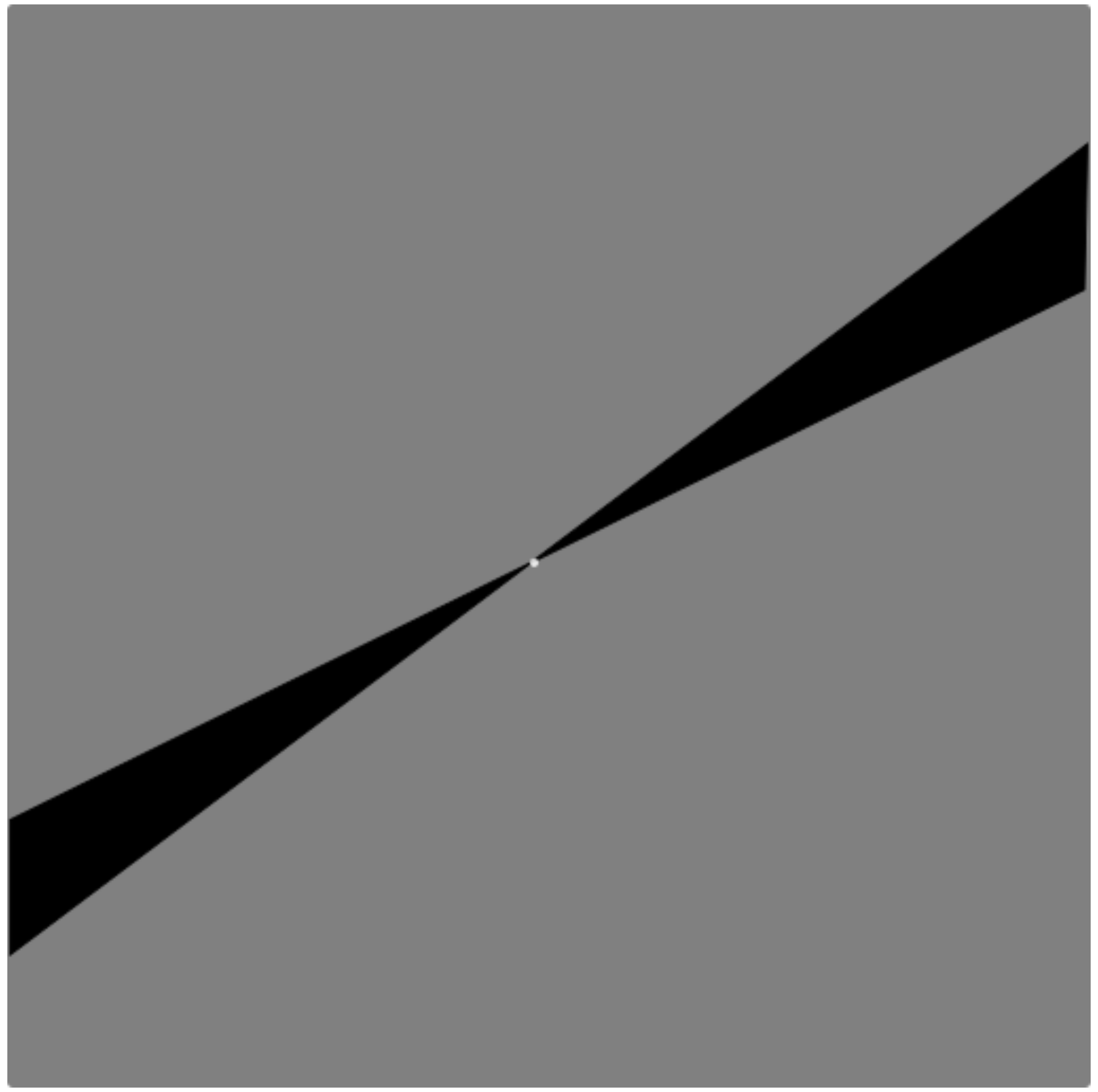


Design Principles for New Sampling Patterns

Integrand Spectrum



Desired Sampling Spectra



In both XU and YV Projections

Thank you for your attention!